Simplifying XML Schema:
Single-type approximations of regular tree languages

Wenbo Lian

National Engineering Research Center of Fundemental Software,
Institute of Software, Chinese Academy of Sciences

May 25, 2013

Outline

Motivation

- XML documents can be exactly modeled by XML Schema Definition
- XML Schema Definition can be as strong as EDTD
- For pratical use,constraints are imposed on XML Schema Definitions(XSD),which may break the closure of boolean operator
  - UPA(Unique Particle Attribution):In G.J Bex et.al.
  - EDC(Element Declaration Consistent):In this paper
- Element Declarations Consistents
  - Elements with the same name in the same content model must have the same type
  - Advantage:Facilites a simple one-pass top-down validation algorithm
  - Disadvantage:Break the closure of XSD under union and set difference
- Approximations is needed in two flavours
  - Upper XSD-approximation:Union of two XSD
  - Lower XSD-approximation:Description of interface in web

Exmaple

The DTD example

```
<!DOCTYPE CONFERENCE [
     <!ELEMENT conference       (track+|(session,break?)+)>
     <!ELEMENT track            (session,break?)+>
     <!ELEMENT session          (chair,talk+)>
     <!ELEMENT talk             ((title,authors)|(title,speaker))>
     <!ELEMENT chair            (#PCDATA)>
     <!ELEMENT break            (#PCDATA)>
     <!ELEMENT title            (#PCDATA)>
]>
```

Exmaple

The XML Schema example

```
<xsd:complexType name="track">
  <xsd:sequence minOccurs="1" maxOccurs="unbounded">
    <xsd:choice>
      <xsd:element name="invSession" type="invSession"
                   minOccurs="1" maxOccurs="1"/>
      <xsd:element name="conSession" type="conSession"
                   minOccurs="1" maxOccurs="1"/>
    </xsd:choice>
    <xsd:element name="break" type="xsd:string"
                 minOccurs="0" maxOccurs="1"/>
  </xsd:sequence>
</xsd:complexType>
```

Contributions and Related work

- Contributions
  - Every EDTD has a unique upper XSD-approximation
  - The approximation of two XSDs union and set difference can be determined in polynomial time
  - Deciding whether S is the minimal upper XSD-approximation of D is complete for PSPACE
    where S is a single-type EDTD,D is an EDTD
- Related work
  - Murata et al. establish a *taxonomy* of XML Schema in terms of tree language
  - Martens et al. characterized ST-REG as the subclass of the regular tree language closed under ancestor-guarded subtree exchange

Outline

Strings,trees,and contexts

Definition. *State-labeled automata* $N(Q,\Sigma,\delta,S,F)$:$\forall q \in Q$,the set $\{a|\exists p \in Q\text{such that}q \in \delta(p,a)\}$ is a *singleton*
$N(w)$:The resulting state set of N after reading w from some state $s \in I$
Definition. Σ-*Tree*: Dom(t)= $\{\varepsilon\} \cup \{iu : 1 \leqslant i \leqslant n, u \in Dom(t)\}$

Σ-*label*:Denotated by $lab^t(v)$

Definition. *ch-str*$^t(v)$: The child string of node v,i.e.,the string $lab^t(v1)\cdots lab^t(vn)$

Definition. *anc-str*$^t(v)$ where node v is $i_1 \ldots i_k$: $lab^t(\varepsilon)lab^t(i_1)$ $\ldots lab^t(i_1 \ldots i_{k-1})$ $lab^t(v)$

Definition. *Context*:A tree with a "hole" marker $\bullet$

XML Schema Languages

Definition 2.1. DTDD:A tuple (Σ, d, S_d),where

- Σ: Finite alphabet
- d:$\Sigma \rightarrow \Sigma^*$
- $S_d \subseteq \Sigma$ is the set of start symbols
- the size of DTD:$|\Sigma| + |S_d| + |d|$
- A tree t accepted by L(D)(or L(d)) if $\forall v \in Dom(t)$, $ch\text{-}str^t(v) \in d(lab^t(v))$

XML Schema Languages

Definition 2.1. DTDD:A tuple (Σ, d, S_d),where

- Σ: Finite alphabet
- d:$\Sigma \to \Sigma^*$
- $S_d \subseteq \Sigma$ is the set of start symbols
- the size of DTD:$|\Sigma| + |S_d| + |d|$

Definition 2.2.$EDTD$D:A tuple $(\Sigma, \Delta, d, S_d, \mu)$

- Δ: A finite type set
- (Δ, d, S_d): A DTD
- μ: $\Delta \to \Sigma$
- A tree accepted by D if $\exists t' \in L(d)$ such that $\mu(t') = t$

XML Schema Languages

Definition 2.1. DTDD:A tuple (Σ, d, S_d),where

- Σ: Finite alphabet
- d:$\Sigma \to \Sigma^*$
- $S_d \subseteq \Sigma$ is the set of start symbols
- the size of DTD:$|\Sigma| + |S_d| + |d|$

Definition 2.2.$EDTD$D:A tuple $(\Sigma, \Delta, d, S_d, \mu)$

- Δ: A finite type set
- (Δ, d, S_d): A DTD
- μ: $\Delta \to \Sigma$

Proviso 2.3.All EDTDs are reduced.

