Model Checking Bounded Continuous-time Extended Linear Duration Invariants

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ABSTRACT

Extended Linear Duration Invariants (ELDI), an important subset of Duration Calculus, extends well-studied Linear Duration Invariants with logical connectives and the chop modality. It is known that the model checking problem of ELDI is undecidable with both the standard continuous-time and discrete-time semantics [12, 13], but it turns out to be decidable if only bounded execution fragments of timed automata are concerned in the context of the discrete-time semantics [36]. In this paper, we prove that this problem is still decidable in the continuous-time semantics, although it is well-known that model-checking Duration Calculus with the continuous-time semantics is much more complicated than the one with the discrete-time semantics. This is achieved by reduction to the validity of Quantified Linear Real Arithmetic (QLRA). Some examples are provided to illustrate the efficiency of our approach.

KEYWORDS

Model Checking, Duration Calculus, ELDI, Timed Automata, Quantified Linear Real Arithmetic.

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1 INTRODUCTION

Duration Calculus (DC) is an Interval Temporal Logic due to Zhou, Hoare and Ravn [18]. It extends Interval Temporal Logic [15] with the notion of *duration*, the integral of state expression over the reference interval. It is designed for specifying and reasoning about real-time and embedded systems at a high abstract level [14], and has been successfully and widely applied in practice [23, 33].

Although the powerful expressiveness of DC is desirable for requirement specification and analysis, it is a burden for automatic verification as satisfiability, validity and model checking of DC are undecidable [34], unless the use of chop (the only modality in DC), and/or negation, and/or the considered models are severely constrained [10, 11, 16, 23, 24, 28, 30, 34, 35].

In [35], a subset of DC formulas of the form $b \leq \ell \leq e \Rightarrow \sum_{s\in S} c_s \int s \leq M$, called Linear Duration Invariants (LDI), was identified, in which a given interval $[t, t']$, $\int s$ stands for the accumulated time for the presence of state s over $[t, t']$, and ℓ for the length of the interval, i.e., $t' - t$. So, an LDI formula says that if the length of the interval satisfies the constraint $b \leq \ell \leq e$, then over the time interval the durations of the state expressions should satisfy the constraint $\sum_{s\in S} c_s \int s \leq M$. LDI is indeed expressive, for instance, in the gas burner example [33], the safety requirement that "the proportion of leak time is no more than one-twentieth of the elapsed time for any time interval over one minute", can be easily specified as $\ell \geq 60 \Rightarrow 20 \int \mathit{Leak} \leq \ell$.

In [35], it was proved that the model checking problem of LDI against real-time automata is decidable. Real-time automata is a specific kind of timed automata with a single clock which is reset at each transition. Following this work, in [3, 20, 21], the authors investigated the model-checking of LDI against more expressive models such as timed automata [1] and hybrid automata [17]. Some more efficient algorithms for mode-checking LDI against timed automata based on

graph search were proposed in [30, 31], and the results were further extended to the networks of timed automata [32].

An interesting problem is whether it is possible to find a larger subset of DC whose model-checking problem is still decidable. It is natural to investigate the extension of LDI with Boolean connectives and chop modality, i.e., ELDI. Unfortunately, the satisfiability and validity of ELDI both are undecidable in the discrete-time and continuous-time (dense-time) settings according to the results in [34]. In [12, 13], Fränzle and Hansen further proved that the model-checking problem of ELDI against finite state machines also turns out to be undecidable both in the discrete time and continuous time settings. Therefore, they proposed an approximation semantics for ELDI, called doubly situation based semantics, and showed its model-checking is decidable in the discrete time setting with the complexity of cubic in the number of states of the model and linear in the size of the formula. However, further observation indicates that such approximation semantics is too coarse to be useful in practice [12]. So, they refined the semantics to another approximation semantics called counting semantics and reduced the model-checking problem of ELDI to Presburger Arithmetic with the complexity of 3-fold exponential [13]. In addition, according to their approach, one can only prove/disprove those formulas that can be approximated to be **true/false** over the given model represented by a Kripke structure, while no conclusion can be drawn in other cases. Therefore, the low efficiency and approximation semantics hinder the application of their approach. Motivated by their work, as an alternative, in [36] Zu *et al.* proved that model-checking bounded ELDI against timed automata in the standard discrete-time semantics is decidable by providing an efficient model-checking algorithm with the complexity of singly exponential in the size of formulas and quadratic in the number of states of the considered model, as a bounded ELDI formula Φ is of the form $b \leq \ell \leq e \Rightarrow \phi$, thus checking whether $\mathcal{A} \models \Phi$ is reduced to check whether any execution of $\mathcal{A}$, whose length is within $[b, e]$, satisfies ϕ, where $\mathcal{A}$ is a timed automaton and e is bounded.

In this paper, we investigate the model-checking problem of bounded ELDI against timed automata in the standard continuous-time semantics. We prove that it is still decidable. The basic idea is that for a given timed automaton $\mathcal{A}$ and a bounded ELDI formula Φ, we first find the set Θ of all execution fragments of $\mathcal{A}$ whose length is in between the lower and upper bounds from the zone graph of $\mathcal{A}$, and the number of such paths is finite. Then, for each path in Θ, we construct a Quantified Linear Real Arithmetic (QLRA) formula by taking Φ into account. Finally, we exploit REDLOG [8], a computer algebra tool based on quantifier elimination [29], to solve the derived QLRA formulas. Although the complexity of REDLOG is doubly exponential in the number of variables in the worst case, REDLOG can handle QLRA formulas quite efficiently in practice, as *virtual substitution* [9] can be applied to formulas that contain only polynomials with degree no more than 4. Our experiments indicate the efficiency and scalability of our approach.

Related work. In [6, 7], the authors considered the verification and synthesis of a class of linear hybrid automata (LHA) called reasonable LHA and their composition against different kinds of properties like safety and bounded reachability, defined by *Bounded Time Logic* (BTL), and proved that they are decidable, yet with different complexity. In general, timed automata is a proper subset of LHA, but not comparable with reasonable LHA. In addition, bounded continuous-time ELDI is not comparable to BTL. Certainly, the general solution used in [6, 7] is similar to ours used in this paper, both by reduction to the decision problem of linear arithmetic, but their technical details are completely different.

The rest of the paper is organized as follows. Section 2 recalls some basic notions of timed automata, zones and ELDI. Section 3 explains how to find all execution fragments with the bounded length for a given timed automaton from its zone graph. The algorithm of constructing a QLRA formula according to a given ELDI formula and an execution fragment is presented in Section 4. Section 5 is devoted to solving the derived QLRA formulas by quantifier elimination, following by the complexity analysis. After presenting the implementation and experiments in Section 6, we draw a conclusion in Section 7.

2 PRELIMINARIES

In this section, we first review timed automata (TA) as a modeling language for real-time systems and zone graph as the symbolic representation of states of TA, then recall ELDI, a subset of DC, as a specification logic for real-time systems. For convenience, we fix a set of propositions $\mathcal{P}$ throughout this paper.

