Lemma 1 *For every semi-deterministic Büchi automaton $\mathcal{A}$ there exists a deterministic Muller automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.*

Proof:

Let $\mathcal{A} = (N \uplus D, I, T, F)$, $d = |D|$, and let D be ordered by $<$. We construct the DMA $(S', \{s_0'\}, T', \mathcal{F})$:

- $S' = 2^N \times \{0, \dots, 2d\} \to D \cup \{\llcorner\!\lrcorner\}$
- $s_0' = (\{N \cap I\}, (d_1, d_2, \dots, d_n, \llcorner\!\lrcorner, \dots, \llcorner\!\lrcorner))$,
  where $d_i < d_{i+1}, \{d_1, \dots, d_n\} = D \cap I\}$.
- $T' = \{((N_1, f_1), \sigma, (N_2, f_2)) \mid N_2 = pr_3(T \cap N_1 \times \{\sigma\} \times N)$
  $D' = pr_3(T \cap N_1 \times \{\sigma\} \times D)$
  $g_1 : n \mapsto d_2 \in D \Leftrightarrow f_1 : n \mapsto d_1 \in D \wedge d_1 \to^{\sigma} d_2$
  g_2: insort the elements of D' in the empty slots of g_1 (using $<$)
  f_2: delete every recurrance (leaving an *empty* slot)
- $\mathcal{F} = \{F' \subseteq S' \mid \exists i \in 1, \dots, 2d$ s.t.
  $f(i) \neq \llcorner\!\lrcorner$ for all $(N', f) \in F'$ and
  $f(i) \in F$ for some $(N', f) \in F'\}$.

<u>$\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{A}')$:</u>
If $\alpha \in \mathcal{L}(\mathcal{A})$, $\mathcal{A}$ has an accepting run $r = n_0 \dots n_{j-1} d_j d_{j+1} d_{j+2} \dots$
where $n_k \in N$ for $k < j$ and $d_k \in D$ for $k \geq j$.
Consider the run $r' = (N_0, f_0), (N_1, f_1), \dots$ of $\mathcal{A}'$ on α.

- $n_k \in N_k$ for all $k < j$,
- for all $k \geq j$, $d_k = f_k(i)$ for some $i \leq 2d$,
- these i's are non-increasing, and hence stabilize eventually.
- for this stable i,
  $f(i) \neq \llcorner\!\lrcorner$ for all $(N', f) \in In(r')$ and $f(i) \in F$ for some $(N', f) \in In(r')$.
- $In(r') \in \mathcal{F}$.

<u>$\mathcal{L}(\mathcal{A}') \subseteq \mathcal{L}(\mathcal{A})$:</u>
For $\alpha \in \mathcal{L}(\mathcal{A}')$, $\mathcal{A}'$ has an accepting run $r' = (N_0, f_0), (N_1, f_1), \dots$.

- We pick an i and an accepting set $F' \in \mathcal{F}$ s.t.
  $f(i) \neq \llcorner\!\lrcorner$ for all $(N', f) \in F'$ and $f(i) \in F$ for some $(N', f) \in F'$.

- We pick a $j \in \omega$ such that $f_n(i) \neq \llcorner\!\lrcorner$ for all $n > j$.
- There is a run $r = s_0 s_1 \dots s_j f_{j+1}(i) f_{j+2}(i) f_{j+3}(i) \dots$ of $\mathcal{A}$ for α.
- r is accepting.

■