- Reduced:for any type $\tau \in \Delta$,there exists a tree $t' \in L(d)$ and a node u such that $lab^{t'}(u) = \tau$
- Any EDTD has an quivalent reduced EDTD and can be computed from a given EDTD in polynomial time
- Similar to CFG,see [J. Albert et.al 2001,W. Martens et.al 2009]

XML Schema Languages

Definition 2.2.*EDTD*D:A tuple $(\Sigma, \Delta, d, S_d, \mu)$

- Δ: A finite type set
- (Δ, d, S_d): A DTD
- μ: $\Delta \to \Sigma$

Definition 2.4.*Single-type EDTD*: An EDTD $(\Sigma, \Delta, d, S_d, \mu)$ with property that no two types τ_1 and τ_2 exists with $\mu(\tau_1) = \mu(\tau_2)$ such that

- $\tau_1, \tau_2 \in S_d$
- there is a type τ such that $w_1\tau_1v_1 \in d(\tau)$ and $w_2\tau_2v_2 \in d(\tau)$ for some strings w_1,v_1,w_2 and v_2.
- ST-REG is the class of regular tree language can be definable by single-type EDTDs.

Definition 2.2.*EDTD*D:A tuple $(\Sigma, \Delta, d, S_d, \mu)$

- Δ: A finite type set
- (Δ, d, S_d): A DTD
- μ: $\Delta \to \Sigma$

Definition 2.4.*Single-type EDTD*: An EDTD $(\Sigma, \Delta, d, S_d, \mu)$ with property that no two types τ_1 and τ_2 exists with $\mu(\tau_1) = \mu(\tau_2)$ such that

- $\tau_1, \tau_2 \in S_d$
- there is a type τ such that $w_1\tau_1v_1 \in d(\tau)$ and $w_2\tau_2v_2 \in d(\tau)$ for some strings w_1,v_1,w_2 and v_2.
- ST-REG is the class of regular tree language can be definable by single-type EDTDs.

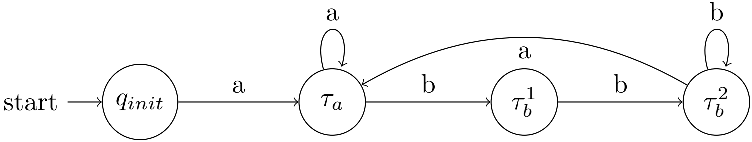
Intuitively use an automaton to help assign type for a tree which may be accepted by EDTD

Definition 2.5.*type automaton* of an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$ a *state-labeled* NFA without final states such that $Q = \Delta \uplus \{q_{init}\}$ and foreach $q \in Q$

- if q=q_{init},then $\delta(q, a) = \{\tau | \mu(\tau) = a \text{ and } \tau \in S_d\}$ and
- otherwise,$\delta(q, a) = \{\tau | \mu(\tau) = a$ and τ occurs in some word in d(q)$\}$

Example 2.6.Given an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$,with
$\Delta = \{\tau_a, \tau_b^1, \tau_b^2\}, S_d = \{\tau_a\}$ and $\mu(\tau_a) = a, \mu(\tau_b^1) = \mu(\tau_b^2) = b$:

$\tau_a \to \tau_a + \tau_b^1,\ \tau_b^1 \to \tau_b^2 + \varepsilon,$
$\tau_b^2 \to \tau_a + \tau_b^2 + \varepsilon$
Construct the type automaton for the EDTD

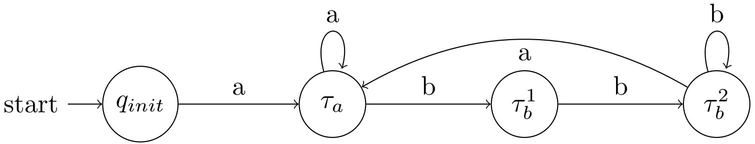


Example 2.6.Given an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$,with
$\Delta = \{\tau_a, \tau_b^1, \tau_b^2\}, S_d = \{\tau_a\}$ and $\mu(\tau_a) = a, \mu(\tau_b^1) = \mu(\tau_b^2) = b$:

$\tau_a \to \tau_a + \tau_b^1$, $\tau_b^1 \to \tau_b^2 + \varepsilon$,

$\tau_b^2 \to \tau_a + \tau_b^2 + \varepsilon$

Construct the type automaton for the EDTD



Observation 2.7

- Given an EDTD,its type automation can be construct in linear time
- For each EDTD,the state q_{init} of its type automaton has no incoming transitions
- The type automaton of an EDTD is a DFA iff D is a single-type EDTD

XML Schema Languages

Definition 2.8. A DFA-based XSD is a pair D=(Σ, A, d, S_d), where A=$(Q, \Sigma, \delta, \{q_{init}\}, \varnothing)$ is a state-labeled DFA with:

- initial state q_{init} and without final states
- d is a function from Q$\backslash\{q_{init}\}$ to regular languages over Σ
- $S_d \subseteq \Sigma$ is the set of start symbols
- A tree t satisfies D if $lab^t(\varepsilon) \in S_d$ and for every node u where $A(anc\text{-}str^t(u))=\{q\}$,$ch\text{-}str^t(u) \in d(q)$

Proposition 2.9. DFA-based XSDs are *expressively equivalent* to single-type EDTDs and one can translate between DFA-based XSDs and single-type EDTDs in linear time

XML Schema Languages

Definition 2.8. A DFA-based XSD is a pair D=(Σ, A, d, S_d), where A=$(Q, \Sigma, \delta, \{q_{init}\}, \varnothing)$ is a state-labeled DFA with:
Proposition 2.9. DFA-based XSDs are *expressively equivalent* to single-type EDTDs and one can translate between DFA-based XSDs and single-type EDTDs in linear time

Proof.