2.1 Timed automata

A timed automaton (TA) [1] is a finite-state automaton which is equipped with a set of clocks, and each location is equipped with a set of propositions from $\mathcal{P}$ that hold at the location. Let X represent the set of clocks and $\Delta(X)$ be the set of *clock constraints* on X, which are conjunctions of the formulas of the form $x \leq c$ or $c \leq x$, where $x \in X$ and $c \in \mathbb{N}$. Additionally, we assume that all TA are strongly *non-Zeno* [1], i.e., there is a non-zero constant $\epsilon \in \mathbb{R}^+$ such that every control cycle takes at least ϵ time units.

Definition 2.1 (Timed Automaton). A TA is a tuple $\mathcal{A} = (L, X, E, \Sigma, l_0, \Lambda, I)$, where L is a finite set of locations, X is a finite set of clocks, $E \subseteq L \times \Sigma \times \Delta(X) \times 2^X \times L$ is a transition relation, Σ is a set of actions, $l_0 \in L$ is the initial

location, Λ is a mapping that assigns a subset of $\mathcal{P}$ to each location l indicating all propositions in $\Lambda(l)$ hold in l, and I is a mapping that assigns each location $l \in L$ with a clock constraint $I(l) \in \Delta(X)$, called *invariant*. Furthermore, an invariant in TA must be downwards closed.

A *clock valuation* is a function $\mathbf{v} : X \to \mathbb{R}^{\geq 0}$. We denote the set of all clock valuations by $\mathcal{H}$. Hence a state of a TA $\mathcal{A}$ is a pair $(l, \mathbf{v}) \in L \times \mathcal{H}$ consisting of a location and a clock valuation. Every subset $\lambda \subseteq X$ induces a reset function $\mathit{Reset}_\lambda : \mathcal{H} \to \mathcal{H}$ defined by

$$\mathit{Reset}_\lambda \mathbf{v}(x) = \begin{cases} 0, & \textbf{if } x \in \lambda \\ \mathbf{v}(x), & \textbf{if } x \notin \lambda \end{cases}$$

We use $\mathbf{1}$ to denote the unit vector(1,...,1) and $\mathbf{0}$ for zero vector. There are two kinds of *steps* of a TA: *discrete step* and *time-delay step*. A discrete step is of the form $(l, \mathbf{v}) \xrightarrow{a} (l', \mathbf{v}')$, if there exists $(l, a, g, \lambda, l') \in E$ such that $\mathbf{v}$ satisfies g and $\mathbf{v}' = \mathit{Reset}_\lambda(\mathbf{v})$, where $a \in \Sigma$. A time-delay step is of the form $(l, \mathbf{v}) \xrightarrow{t} (l, \mathbf{v} + t\mathbf{1})$, such that for any $t' \in [0, t]$, $\mathbf{v} + t'\mathbf{1}$ satisfies $I(l)$, where $t \in \mathbb{R}^{\geq 0}$.

Definition 2.2 (Run and Behaviour). Let $\mathcal{A}$ be a TA.

(1) A run r of $\mathcal{A}$ is an infinite sequence of the form

$$r \ : \ (l_0, \mathbf{v}_0) \xrightarrow[\delta_0]{a_0} (l_1, \mathbf{v}_1) \xrightarrow[\delta_1]{a_1} (l_2, \mathbf{v}_2) \xrightarrow[\delta_2]{a_2} \cdots$$

where $(l_0, \mathbf{v}_0)$ is the initial state, and δ_i is the time $\mathcal{A}$ staying in the location l_i. If $l_i = l_{i+1}$ and a_i is an empty action, the step is a time-delay step and $\mathbf{v}_{i+1} = \mathbf{v} + \delta_i \mathbf{1}$; otherwise, the step is a discrete step with $\delta_i = 0$ and $\mathbf{v}_{i+1} = \mathit{Reset}_\lambda(\mathbf{v_i})$, for $i \geq 0$.

(2) A behaviour β corresponding to the run, is the infinite sequence of timed locations

$$\beta \ : \ (l_0, t_0)(l_1, t_1) \cdots (l_k, t_k) \cdots$$

that satisfies the following conditions: (1) $t_0 = 0$; (2) for any $T \in \mathbb{R}^+$, there is some $i \geq 0$ such that $t_i \geq T$; (3) t_i is the instant when $\mathcal{A}$ enters l_i, which implies $\delta_i = t_{i+1} - t_i$, and $\mathcal{A}$ stays in l_i for δ_i time units.

A *zone* is a clock constraint [2]. For a location, a zone is the maximal set of clock valuations satisfying the constraint. In a zone-graph, zones are used to denote symbolic states. Zones and their representations based on Difference Bounded Matrices (DBMs) are the standard data structures which have been implemented in several verification tools for TA, e.g., UPPAAL [19]. Operations over zones are well defined in [2]. Note that zone provides a more efficient representation of symbolic states than *region graph* [1], therefore, model-checking algorithms for TA based on zone is more efficient than those based on region graph.

2.2 Extended linear duration invariants

ELDI with the set $\mathcal{P}$ of state variables consists of three syntactic categories, which are state expressions S, linear duration formulas (LDFs) $\mathcal{D}$, and ELDI formulas ϕ. The BNFs for them are given as follows:

$$\begin{aligned} S &::= 0 \mid P \mid \neg S \mid S_1 \vee S_2 \\ \mathcal{D} &::= \sum_{i \in \Omega} c_i \textstyle\int S_i \leq c \\ \phi &::= \mathcal{D} \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 ; \phi_2 \end{aligned}$$

where $P \in \mathcal{P}$ stands for a state variable, interpreted as a Boolean function over time, c_is and c are real numbers, and Ω is a finite set of indices.

As the convention of DC, ℓ is defined as $\int 1$, denoting the length of the reference interval. The Boolean value **true**, denoted by $\top$, is defined by $\ell \geq 0$, falling in ELDI. Obviously, each ELDI formula can be represented by the form $b \leq \ell \leq e \Rightarrow \phi$, where $b \in \mathbb{R}^{\geq 0}$, $e \in \mathbb{R}^{\geq 0} \cup \{\infty\}$, and ϕ is defined as above. In this paper, we only focus on the case when e is bounded, i.e., in $\mathbb{R}^{\geq 0}$, and will represent an ELDI of this form by $\Phi, \Psi, \cdots$, possibly with superscript and subscript in the sequel. Therefore, we also call ELDI with such restriction *bounded ELDI*.