From XSD to Single-type EDTD,intuitively as the reverse of construction of type automaton.
From DFA-based XSD D=(Σ, A, d, S_d),where A=$(Q, \Sigma, \delta, \{q_{init}\}, \varnothing)$ to single-type EDTD E=$(\Sigma, \Delta, d', S'_d, \mu)$

- $\Delta = \{(a,q) \in \Sigma \times Q | \exists p : \delta(p,a) = q \in A\}$
- $S'_d = \{(a,q) | a \in S_d$ and $\delta(q_{init}, a) = q \in A\}$
- $\mu((a,q)) = a$ for every (a,q) $\in \Delta$,and
- for each (a,q)$\in \Delta$,we define $d'((a,q))$ to be the language $\{(a_1,q_1)\cdots(a_n,q_n) \in \Delta^* | a_1\cdots a_n \in d(q)\}$ and,for each a_i,$\delta(q,a_i) = q_i$ in A

□

XML Schema Languages

Definition 2.8. A DFA-based XSD is a pair D=(Σ, A, d, S_d), where A=$(Q, \Sigma, \delta, \{q_{init}\}, \varnothing)$ is a state-labeled DFA with:
Proposition 2.9. DFA-based XSDs are *expressively equivalent* to single-type EDTDs and one can translate between DFA-based XSDs and single-type EDTDs in linear time

Proof.

From single-type EDTD E=$(\Sigma, \Delta, d, S_d, \mu)$

- $S'_d = \{\mu(\tau) | \tau \in S_d\}$
- A=$(Q, \Sigma, \delta, \{q_{init}\})$ is the type automaton of E
- for each $\tau \in \Delta$,we define $d'(\tau) = \mu(d(\tau))$, where $\mu(d(\tau))$denotes the homomorphic extension of μ to string langues

□

Definition 2.10.A tree language T is *closed under ancestor-guarded subtree exchange* if the following property holds. Whenever for two $t_1, t_2 \in T$ with nodes v_1, v_2 respectively, $\textit{anc-str}^{t_1}(v_1)=\textit{anc-str}^{t_2}(v_2)$ then

$$t_1[v_1 \leftarrow \textit{subtree}^{t_2}(v_2)] \in T$$

Definition 2.10.A tree language T is *closed under ancestor-guarded subtree exchange* if the following property holds. Whenever for two $t_1, t_2 \in T$ with nodes v_1, v_2 respectively, $\textit{anc-str}^{t_1}(v_1) = \textit{anc-str}^{t_2}(v_2)$ then

$$t_1[v_1 \leftarrow \textit{subtree}^{t_2}(v_2)] \in T$$

Theorem 2.11.A regular language T is definable by a single-type EDTD iff it is *closed under ancestor-guarded subtree exchange*.

XML Schema Languages

Definition 2.10.A tree language T is *closed under ancestor-guarded subtree exchange* if the following property holds. Whenever for two $t_1, t_2 \in T$ with nodes v_1, v_2 respectively, $anc\text{-}str^{t_1}(v_1) = anc\text{-}str^{t_2}(v_2)$ then

$$t_1[v_1 \leftarrow subtree^{t_2}(v_2)] \in T$$

Theorem 2.11.A regular language T is definable by a single-type EDTD iff it is *closed under ancestor-guarded subtree exchange*.

Definition 2.12. *minimal upper XSD-approximation of an EDTD D*:A Single-type EDTD D_1 where $L(D) \subseteq L(D_1)$ and no D' exists such that $L(D) \subseteq L(D') \subset L(D_1)$

XML Schema Languages

Definition 2.10.A tree language T is *closed under ancestor-guarded subtree exchange* if the following property holds. Whenever for two $t_1, t_2 \in T$ with nodes v_1, v_2 respectively, $anc\text{-}str^{t_1}(v_1) = anc\text{-}str^{t_2}(v_2)$ then

$$t_1[v_1 \leftarrow subtree^{t_2}(v_2)] \in T$$

Theorem 2.11.A regular language T is definable by a single-type EDTD iff it is *closed under ancestor-guarded subtree exchange.*

Definition 2.12. *minimal upper XSD-approximation of an EDTD D*:A Single-type EDTD D_1 where $L(D) \subseteq L(D_1)$ and no D' exists such that $L(D) \subseteq L(D') \subset L(D_1)$

Theorem 2.13. The universality problem for EDTDs,i.e.,deciding whether $\mathcal{T}_\Sigma \subseteq L(D)$ for an EDTD D,is EXPTIME-complete.

Single-type closure and derivation trees

Prove some basic properties about languages definable by single-type EDTDs and their closure properties

Definition 2.14.Let T be a tree language. closure(T) means the smallest tree language which contains T and which is closed under ancestor-guarded subtree exchange.

Lemma 2.15.Let $(X_i)_{i\in I}$ be an arbitary family of tree languages where each X_i is closed under ancestor-guarded subtree exchange.Then the intersection $\bigcap_{i\in I} X_i$ is also closed under ancestor-guarded subtree exchange.

Single-type closure and derivation trees

Prove some basic properties about languages definable by single-type EDTDs and their closure properties

Definition 2.14.Let T be a tree language. closure(T) means the smallest tree language which contains T and which is closed under ancestor-guarded subtree exchange.