Definition 2.3 (Interpretation $\mathcal{I}_\beta$ of ELDI). Given a TA $\mathcal{A}$ and one of its behaviours β, define an interpretation $\mathcal{I}_\beta$ of ELDI with continuous-time semantics as follows:

state expressions: given a time point $t \in \mathbb{R}^{\geq 0} \cup \{0\}$
$\mathcal{I}_\beta(0)(t) = 0$ and $\mathcal{I}_\beta(1)(t) = 1$;
$$\mathcal{I}_\beta(P)(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \wedge P \in l_i \\ & \quad \text{for some } i > 0 \\ 0 & \text{otherwise} \end{cases};$$
$\mathcal{I}_\beta(\neg S)(t) = 1 - \mathcal{I}_\beta(S)(t)$;
$\mathcal{I}_\beta(S_1 \vee S_2)(t) = \max\{\mathcal{I}_\beta(S_1)(t), \mathcal{I}_\beta(S_2)(t)\}$.

durations: given an interval $[t_1, t_2]$, where $t_1, t_2 \in \mathbb{R}^{\geq 0} \cup \{0\}$ and $t_1 \leq t_2$, $\int S$ is interpreted by $\mathcal{I}_\beta(\int S)([t_1, t_2]) = \int_{t_1}^{t_2} \mathcal{I}_\beta(S)(t)dt$.

formulas: given an interval $[t_1, t_2]$, an ELDI formula ϕ is interpreted by
$\mathcal{I}_\beta, [t_1, t_2] \models \sum_{i \in \Omega} c_i \int S_i \leq c$ iff $\sum_{i \in \Omega} c_i \mathcal{I}_\beta(\int S_i)([t_1, t_2]) \leq c$;
$\mathcal{I}_\beta, [t_1, t_2] \models \neg\phi$ iff $\mathcal{I}_\beta, [t_1, t_2] \not\models \phi$;
$\mathcal{I}_\beta, [t_1, t_2] \models \phi_1 \vee \phi_2$ iff $\mathcal{I}_\beta, [t_1, t_2] \models \phi_1$ or $\mathcal{I}_\beta, [t_1, t_2] \models \phi_2$;
$\mathcal{I}_\beta, [t_1, t_2] \models \phi_1 ; \phi_2$ iff $\mathcal{I}_\beta, [t_1, t] \models \phi_1$ and $\mathcal{I}_\beta, [t, t_2] \models \phi_2$ for some $t \in [t_1, t_2]$.

2.3 Quantified linear real arithmetic

Quantified linear real arithmetic (QLRA) is a theory of first order logic, with the specific signature $\langle \mathbb{R}, 0, +, =, < \rangle$, i.e.,

in which all terms are linear. Thus, formulas of QLRA are defined according to the following syntax:

$$\phi \quad ::= \quad c_0 + c_1 x_1 + \cdots + c_n x_n \rhd 0 \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \forall x.\phi$$

where $c_0, c_1, \ldots, c_n \in \mathbb{R}, \rhd \in \{=, <\}$. QLRA is interpreted in the standard way. Other notations of first-order logic and real arithmetic are defined as usual.

3 FINDING ALL POSSIBLE BOUNDED EXECUTION FRAGMENTS

In this section, we consider given a TA $\mathcal{A}$ and an interval $[b, e]$ with $b, e \in \mathbb{R}^{\geq 0}$ and $b \leq e$, how to find all execution fragments of $\mathcal{A}$ whose lengths are in between b and e.

As discussed in [2], *ZoneG*($\mathcal{A}$) is a transition graph derived from $\mathcal{A}$, can be represented by $\langle Z, z_0, \mapsto \rangle$, where

- Every $z \in Z$ stands for a zone, which is a pair $(l, \Delta(l))$, where $l \in \mathcal{A}.L$, and $\Delta(l)$ is a timing constraint on $\Delta(\mathcal{A}.X)$[1], denoting a symbolic state of $\mathcal{A}$, i.e., a set of clock valuations satisfying $\Delta(l)$.
- $z_0 = (l_0, \wedge_{x \in \mathcal{A}.X} x = 0)$, is the initial zone.
- $\mapsto = \uparrow \cup \cup_{a \in \Sigma} \xrightarrow{g,a,\lambda}$, where
  - $(l_1, \Delta(l_1)) \uparrow (l_2, \Delta(l_2))$ iff $l_1 = l_2 \wedge \Delta(l_2) = \{u + d \in I(l_1) \mid u \in \Delta(l_1) \wedge d \in \mathbb{R}^+\}$. $\uparrow$ stands for a delay transition.
  - $(l_1, \Delta(l_1)) \xrightarrow{g,a,\lambda} (l_2, \Delta(l_2))$ iff $(l_1, g, a, \lambda, l_2) \in \mathcal{A}.E \wedge \Delta(l_2) = \{u[\lambda] \mid u \in \Delta(l_1) \wedge g(u)\}$. $\xrightarrow{g,a,\lambda}$ stands for a possible discrete transition over the zone graph derived from $\mathcal{A}$.

Given a transition $\tau \in \mapsto$, $\tau.preZ$ and $\tau.postZ$ respectively denote the pre- and post-zone of the transition. Given a discrete transition τ, $\tau.g$, $\tau.a$ and $\tau.\lambda$ denote its guard, action and the set of reset clocks, respectively. Given a zone z, we use $z.\tau$ to stand for the set of transitions outgoing from z, and $Post^a(z)$ for the set of zones which can be reached from z via a discrete transition, i.e., $\{z' \mid (z, g, a, \lambda, z') \in \mapsto$ for some $g \in \Delta(X), a \in \Sigma,$ and $\lambda \subseteq X\}$.

Given a TA $\mathcal{A}$, Algorithm 1 presents a procedure for finding all possible execution fragments of $\mathcal{A}$ whose lengths are within the given interval $[b, e]$. The basic idea of Algorithm 1 is as follows:

- Firstly, according to the descriptions of zone in [2, 26, 27], we construct a zone graph of $\mathcal{A}$ with k-normalization, denoted by *ZoneG*($\mathcal{A}$), the resulting zone graph. As $\mathcal{A}$ has no time difference constraints as guards, thus *ZoneG*($\mathcal{A}$) is *sound* and *complete* w.r.t. the standard operational semantics of $\mathcal{A}$ as discussed in the previous section. Moreover, its transition relation is finite [26, 27].

[1] Unlike $\Delta(X)$, $c \leq x - y$ and $x - y \leq c$ are allowed in $\Delta(l)$, where x, y are clock variables and c is a natural number.

Algorithm 1: PEF(*ZoneG*($\mathcal{A}$), b, e) /* Finding all possible execution fragments satisfying the length bound */

input : *ZoneG*($\mathcal{A}$), the zone graph of $\mathcal{A}$; $[b, e]$, an interval
output : Θ, the set of possible execution fragments whose lengths are within $[b, e]$

```
1  begin
2      Θ := ∅ ;
3      foreach z ∈ Z do
4          ρ := ε ;
5          STK.push(z) ;
           /* STK is a stack, storing all zones
              to be explored                    */
6          while STK ≠ ∅ do
7              currentZ := STK.pop;
8              if ↑∈ currentZ.τ then
9                  currentZ := currentZ ↑ ;
                   /* t is an additional clock added
                      to the DBMs, reset at the
                      beginning of ρ             */
10             if currentZ ∧ (b ≤ t < e) ≠ ∅ then
11                 ρ := ρ ∘ ⟨currentZ⟩; Θ := Θ ∪ {ρ};
12                 if Post^a(currentZ) ≠ ∅ then
13                     foreach z' ∈ Post^a(currentZ) do
14                         STK.push(z');
15                 else
16                     ρ := remove(ρ, currentZ);
17                     while Post^a(Lastzone(ρ)) have been
                       popped out do
18                         ρ := remove(ρ, Lastzone(ρ));
19             else if currentZ ∧ (t < b) ≠ ∅ then
20                 ρ := ρ ∘ ⟨currentZ⟩;
21                 if Post^a(currentZ) ≠ ∅ then
22                     foreach z' ∈ Post^a(currentZ) do
23                         STK.push(z');
24                 else
25                     ρ := remove(ρ, currentZ);
26                     while Post^a(Lastzone(ρ)) have been
                       popped out do
27                         ρ := remove(ρ, Lastzone(ρ));
28             else if currentZ ∧ (t ≥ e) ≠ ∅ then
29                 ρ := ρ ∘ ⟨currentZ⟩; Θ := Θ ∪ {ρ};
30                 ρ := remove(ρ, currentZ) ;
31                 while Post^a(Lastzone(ρ)) have been popped
                   out do
32                     ρ := remove(ρ, Lastzone(ρ));
33     return Θ
```

- Starting from each symbolic state in *ZoneG*($\mathcal{A}$), we find all execution fragments whose lengths are between b and e using depth first search. With an implicit extra clock t added to the DBMs, we can easily determine whether the length of the current execution fragment ρ can reach the interval $[b, e]$ through checking whether *currentZ* $\wedge$ $(b \leq t \leq e)$ holds. *STK* is a stack to store the zones to be explored. The algorithm pushes a zone z into *STK* as long as the length of the derived execution fragment by appending Z to the end of the considered execution fragment is no more than the given upper bound e, otherwise, the head of *STK* will be popped. Suppose the number of the locations of $\mathcal{A}$ is N, then the number of such execution fragments is at most $N^{1+N*(1+\lceil \frac{e}{\epsilon} \rceil)}$, where ϵ is the least dwelling time in each control cycle of $\mathcal{A}$.

THEOREM 3.1 (CORRECTNESS OF ALGORITHM 1). *For any TA $\mathcal{A}$ and $[b, e]$, Algorithm 1 is correct, i.e.,*

Termination: *the algorithm terminates;*
Soundness: *if $\rho \in \Theta$, then ρ is a real execution fragment of zones in ZoneG($\mathcal{A}$) with length in $[b, e]$; and*
Completeness: *if ρ is a real execution fragment of zones in ZoneG($\mathcal{A}$) with length in $[b, e]$, then $\rho \in \Theta$.*

PROOF. For **termination**, we only need to prove the **while** loop terminates as it is obvious that the outermost **for** loop and the three innermost **for** loops terminate. For a given zone z, suppose there is an execution fragment ρ starting from z with $N * (1 + \lceil \frac{e}{\epsilon} \rceil)$ transitions, whose length is within $[b, e]$, where N is the number of zones. By Pigeonhole principle, there is a zone z' with more than $1 + \lceil \frac{e}{\epsilon} \rceil$ occurrences in ρ, which implies there are $\lceil \frac{e}{\epsilon} \rceil$ control cycles in ρ at least. Hence, the execution time of ρ is more than e from the assumption that each control cycle has ϵ dwelling time at least. It follows that any execution fragment starting from z has $N * (1 + \lceil \frac{e}{\epsilon} \rceil)$ transitions at most if its length is within $[b, e]$, and therefore, all such execution fragments can be found in $N^{N*(1+\lceil \frac{e}{\epsilon} \rceil)}$ iterations at most. After finding out all such execution fragments, no zone can be pushed in *STK* any more, but at least one zone is popped out from *STK* in each iteration (see lines 10-32). This implies that the **while** loop must terminate.

For **soundness**, suppose $\rho \in \Theta$. Obviously, ρ is an execution fragment of *ZoneG*($\mathcal{A}$) by Algorithm 1. Additionally, from line 10-32, it follows that ρ's length is within $[b, e]$, because on the one hand, ρ's length cannot be less than b as $\rho \notin \Theta$ otherwise; on the other hand, ρ's length cannot be greater than e, as the last zone in ρ cannot be appended otherwise according to line 28-32.

For **completeness**, suppose $\rho = (l_1, \Delta(l_1)), ..., (l_n, \Delta(l_n))$ is a real execution fragment of *ZoneG*($\mathcal{A}$) whose length is within $[b, e]$. Thus, we can construct a $\rho' \in \Theta$ as follows. Let ρ' start from $(l_1, \Delta(l_1))$. Obviously, it is doable by line 3. Because ρ is a real execution fragment of *ZoneG*($\mathcal{A}$), the second zone $(l_2, \Delta(l_2))$ in ρ must be a successor of $(l_1, \Delta(l_1))$, and the length of $(l_1, \Delta(l_1)), (l_2, \Delta(l_2))$ is less than e, so ρ' can be extended by appending $(l_2, \Delta(l_2))$ at the end from line 10-27. We can repeat the above procedure until $(l_n, \Delta(l_n))$ is appended to the end of ρ'. Clearly, from line 10-14 and line 28-29, $\rho' \in \Theta$. □

4 REDUCTION TO QRLA

In this section, we present a translation from a given possible execution fragment whose length is within the given interval and an ELDI formula into a QLRA formula equivalently in the sense that the execution fragment satisfies the ELDI formula iff the resulting QLRA formula is valid.

Note that for any execution fragment generated by Algorithm 1 which is a sequence of zones, we can remove those zones without dwelling time, because $(\phi; \ell = 0) \Leftrightarrow (\ell = 0; \phi) \Leftrightarrow \phi$ always holds in DC. So, in what follows, we only consider such refined execution fragments, and denote them by $z_1, z_2, \ldots, z_k$. Moreover, for each zone z_i, we introduce a variable δ_i to indicate the real duration on which the automaton dwells. Thus, we will denote the refined execution fragment as $(z_1, \delta_1)(z_2, \delta_2), \ldots, (z_k, \delta_k)$.

Given the ELDI formula $(b \leq \ell \leq e \Rightarrow \phi)$ and a refined execution fragment $\rho = (z_1, \delta_1), (z_2, \delta_2), \ldots, (z_k, \delta_k)$, the encoded QLRA formula is of the form $L(\rho) \Rightarrow L(\phi)$, where $L(\rho)$ is a QLRA formula translated from ρ, which entails $b \leq \sum_{i=1}^{k} \delta_i \leq e$, and $L(\phi)$ is obtained from ϕ. We will explain the details of the translation below.