Lemma 2.15.Let $(X_i)_{i \in I}$ be an arbitary family of tree languages where each X_i is closed under ancestor-guarded subtree exchange.Then the intersection $\bigcap_{i \in I} X_i$ is also closed under ancestor-guarded subtree exchange.

Proof.

Let X=$\bigcap_{i \in I} X_i$.

- Let t_1, t_2 from X with nodes v_1, v_2 where $anc\text{-}str^{t_1}(v_1)$=$anc\text{-}str^{t_2}(v_2)$
- With t=$t_1[v_1 \leftarrow subtree^{t_2}(v_2)] \in X$

$\square$

Single-type closure and derivation trees

Prove some basic properties about languages definable by single-type EDTDs and their closure properties

Definition 2.14.Let T be a tree language. closure(T) means the smallest tree language which contains T and which is closed under ancestor-guarded subtree exchange.

Definition 2.16.Let X be a tree language and t a tree from closure(X). A *derivation tree* of t w.r.t X is a (finite) binary tree ϑ labeled with tree from closure(X) such that:

- The root of ϑ is labeled with t:$lab^{\vartheta}(\varepsilon)$=t
- For each leaf $v \in Dom(\vartheta)$,we have $lab^{\vartheta}(v) \in X$.
- For each internal node $v \in Dom(\vartheta)$ and $i \in \{1, 2\}$,let $t_i = lab^{\vartheta}(v_i)$. Then there are nodes $u_i \in Dom(t_i)$ such that $anc\text{-}str^{t_i}(u_1) = anc\text{-}str^{t_2}(u_2)$

Single-type closure and derivation trees

Lemma 2.17. Let X be a tree language and t a tree.Then $t \in closure(X)$ iff t has a derivation tree w.r.t X.

Proof.

For if part. Whenever t has a derivation tree w.r.t. X,then $t \in closure(X).Immediately$

The only if part.

- T_i the set of trees from closure(X) which have a derivation tree of height i
- T_0 is X. suppose $t \in T_i$ and ϑ is the derivation tree of t,then $t(\vartheta, \vartheta) \in T_{i+1}$,so $T_i \subseteq T_{i+1}$
- T=$\bigcup_{i \in \mathbb{N}} T_i$
- for $t_1, t_2 \in T$,there exist n_1, n_2 such that $t_1 \in T_{n_1}, t_2 \in T_{n_2}$, any tree t obtained by applying ancestor-guarded subtree exchange to t_1,t_2 is in $T_{max(n_1,n_2)+1} \subseteq T$, so T is closed under ancestor-guarded subtree exchange and contains X.
- closure(X) is the *smallest set closed under ancestor-subtree exchange* which contains X,$closure(X) \subseteq T$.

Outline

1. Motivation

2. Preliminaries

3. Upper XSD-approximations
   - Single-type up approximations of EDTDs
   - Unions of XSDs
   - Intersection of XSDs
   - Complements of XSDs

Outline

Minimal Upper XSD-approximatioins of an EDTD

Construction 3.1.(*Minimal upper approximation of an EDTD*).
D=$(\Sigma, \Delta, d, S_d, \mu)$ an EDTD.Let N=$(Q_N, \Sigma, \delta_N, \{q_{init}\})$ the *type automaton* of D ,and let $A_N = (Q, \Sigma, \delta, \{\{q_{init}\}\})$ be the DFA obtained from N by performing the standard subset construction.

- $Q \subseteq 2^{Q_N}$ is the *smallest* set such that $\{q_{init}\} \in Q$ and whenever $S \in Q$ then for every $a \in \Sigma$ then for every $a \in \Sigma$ we have $\bigcup_{q \in S} \delta_N(q, a) \in Q$
- each non-initial state consists of a set of types S of D in which, for every $\tau, \tau' \in S$
- DFA-based XSD (Σ, A_N, d', S'_d) where
  - $S'_d = \{a \in \Sigma | \tau \in S_d, \mu(\tau) = a\}$
  - $d'(S) := \bigcup_{\tau \in S} \mu(d(\tau))$ for every $S \in Q$
- μ canonically extended to languages

Minimal upper approximation of EDTD

Theorem 3.2.The minimal upper XSD-approximation of an EDTD is *unique* and can be compute in *exponential* time.There is a family of EDTDs $(D_n)_{n\geqslant 2}$,such that the size of every D_n is $O(n)$ but the type-size of the minimal upper XSD-approximation is $\Omega(2^n)$

Proof.

First prove that an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$,determinizing its type automaton result a DFA-based XSD D'=(Σ, A, d', S'_d)which is the unique minimal upper XSD-approximation

- $L(D) \subseteq L(D')$

$\square$

Minimal upper approximation of EDTD

Theorem 3.2.The minimal upper XSD-approximation of an EDTD is *unique* and can be compute in *exponential* time.There is a family of EDTDs $(D_n)_{n\geqslant 2}$,such that the size of every D_n is $O(n)$ but the type-size of the minimal upper XSD-approximation is $\Omega(2^n)$

Proof.