Deriving timing constraints from refined execution fragments: Given an execution fragment $\rho = (z_1, \delta_1), (z_2, \delta_2), \ldots, (z_k, \delta_k)$ and initial value $\mathbf{x}_0 = (x_{01}, \cdots, x_{0m})$ (suppose $X = \{x_1, \ldots, x_m\}$), Algorithm 2 derives a QLRA formula to stand for the timing constraint on the dwelling times δ_is and the initial value $\mathbf{x}_0$ derived from ρ. Essentially, the constraint on the dwelling times δ_is is derived by considering the following three aspects:

- each δ_i should be non-negative;
- their sum should be within the length interval of considered execution fragments, i.e., $b \leq \sum_{i=1}^{k} \delta_i \leq e$;
- the constraint derived from the corresponding zone by taking the initial values of clocks into account.

Encoding ELDI formula: Given an execution fragment $\rho = (z_1, \delta_1), \ldots, (z_n, \delta_n)$ and an ELDI formula ϕ, the encoding procedure is done by the structure of ϕ as follows:

- If ϕ is an atomic formula of the form $b \leq \ell \leq e \Rightarrow \mathcal{D}$, we mainly focus on how to encode $\mathcal{D}$. Suppose $\mathcal{D}$ contains d duration expressions $\int S_1, \ldots, \int S_d$. We use e_{ij} to denote the duration that S_i holds at z_j, for

Algorithm 2: $LP(\rho, \mathbf{x}_0)$

input : ρ, a refined execution fragment $(z_1, \delta_1), \ldots, (z_k, \delta_k)$; $\mathbf{x}_0$, the initial value of clocks at the beginning of ρ.
output : Γ, the timing constraint derived from the execution fragment.

1 **begin**
2 $z'_1 = z_1[x_{01} + \delta_1/x_1, \cdots, x_{0m} + \delta_1/x_m]$;
3 **foreach** $i \in \{2, \ldots, k\}$ **do**
4 **foreach** $j \in \{1, \ldots, m\}$ **do**
5 **if** $x_j \in a_i.\lambda$ **then**
/* a_i is the transition from z_{i-1} to z_i */
6 $e_j := \delta_i$;
/* x_j should be reset */
7 **else**
8 $e_j := z_{i-1}.x_j + \delta_i$;
/* $z_{i-1}.x_j$ stands for the value of clock x_j at the previous zone */
9 $z'_i = z_i[e_1/x_1, \cdots, e_m/x_m]$;
10 $\Gamma = \wedge_{i=1}^{k}(z'_i \wedge \delta_i \geq 0) \wedge b \leq \sum_{i=1}^{k} \delta_i \leq e$;
11 **return** Γ ;

$i = 1, \ldots, d$ and $j = 1, \ldots, n$. Clearly, if z_j satisfies S_i, then e_{ij} is δ_j, otherwise 0. Thus, the total duration of S_i holding on ρ should be $\sum_{j=1}^{n} e_{ij}$. Hence, we just need to replace each S_i with $\sum_{j=1}^{n} e_{ij}$ in $\mathcal{D}$. Therefore, the translated QLRA formula is

$$LP(\rho, \mathbf{0}) \Rightarrow (\mathcal{D}[\sum_{i=1}^{n} e_{i1}/\textstyle\int S_1, \cdots, \sum_{i=1}^{n} e_{id}/\textstyle\int S_d]),$$

where $LP(\rho, \mathbf{0})$ stands for the QLRA formula translated from ρ by applying Algorithm 2 with initial clock values $\mathbf{0}$, and $\phi[e_1/e_2]$ stands for replacing each occurrence of e_2 by e_1 in ϕ.

- When $\phi = \neg\phi_1, \phi_1 \wedge \phi_2, \phi_1 \vee \phi_2$, it is easy to obtain by revoking the procedure recursively.
- If $\phi = \phi_1; \phi_2$, then we consider the $n + 2$ cases where the chop point is taken respectively before z_1, at z_1, ..., at z_n, after z_n. Subsequently, we recursively recall the translation procedure to the resulted corresponding sub-problems.
- Finally, quantify the resulted formula with the respective quantifications to the corresponding introduced fresh variables (line 20).

The above procedure is implemented by Algorithm 3, and the returned formula is a closed QLRA formula.

THEOREM 4.1 (CORRECTNESS OF ALGORITHM 3). *Algorithm 3 is correct, i.e.,*

Termination: *the algorithm terminates;*
Soundness: *if $\rho \models \phi$ then $LF(\phi, \rho, \mathbf{x}_0)$ is satisfiable (valid);*
Completeness: *if $LF(\phi, \rho, \mathbf{x}_0)$ is satisfiable (valid), then $\rho \models \phi$.*

PROOF. Regarding **termination**, it can be simply done by induction on the structure of ϕ. Regarding **soundness** and **completeness**, we still proceed by induction on the structure of ϕ as follows:

- The basic case, i.e., ϕ is a formula of the form $b \leq \ell \leq e \Rightarrow \mathcal{D}$, where $D = \sum_{i\in\Omega} c_i \textstyle\int S_i \leq c$. Then $\mathcal{I}_\rho, [\rho.b, \rho.e] \models b \leq \ell \leq e \Rightarrow \sum_{i\in\Omega} c_i \int S_i \leq c$ iff $b \leq \rho.e - \rho.b \leq e \Rightarrow \sum_{i\in\Omega} c_i \mathcal{I}_\rho(\int S_i)([\rho.b, \rho.e]) \leq c$, where $\rho.b$ and $\rho.e$ stands for the starting and ending points of ρ. Obviously,

$$\forall \delta_1, \cdots, \delta_n, \mathcal{Q}. \left(\begin{array}{c} LP(\rho, \mathbf{x}_0) \\ \Rightarrow \mathcal{D}[\sum_{i=1}^{n} e_{i1}/\textstyle\int S_1, \cdots, \sum_{i=1}^{n} e_{id}/\int S_d] \end{array} \right) \quad (1)$$