First prove that an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$,determinizing its type automaton result a DFA-based XSD D'=(Σ, A, d', S'_d)which is the unique minimal upper XSD-approximation

- $L(D) \subseteq L(D')$
  - Suppose $t \in L(D)$,according to the *definition* of EDTD $\exists t' \in L(d)$ and $\mu(t') = t$,such that $\forall v \in Dom(t')$,$ch\text{-}str^{t'}(v) \in d(lab^{t'}(v))$.
  - Let $v \in Dom(t)$ and S=A$(anc\text{-}str^{t}(v))$.
  - According to Construction of D',$lab^{t'}(v) \in S$ and $ch\text{-}str^{t}(v) \in d'(S)$
  - As this holds for all nodes of $t, t \in L(D')$

$\square$

Minimal upper approximation of EDTD

Theorem 3.2.The minimal upper XSD-approximation of an EDTD is *unique* and can be compute in *exponential* time.There is a family of EDTDs $(D_n)_{n \geqslant 2}$,such that the size of every D_n is $O(n)$ but the type-size of the minimal upper XSD-approximation is $\Omega(2^n)$

Proof.

First prove that an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$,determinizing its type automaton result a DFA-based XSD D'=(Σ, A, d', S'_d)which is the unique minimal upper XSD-approximation

- $L(D') \subseteq closure(L(D))$
  - iterate over the nodes of $t \in L(D')$ in a breadth first manner,such that when we reach a node v,construct a tree t_v which satisfies
    - $t_v \in closure(L(D))$
    - the parts of t and t_v up to v(breadth first manner) and their children are isomorphic

□

Minimal upper approximation of EDTD

Theorem 3.2.The minimal upper XSD-approximation of an EDTD is *unique* and can be compute in *exponential* time.There is a family of EDTDs $(D_n)_{n\geqslant 2}$,such that the size of every D_n is $O(n)$ but the type-size of the minimal upper XSD-approximation is $\Omega(2^n)$

Proof.

First prove that an EDTD D=$(\Sigma, \Delta, d, S_d, \mu)$,determinizing its type automaton result a DFA-based XSD D'=(Σ, A, d', S'_d)which is the unique minimal upper XSD-approximation

- $L(D') \subseteq closure(L(D))$
  - construct the sequence of trees t_v
    - $\forall v \in Dom(t)$ assign a type τ_v such that $ch\text{-}str^t(v) \in \mu(d(\tau_v))$
    - Iterate $v \in Dom(t)$ in breadth first manner
    - When v is root node,$t_v \in L(D)$ can be construct as $L(D)$ is reduced therefore $t_v \in closure(L(D))$
    - $t_u \in closure(L(D))$ was constructed,v next node to iterate,$\exists t'_v \in L(D)$ by assigning τ_v to t'_v where $anc\text{-}str^{t'_v}(v)$=$anc\text{-}str^t(v)$ and $ch\text{-}str^{t'_v}(v)$=$ch\text{-}str^t(v)$ as D is reduced and the construction of D'
    - $t_u[v \leftarrow \text{subtree}^{t'_v}(v)] \in closure(L(D))$ can be the t_v in the sequence

Minimal upper approximation of EDTD

Theorem 3.2.The minimal upper XSD-approximation of an EDTD is *unique* and can be compute in *exponential* time.There is a family of EDTDs $(D_n)_{n\geqslant 2}$,such that the size of every D_n is $O(n)$ but the type-size of the minimal upper XSD-approximation is $\Omega(2^n)$

Proof.

Now prove that the exponential type size cannot be avoid

Can consider the unary tree,each node has at most one child,such a tree can be view as a *regular word*

EDTDs and stEDTDs can intuitively correspond to NFAs and DFAs

Need to translate between stEDTDs and DFAs

□

Minimal upper approximation of EDTD

Theorem 3.2.The minimal upper XSD-approximation of an EDTD is *unique* and can be compute in *exponential* time.There is a family of EDTDs $(D_n)_{n \geqslant 2}$,such that the size of every D_n is $O(n)$ but the type-size of the minimal upper XSD-approximation is $\Omega(2^n)$

Proof.

Prove that exists a family of EDTDs.
$L_n = (a+b)^* a (a+b)^n$

- Property:the uniqure node at distance n of the leaf node is a
- let EDTD D_n accepts L_n,$D_n$can be easily construct of size linear in n
- $D'_n = (\Sigma, A, d, S_d)$ a DFA-based XSD such that $L(D_n) = L(D'_n)$
- A_n is a DFA obtained from A by
  - $\forall q \in A$ where $\varepsilon \in L(d(q))$,mark q final
  - remove all transitions (q, σ, q') where $\sigma \notin L(d(q))$
- $L(A_n) = L_n$and the DFA which accepts L_n is of size exponential in n

$\square$

Minimal upper approximation of EDTD

Lemma 3.3 Let D_1 be an EDTD and let D_2 be a single-type EDTD.Testing whether $L(D_1) \subseteq L(D_2)$ is in PTIME

Proof.

Let $D_1 = (\Sigma, \Delta_1, d_1, S_{d_1}, \mu_1)$ and $D_2 = (\Sigma, \Delta_2, d_2, S_{d_2}, \mu_2)$ let for each $i \in \{1, 2\}$,$A_i = (Q_i, \Sigma, \delta_i, I_i)$.
$L(D_1) \notin L(D_1)$ iff there exitsts a type $\tau_2 \in \Delta_2$ for which there exists a string w which

- $A_2(w) = \{\tau_2\}, A_1(w) = S_1$,and
- there exists a $\tau_1 \in S_1$ and a string $v \in d_1(\tau_1)$ such that $\mu_1(v) \notin \mu_2(d_2(\tau_2))$

□

Minimal upper approximation of EDTD

Lemma 3.3 Let D_1 be an EDTD and let D_2 be a single-type EDTD.Testing whether $L(D_1) \subseteq L(D_2)$ is in PTIME

Proof.

Provide a PTIME algorithm for the complement of the problem

- (1)Compute the binary relation $R = \{(\tau_1, \tau_2) | \exists w$ such that $\tau_1 \in A_1(w)$and $A_2(w) = \{\tau_2\}\}$
- (2)Test whether exists a pair (τ_1, τ_2) in R for which $\mu_1(d_1(\tau_1)) \notin \mu_2(d_2(\tau_2))$
- the step(1) can be computed in polynomial time by considering product automation $A_1 \times A_2$
- the step(2) is in PTIME since both $\mu_1(d_1(\tau_1))$ and $\mu_2(d_2(\tau_2))$ can be represented by polynomial-size DFAs

□

Minimal upper approximation of EDTD

Theorem 3.4(See [25].) The complexity of the language inclusion problem $L(X) \subseteq L(Y)$ is PSPACE-complete when X and Y are given as regular expressions or NFAs

Minimal upper approximation of EDTD

Theorem 3.5 Deciding whether a single-type EDTD is a minimal upper XSD-approximation of a given EDTD is PSPACE-complete.

Proof.

For the upper bound,let $D_1 = (\Sigma, \Delta_1, d_1, S_{d_1}, \mu_1)$ a single-type EDTD,D an EDTD.

- First,test whether $L(D) \subseteq L(D_1)$
- Let D_2 be the minimal upper XSD-approximation of D according to Theorem 3.2,claim that
  - D_1 is the minimal upper XSD-approximation of D iff $L(D_1) \subseteq L(D_2)$:Easy to proof
  - Can test whether $L(D_1) \subseteq L(D_2)$ in PSPACE without fully constructing D_2

□

Minimal upper approximation of EDTD

Theorem 3.5 Deciding whether a single-type EDTD is a minimal upper XSD-approximation of a given EDTD is PSPACE-complete.

Proof.

For the upper bound,let $D_1 = (\Sigma, \Delta_1, d_1, S_{d_1}, \mu_1)$ a single-type EDTD,D an EDTD.

- According to the proof of [W. Martens et.al 2007],testing $L(D_1) \subseteq L(D_2)$ reduces to
  - Computing a correspondence relation $R \subseteq \Delta_1 \times \Delta_2$ between their types
  - For each pair $(\tau_1, \tau_2) \in R$,testing the inclusion $\mu_1(d_1(\tau_1)) \subseteq \mu_2(d_1(\tau_2))$
- In other words,$L(D_1) \nsubseteq L(D_2)$ iff

$$\textit{there is a } (\tau_1, \tau_2) \in R \textit{ such that } \mu_1(d_1(\tau_1)) \nsubseteq \mu_2(d_2(\tau_2))$$

□

Minimal upper approximation of EDTD

Theorem 3.5 Deciding whether a single-type EDTD is a minimal upper XSD-approximation of a given EDTD is PSPACE-complete.

Proof.

For the upper bound,let $D_1 = (\Sigma, \Delta_1, d_1, S_{d_1}, \mu_1)$ a single-type EDTD,D an EDTD.

- PSPACE procedure consist of following steps:
  - Guess w and keep track of $(A_1(w), A_2(w))$ without constructing A_2 itself.A_1, A_2 the type automaton corresponding to D_1, D_2
  - Whether $\mu_1(d_1(\tau_1)) \notin \mu_2(d_1(\tau_2))$is the same as $\mu_1(d_1(\tau)) \notin \mu(d(\tau_1)) + \cdots + \mu(d(\tau_k))$
- Intuitively,we guess the path instead of constructing all the possible states

□

Minimal upper approximation of EDTD

Theorem 3.5 Deciding whether a single-type EDTD is a minimal upper XSD-approximation of a given EDTD is PSPACE-complete.

Proof.

The PSPACE for the lower bound can be obtained from the fact that testing $L(A) \subseteq L(A_1) \cup \cdots \cup L(A_n)$ for DFAs $A, A_1, \cdots, A_n$ is PSPACE-complete

Construct an EDTD D which takes $\tau_r^1 \to L(A_1), \cdots, \tau_r^n \to L(A_n)$ as the content model where $\forall 1 \leqslant i \leqslant n, \mu_1(\tau_r^i) = r$

Construct single-type D_1 as which takes $\tau_r \to L(A)$ as the content model where $\tau_r = r$

The problem for testing $L(A) \subseteq L(A_1) \cup \cdots \cup L(A_n)$ will reduce to test whether D_1 is the minimal XSD-approximation of D □

Outline

Unions of XSDs

Theorem 3.6. Let D_1 and D_2 be two single-type EDTDs.

- The minimal upper XSD-approximation of $L(D_1) \cup L(D_2)$ is unique and can be computed in time $O(|D_1||D_2|)$.
- There exists a family of single-type EDTDs $(D_1^n, D_2^n)_{n \geqslant 1}$, such that the size of every D_1^n and D_2^n is $O(n)$ but the type-size of the minimal upper XSD-approximation for $L(D_1^n) \cup L(D_2^n)$ is $\Omega(n^2)$

Proof.