  is unsatisfiable implies $\rho \not\models \phi$ (soundness), and that (1) holds implies $\rho \models \phi$ (completeness) according to Algorithm 2 and Algorithm 3.
- $\phi = \neg\phi_1$
  For soundness, suppose $LF(\rho, \neg\phi_1)$ is unsatisfiable, i.e.,$\neg LF(\phi_1, \rho, \mathbf{x}_0)$ is unsatisfiable by Algorithm 3, which implies $LF(\phi_1, \rho, \mathbf{x}_0)$ is valid. By the induction hypothesis, we have $\rho \models \phi_1$. Therefore, $\rho \not\models \neg\phi_1$.
  For completeness, suppose $LF(\neg\phi_1, \rho, \mathbf{x}_0)$ is valid, which derives $LF(\phi_1, \rho, \mathbf{x}_0)$ is unsatisfiable by Algorithm 3. By the induction hypothesis, we have $\rho \not\models \phi_1$. Hence, $\rho \models \neg\phi_1$.
- $\phi = \phi_1 \vee \phi_2$
  For soundness, suppose $LF(\phi_1 \vee \phi_2, \rho, \mathbf{x}_0)$ is unsatisfiable, i.e., $LF(\phi_1, \rho, \mathbf{x}_0) \vee LF(\phi_2, \rho, \mathbf{x}_0)$ is unsatisfiable by Algorithm 3, which implies $LF(\phi_1, \rho, \mathbf{x}_0)$ and $LF(\phi_2, \rho, \mathbf{x}_0)$ both are unsatisfiable. By the induction hypothesis, it follows that $\rho \not\models \phi_1$ and $\rho \not\models \phi_2$. Therefore, $\rho \not\models \phi_1 \vee \phi_2$.
  For completeness, suppose $LF(\phi_1 \vee \phi_2, \rho, \mathbf{x}_0)$ is valid, which implies $\neg LF(\phi_1, \rho, \mathbf{x}_0)$ is unsatisfiable or $\neg LF(\phi_2, \rho, \mathbf{x}_0)$ is unsatisfiable by Algorithm 3. By the induction hypothesis, we have either $\rho \not\models \neg\phi_1$ or $\rho \not\models \phi_2$, hence $\rho \models \phi_1 \vee \phi_2$.
- $\phi = \phi_1; \phi_2$
  It is clear that $\rho \models \phi_1; \phi_2$ iff that ρ can be split into two parts ρ_1 and ρ_2 such that $\rho = \rho_1 \circ \rho_2$, and $\rho_1 \models \phi_1$ and $\rho_2 \models \phi_2$, iff there exists $0 \leq i \leq n$ such that zone z_i can be split two parts, i.e., (z_i, δ_{i1}) and (z_i, δ_{i2}) with $\rho_1 = (z_1, \delta_1), \cdots, (z_i, \delta_{i1}) \models \phi_1$ and $\rho_2 = (z_i, \delta_{i2}), \cdots, (z_n, \delta_n) \models \phi_1$ with $\delta_i = \delta_{i1} + \delta_{i2}$, where δ_{i1} and δ_{i2} are fresh variables, which is exactly

equivalent to

$$\forall \delta_1, \cdots, \forall \delta_n, \mathcal{Q}. \left(\begin{array}{ll} & \mathit{LF}(\phi_1, \langle (z_1, \delta_1), \cdots, (z_i, \delta_{i1}) \rangle, \mathbf{x}_0) \\ \wedge & \mathit{LF}(\langle (\phi_2, z_i, \delta_{i2}), \cdots, (z_n, \delta_n) \rangle, \\ & \qquad \mathbf{x}_0 + \sum_{j=1}^{i-1} \delta_j + \delta_{i1}) \\ \wedge & \delta_i = \delta_{i1} + \delta_{i2} \end{array} \right)$$

by Algorithm 3 and the induction hypothesis. □

Algorithm 3: $\mathit{LF}(\phi, \rho, \mathbf{x}_0)$

input : $\rho = (\underline{z_1}, \underline{\delta_1}), (\underline{z_2}, \underline{\delta_2}), \ldots, (\underline{z_n}, \underline{\delta_n})$, the underlined execution fragment;
$\mathbf{x}_0$, the initial value of clocks at the beginning of ρ;
ϕ, the considered ELDI formula.
output : Γ, the derived QLRA formula.

begin
  $\mathcal{Q} := \varepsilon$;
  /* $\mathcal{Q}$ records existential quantifications over introduced fresh variables */
  case $\phi := b \leq \ell \leq e \Rightarrow \mathcal{D}$
    foreach $i \in \{1, \ldots, n\}$ **do**
      foreach $j \in \{1, \ldots, d\}$ **do**
        if $z_i \models S_j$ **then**
          $e_{ij} := \delta_i$;
        else
          $e_{ij} := 0$
    $\Gamma := \mathit{LP}(\rho, \mathbf{x}_0) \Rightarrow (\mathcal{D}[\sum_{i=1}^{n} e_{i1} / \int S_1, \cdots, \sum_{i=1}^{n} e_{id} / \int S_d])$;
  case $\phi := \neg \phi_1$
    $\Gamma := \neg \mathit{LF}(\phi_1, \rho, \mathbf{x}_0)$;
  case $\phi := \phi_1 \wedge \phi_2$
    $\Gamma := \mathit{LF}(\phi_1, \rho, \mathbf{x}_0) \wedge \mathit{LF}(\phi_2, \rho, \mathbf{x}_0)$;
  case $\phi := \phi_1 \vee \phi_2$
    $\Gamma := \mathit{LF}(\phi_1, \rho, \mathbf{x}_0) \vee \mathit{LF}(\phi_2, \rho, \mathbf{x}_0)$;
  case $\phi := \phi_1; \phi_2$
    $\Gamma :=$

$$\bigvee_{i=0}^{n+1} \left(\begin{array}{ll} & \mathit{LF}(\phi_1, ((z_1, \delta_1), \cdots, (z_i, \delta_{i1})), \mathbf{x}_0) \\ \wedge & \mathit{LF}(\phi_2, ((z_i, \delta_{i2}), \cdots, (z_n, \delta_n)), \\ & \qquad \mathbf{x}_0 + \sum_{j=1}^{i-1} \delta_j + \delta_{i1}) \\ \wedge & \delta_i = \delta_{i1} + \delta_{i2} \end{array} \right)$$

    ;
    $\mathcal{Q} := \mathcal{Q}, \exists \delta_{11}, \exists \delta_{12}, \cdots, \exists \delta_{n1}, \exists \delta_{n2}$;
    /* $\delta_{11}, \delta_{12}, \ldots, \delta_{n1}, \delta_{n2}$ are fresh variables */
  return $\forall \delta_1, \cdots, \forall \delta_n, \mathcal{Q}.\Gamma$;

5 SOLVING DERIVED QLRA FORMULAS AND COMPLEXITY ANALYSIS

In this section, we further discuss how to solve the resulted QLRA formulas and the complexity of our approach.

5.1 Solving derived QLRA formulas

By Theorem 3.1 and Theorem 4.1, given a TA $\mathcal{A}$, and a bounded ELDI formula Φ with lower bound b and upper bound e, model-checking whether Φ is satisfied by $\mathcal{A}$ is reduced to whether a QLRA formula is valid, i.e.,

THEOREM 5.1. *Given a TA $\mathcal{A}$ and a bounded ELDI formula Φ, $\mathcal{A} \models \Phi$ iff $\wedge_{\rho \in \mathit{PEF}(\mathit{ZoneG}(\mathcal{A}), b, e)} \mathit{LF}(\Phi, \rho, \mathbf{0})$ is valid.*

PROOF. Directly by Theorem 3.1 and Theorem 4.1. □

According to Tarski's result, the satisfiability and validity of QLRA both are decidable [29], as QLRA admits the property of quantifier elimination (QE). So, an immediate result of Theorem 5.1 is that

COROLLARY 5.2. *Given a timed automaton $\mathcal{A}$ and an ELDI formula Φ, $\mathcal{A}, [b, e] \models \Phi$ is decidable.*

Tarski's original QE algorithm for real arithmetic is non-elementary [29]. But in the 1970s, Collins invented a new algorithm for QE based on *cylindrical algebraic decomposition* (CAD) [5], which is double exponential in the number of variables. CAD has been implemented in many computer algebra tools such as REDLOG [8] and QEPCAD [4]. Particularly, all formulas of QLRA are linear, therefore the QE of QLRA can be achieved more efficiently by *virtual substitution* due to Weinspfenning [9], which has been implemented in REDLOG, recently in Z3, although the worst case is still double exponential.