$D_1 = (\Sigma, \Delta_1, d_1, S_{d_1}, \mu_1)$ and $D_2 = (\Sigma, \Delta_2, d_2, S_{d_2}, \mu_2)$
D:EDTD D where $L(D) = L(D_1) \cup L(D_2)$ obtained by computing the cross-product of D_1 and D_2
Type automaton of D is product of type automata of D_1 and D_2
Product of deterministic automata is deterministic.Determinization is trival and perform in time $O(|D_1||D_2|)$
The type-size of the minimal upper XSD-approximation D' for $L(D_1) \cup L(D_2)$ is $O(|D_1||D_2|)$
From the Theorem 3.2,this XSD-approximation is unique minimal XSD-approximation $\square$

Unions of XSDs

Theorem 3.6. Let D_1 and D_2 be two single-type EDTDs.

- The minimal upper XSD-approximation of $L(D_1) \cup L(D_2)$ is unique and can be computed in time $O(|D_1||D_2|)$.
- There exists a family of single-type EDTDs $(D_1^n, D_2^n)_{n \geqslant 1}$, such that the size of every D_1^n and D_2^n is $O(n)$ but the type-size of the minimal upper XSD-approximation for $L(D_1^n) \cup L(D_2^n)$ is $\Omega(n^2)$

Proof.

Prove the second goal

Fix n and consider the following single-type EDTD D_1 with $S_d = \{\tau_a^0, \tau_b^0\}$

$$\begin{aligned}
&\tau_a^i \to \tau_a^{i+1} + \tau_b^{i+1} + \varepsilon \ \textit{(for all } 0 \leqslant i < n-1\textit{)} \\
&\tau_b^i \to \tau_a^i + \tau_b^i + \varepsilon \ \textit{(for all } 0 \leqslant i < n\textit{)}, \\
&\tau_a^{n-1} \to \tau_b^n + \varepsilon \\
&\tau_b^n \to \tau_b^n + \varepsilon
\end{aligned}$$

The language $L(D_1)$ consist of unary trees which contains at most n node labeled with a.By changing the roles of a and b, define D_2 such that $L(D_2)$ consists of unary trees which contain at most n nodes labeled with b. □

Unions of XSDs

Theorem 3.6. Let D_1 and D_2 be two single-type EDTDs.

- The minimal upper XSD-approximation of $L(D_1) \cup L(D_2)$ is unique and can be computed in time $O(|D_1||D_2|)$.
- There exists a family of single-type EDTDs $(D_1^n, D_2^n)_{n \geqslant 1}$, such that the size of every D_1^n and D_2^n is $O(n)$ but the type-size of the minimal upper XSD-approximation for $L(D_1^n) \cup L(D_2^n)$ is $\Omega(n^2)$

Proof.

Let D' be the minimal upper XSD-approximation of $L(D_1) \cup L(D_2)$. Now show that type-size of D' is $\Omega(n^2)$

- N' is the type automaton for D'.let $\tau_{k,l} = N'(a^k b^l)$ for $1 \leqslant k, l \leqslant n$.
- Consider types for $(k,l) \neq (k',l')$ and assume $\tau_{k,l} = \tau_{k',l'}, k > k'$. Both $t = a^k b^{2n} a^{n-k}$ and $t' = a^{k'} b^{2n} a^{n-k'}$ are in $L(D')$
- Applying ancestor-type-guarded subtree exchange to node $v = 1^{k+l-1}$ in Dom(t) and node $v' = 1^{k'+l'-1}$ get a tree $t'' = t[v \leftarrow subtree^{t'}(v')] = a^k b^{l+2n-l'} a^{n-k}$ also belongs to $L(D')$
- Since $a^k b^{l+2n-l'} a^{n-k} \notin L(D_1) \cup L(D_2)$.A contradiction.

Outline

Intersection of XSDs

Proposition 3.7.Let D_1 and D_2 be single-type EDTDs.The intersection $L(D_1) \cap L(D_2)$ is definable by a single-type EDTD

Proof.

From Lemma 2.15
Regular languages closed under intersection
Theorem 2.11:stEDTD=rugular tree language+*ancestor-guarded subtree exchange* □

Intersection of XSDs

Theorem 3.8.Let D_1 and D_2 be two single-type EDTDs. The minimal upper XSD-approximation of $L(D_1) \cap L(D_2)$ is unique,defines precisely $L(D_1) \cap L(D_2)$ and can be computed in time $O(|D_1||D_2|)$.There is a family of pairs of single-type EDTDs $(D_1^n, D_2^n)_{n \geqslant 1}$,such that the size of every D_1^n and D_2^n is at least n and the type-size of the minimal upper XSD-approximation for $L(D_1) \cap L(D_2)$ is $\Omega(|D_1^n||D_2^n|)$

Proof.

The construction of the intersection of D_1 and D_2 is analogous to the construction in the proof of Theorem for Union of XSDs.
It is different that we need to construct the intersection of the two internal DFAs.
Use the standard product construction of DFAs.It's possible to construct in $O(|D_1||D_2|)$
To prove the second part of the theorem,take the unary trees as an example

> *let D_1^n and D_2^n accept unary trees of the form a^{k*p_1} and a^{k*p_2}, where $1 \leqslant k$ and $p_1 \neq p_2$ are two smallest primer numbers larger than n*

□

Outline

1. Motivation
2. Preliminaries
3. Upper XSD-approximations
   - Single-type up approximations of EDTDs
   - Unions of XSDs
   - Intersection of XSDs
   - Complements of XSDs

Complements of XSDs

Theorem 3.9. Let D be a single-type EDTD.The minimal upper XSD-approximation for the complement of D is unique and can be computed in time polynomial in $|D|$.

Proof.

Let D=$(\Sigma, \Delta, d, S_d, \mu)$ and let E=(Σ, A, f, S'_d) be the DFA-based XSD equivalent to D with A=$(\Delta, \Sigma, \delta, \{q_{init}\})$. Prove in two steps:
First construct an EDTD D_c for the complement of D
$D_c = (\Sigma, \Delta_c, d_c, S_{d_c}, \mu_c)$,use two set of types:$\Delta$ and Σ

- $\Delta_c = \Delta \uplus \Sigma$
- for every $\tau \in \Delta, \mu_c(\tau) = \mu(\tau)$ and ,for every $a \in \Sigma, \mu_c(a) = a$
- $S_{d_c} = S_d \uplus (\Sigma \backslash \mu(S_d))$;
- for every $\tau \in \Delta, d_c(\tau) = (\Sigma^* \backslash f(\tau)) + \Sigma^* \cdot \bigcup_{a\in\Sigma} \delta(\tau, a) \cdot \Sigma^*$;
- for every $a \in \Sigma, d_c(a) = \Sigma^*$

The EDTD D_c accepts $\mathcal{T}_\Sigma \backslash L(D)$ and $|D_c|$=$O(|\Sigma||D|)$

□

Complements of XSDs

Theorem 3.9. Let D be a single-type EDTD.The minimal upper XSD-approximation for the complement of D is unique and can be computed in time polynomial in $|D|$.

Proof.

Let D=$(\Sigma, \Delta, d, S_d, \mu)$ and let E=(Σ, A, f, S'_d) be the DFA-based XSD equivalent to D with A=$(\Delta, \Sigma, \delta, \{q_{init}\})$. Prove in two steps:
Then the minimal upper approximation of D_c can be constructed in polynomial time.

- Determinizing the type automaton of D_c using subset construction can be done in polynomial time
- Type automaton N_c of D_c contains the type automaton A of D as a sub-automaton
- The subset construction result in an automaton in which every state is a state of $\{\tau, a\}$

□

Complements of XSDs

Theorem 3.10.Let D_1 and D_2 be single-type EDTDs.The minimal upper approximation of $L(D_1)\backslash L(D_2)$ can be computed in time polynomial in $|D_1|+|D_2|$

Proof.

Let,for each $i \in \{1,2\}$,$D_i = (\Sigma, \Delta_i, d_i, S_{d_i}, \mu_i)$.Prove the theorem in two steps:

- Construct an EDTD D_c for the language $L(D_1)\backslash L(D_2)$
- Its minimal upper approximation can be constructed in polynomial time

let $A_1 = (\Delta_1 \uplus \{q^1_{init}\}, \Sigma, \delta_1, \{q^1_{init}\})$ be the type automaton of D_1
let $E_2 = (\Sigma, A_2, f_2, S'_{d_2})$ be the DFA-based XSD equivalent to D_2 obtained by the construction in Proposition 2.9. $A_2 = (\Delta_2 \uplus \{q^2_{init}\}, \Sigma, \delta_2, \{q^2_{init}\})$ is the type automaton of E_2
$L(D_2) = L(E_2)$, $t \in L(D_1)\backslash L(D_2)$ iff $t \in L(D_1)\backslash L(E_2)$
Given a tree t,the EDTD D_c for $L(D_1)\backslash L(E_2)$ tests whether $t \in L(D_2)$ and,in parallel, guesses the path towards such a node v and test whether $ch\text{-}str^t(v) \notin f_2(\tau)$

$\square$

Complements of XSDs

Theorem 3.10.Let D_1 and D_2 be single-type EDTDs.The minimal upper approximation of $L(D_1)\backslash L(D_2)$ can be computed in time polynomial in $|D_1|+|D_2|$

Proof.

Use two sets of types Δ_1 and $\Delta_1 \times \Delta_2$. Use the types $\Delta_1 \times \Delta_2$ for the path from root to v,use Δ_1 to type all other nodes. Let $P = \{(\tau_1, \tau_2) \in \Delta_1 \times \Delta_2 | \mu_1(\tau_1) = \mu_2(\tau_2)\}$,define

- $\Delta_c = \Delta_1 \uplus P$
- for every $\tau \in \Delta_1, \mu_c(\tau) = \mu(\tau)$ and ,for every $(\tau_1, \tau_2) \in P, \mu_c((\tau_1, \tau_2)) = \mu_1(\tau_1)$
- $S_{d_c} = (P \cap (S_{d_1} \times S_{d_2})) \uplus \{\tau_1 \in S_{d_1} | \nexists \tau_2 \in S_{d_2}$ with $\mu(\tau_2) = \mu(\tau_1)\}$
- for every $(\tau_1, \tau_2) \in P$, $d_c((\tau_1, \tau_2)) = \{w \in d_1(\tau_1) | \mu_1(w) \notin f_2(\tau_2)\} \cup \{w_1(\tau_1', \tau_2')w_2 | w_1\tau_1'w_2 \in d_1(\tau_1), \mu_1(\tau_1') = \mu_2(\tau_2') = a, \mu_1(w_1\tau_1'w_2) \in f_2(\tau_2), \delta_1(\tau_1, a) = \tau_1'$ and $\delta_2(\tau_2, a) = \tau_2'\}$
- for every $\tau \in \Delta_1, d_c(\tau) = d_1(\tau)$