5.2 Complexity

As discussed above, model checking ELDI against bounded behaviours of timed automata consists of three procedures:

- The first one is *PEF* to find out all execution fragments whose length is within the bounded interval $[b, e]$ for a given TA $\mathcal{A}$. The number of such execution fragments is at most $N^{1+N*(1+\lceil \frac{e}{\epsilon} \rceil)}$ as we discussed before, where N is the number of the zones generated from $\mathcal{A}$ and ϵ is the least dwelling time in every control cycle of $\mathcal{A}$. This step can be done in $O(N^{N*(1+\lceil \frac{e}{\epsilon} \rceil)})$.
- The second one is *LF*, which translates whether a given execution fragment ρ satisfies a considered ELDI formula Φ into a QLRA formula. The Algorithm *LF* itself can be done in the linear of the size of Φ, but the size of the generated QLRA formula could be $O((N(1 + \lceil \frac{e}{\epsilon} \rceil))^d * |\Phi|)$ in the worst case, where d is the number of nested chops. In addition, we need to

introduce $d*M$ fresh variables and their corresponding quantifications, where M is the number of zones in a bounded observation interval, $N*(1+\lceil\frac{e}{\epsilon}\rceil)$ at most.

- The last one is a QE tool. Here we adopt REDLOG, whose complexity is double exponential in the number of variables, i.e., $O(2^{2^{dN*(1+\lceil\frac{e}{\epsilon}\rceil)}})$ to check whether each of such execution fragments satisfies the considered ELDI formula Φ.
- So, the total complexity to check whether $\mathcal{A} \models \Phi$ is $O(N^{N*(1+\lceil\frac{e}{\epsilon}\rceil)} * 2^{2^{dN*(1+\lceil\frac{e}{\epsilon}\rceil)}})$. Moreover, it is well known that N, the number of zones of *ZoneG*($\mathcal{A}$), is exponential in n, the number of the locations of $\mathcal{A}$ [2], in the worst case. So, the complexity of our approach is 3-fold exponential in the size of $\mathcal{A}$ and 2-fold exponential in the number of nested chops in Φ.

Although the theoretical complexity of our approach is quite high as analyzed above, in practice, the worst cases happen with quite low possibility. We believe that REDLOG can handle QLRA formulas in polynomial time in their sizes in most cases. The below experiments will justify this.

6 IMPLEMENTATION AND EXPERIMENTS

Based on the DBM library of UPPAAL, we developed a prototypical tool for our approach on Linux with two input files: the first is a .xml file to represent the timed automaton under consideration in UPPAAL format, and the other contains the ELDI formula to be verified. Our tool outputs a text file, which represents the generated QLRA formula as the input of REDLOG (Reduce), the QE tool we used. REDLOG will return *true* or *false* to the problem whether the ELDI formula is satisfied by the timed automaton on all bounded execution fragments. Besides, our tool also provides the function to check whether a given bounded execution fragment satisfies the ELDI formula. Therefore, on the one hand, the size of the

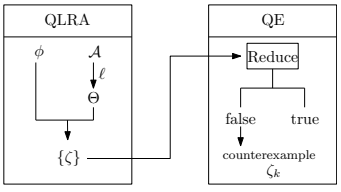


Figure 1: Overall structure.

	r	d	$[e^-, e^+]$
J_1	0	10	5
J_2	0	10	$[2, 6]$
J_3	4	15	8
J_4	0	20	10

Figure 2: Job description.

generated QLRA formula could decrease dramatically, which can scale up our approach very much; on the other hand, it can tell which execution fragment is a counterexample as well, when model-checking the problem is done by enumerating all possible execution paths. The overall structure of our tool is depicted as Fig. 1.

The following case studies are used to illustrate the efficiency of our approach in practice, although it has a quite high theoretical complexity. The experiments are conducted on a laptop with Inter Core i3-5005U at 2.0GHz and 4GB DDR3L-1600MHz RAM.

Example 6.1. In this example, we consider the anomalous behaviour of priority-driven systems given in [22].

The simple system contains four independent jobs, which are scheduled on two identical processors P_1 and P_2 in a priority-driven manner. P_1 and P_2 maintain a common priority order of jobs $J_1 > J_2 > J_3 > J_4$. These jobs may be preempted, but never be migrated, which means that once it begins to be executed on a processor, the job has to be executed on that processor until completion. The release times (r), deadlines (d), and execution times of the jobs are listed in Fig. 2. J_1, J_2, J_4 are released at 0, while J_3 is released at 4. The execution times of J_1, J_3, J_4 are fixed, while J_2's is varied in $[2, 6]$. In addition, J_1 is required to be executed on P_1, J_2 is required to be executed on P_2, while J_3 and J_4 can be executed on either of P_1 and P_2. The property which should be satisfied by P_2 on $[0, 20]$ can be represented by the following ELDI formula:

$$20 \le \ell \le 20 \Rightarrow [(2 \le \int run_{J_2} \le 6 \wedge \int run_{J_2} - \int 1 = 0)$$
$$; ((\int run_{J_3} = 0 \vee \int run_{J_3} = 8) \wedge (\int run_{J_4} = 0 \vee \int run_{J_4} = 10) \wedge$$
$$(0 < \int run_{J_3} + \int run_{J_4} \le 18))].$$

The tool verifies the formula in 2.4 seconds, and returns *false*. This implies that some jobs cannot catch the deadline on $[0, 20]$, a counterexample is provided in Fig. 3.

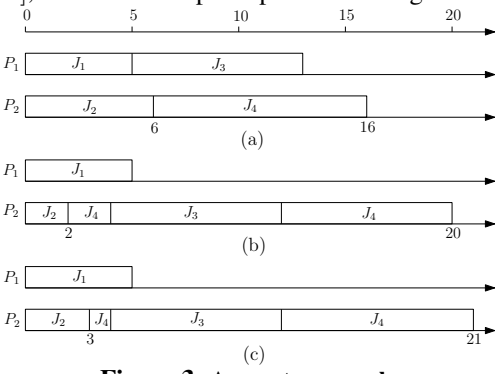
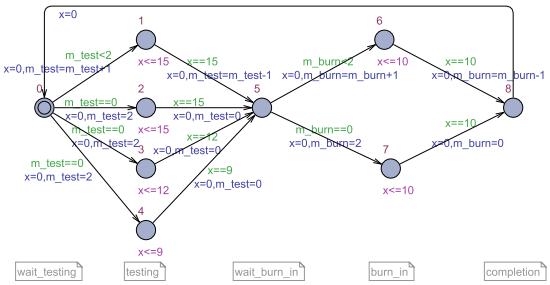


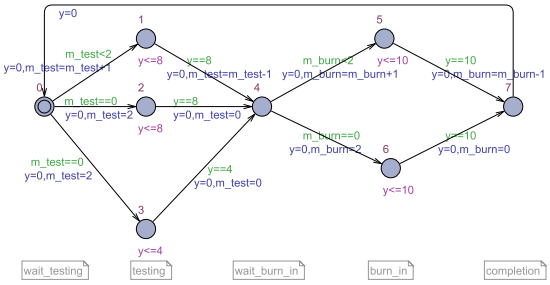
Figure 3: A counterexample

Example 6.2. Now consider an example of final testing of integrated circuits, which is a simplified version of a real-world problem reported in [25]. In this simplified version, all jobs are clustered into two product types A and B. Both of A and B jobs are processed in two stages, i.e., `testing` and `burn_in`. A and B jobs are grouped into 5 and 2 lots respectively. In `testing` stage, all A and B lots are processed serially on several parallel machines, and it takes 3 time units to handle each A lot, and 4 time units to handle each B lot by one machine. In `burn_in` stage, all A and B lots are processed in a batch manner on several parallel machines, and it takes 10 time units to finish a batch by one

machine. The maximum batch size for a machine is 5. For simplicity, here we assume there are two machines on each stage. So, we introduce two integer variables *m_test* $\in [0,2]$ and *m_burn* $\in [0,2]$ to stand for how many machines have been used at `testing` and `burn_in` stages, respectively. At `testing`, no machine can be allocated to B product (dually A product) unless all A jobs (dually B jobs) have been finished in case it has been allocated to A product (dually B product). Additionally, any type product can switch from `testing` to `burn_in` immediately whenever all its lots have been processed at `testing`. Thus, A and B products can be modelled by TA *PA* and *PB* in Fig. 4.(a) and Fig. 4.(b) correspondingly.



(a)



(b)

Figure 4: (a) *PA*: TA for product A; (b) *PB*: TA for product B.

PA has nine locations $0, \ldots, 8$. 0 stands for waiting for testing, 1 for one machine allocated to test A product, $2, 3, 4$ for both of the two machines allocated to test A product, but with different scheduling policies. In 2, one machine has 5 lots and the other has no jobs; in 3, one machine has 4 lots and the other has 1 lot; in 4, one machine has 3 lots and the other has 2 lots. 5 for waiting for burning, $6, 7$ for A lots under burn respectively with one machine and two machines, 8 for completion. The above procedure can be repeated. *PB* can be understood similarly. Note that there are only two scheduling policies in *PB* at `testing` when both of the two machines are allocated to it. x and y are two clock variables. The product automaton of *PA* and *PB* is shown in Fig. 5 .

The timing constraints on the products A and B are given as follows: The deadlines for A and B are 30 and 36, respectively. Now, the question is wether we can find a feasible schedule to meet these requirements within 36 to 40 time units, i.e., whether the following formula holds.

$$
\begin{aligned}
&36 \le \ell \le 40 \Rightarrow \\
&\left(\begin{array}{l} \int(PA.wait\_testing, *) + \int(PA.testing, *) + \\ \int(PA.wait\_burn\_in, *) + \int(PA.burn\_in, *) \le 30 \\ \wedge [9 \le \int(PA.testing, *) \le 15; \int(PA.burn\_in, *) = 10] \end{array} \right) \\
&\wedge \left(\begin{array}{l} \int(*, PB.wait\_testing) + \int(*, PB.testing) + \\ \int(*, PB.wait\_burn\_in) + \int(*, PB.burn\_in) \le 36 \\ \wedge [4 \le \int(*, PB.testing) \le 8; \int(*, PB.burn\_in) = 10] \end{array} \right)
\end{aligned}
$$

where $(PA.l, *)$ stands for a state expression, which means that in $PA \times PB$, PA stays in location l at time t if $(PA.l, *)(t) = 1$, $(*, PB.l)$ can be understood symmetrically.

It takes 126 minutes totally to check the above property with our tool, which finds several feasible schedules by solving the resulting QLRA formula. For example, in Fig. 5, the path $00 \to 10 \to 11 \to 14 \to 54 \to 55 \to 65 \to 67 \to 87$ with chop points at 54 is a feasible schedule.

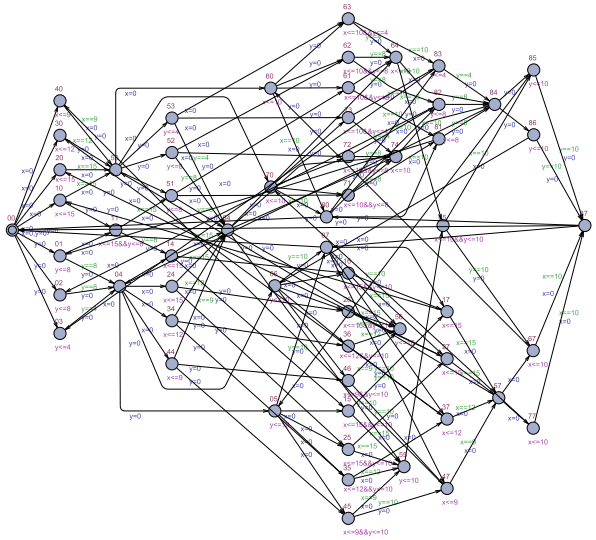


Figure 5: *PA* $\times$ *PB*, the product of PA and PB

Considering the size of the example, the execution time is acceptable. The zone graph of the product automaton has more than 660 zones which are the beginning zones of the search process. What's more, the loops of the product automaton increase the number of the paths and the path length dramatically. The size of the generated file storing QLRA formulas is 3.1GB and it was divided into 187 files as inputs for REDLOG due to the limit of the memory size.

Note that *virtual substitution* has been implemented in the latest release of Z3. Z3 takes 0.24 seconds and 114 minutes respectively to solve the resulting QLRA formulas in *Example 6.1* and *Example 6.2*. This indicates that the efficiencies of Z3 and REDLOG are nearly same on solving such formulas.

7 CONCLUSION

In this paper, we investigate the model-checking of timed automata against bounded ELDI in the context of the continuous-time semantics. This is achieved through the following steps: firstly, we compute all execution fragments of a given timed automaton $\mathcal{A}$ whose length is within the given bounded interval according to $\mathcal{A}$'s zone graph; secondly, we encode whether each execution fragment obtained in the first step satisfies the given bounded ELDI formula Φ into a QLRA formula; finally, we invoke REDLOG, a computer algebra tool, or Z3, an SMT solver, to solve the resulting QLRA formula. The complexity of our approach is triple exponential in the number of $\mathcal{A}$'s locations and double exponential in the number of nested chops in Φ in the worst case. But in practice, it is very efficient as in REDLOG and Z3, *virtual substitution* can be applied to QLRA formulas. We have implemented a prototypical tool, and some case studies are provided to illustrate our approach[2].

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