Well-Supported Semantics for Description Logic Programs

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IJCAI 2011, Barcelona, Spain

Outline

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III. Well-Supported Models

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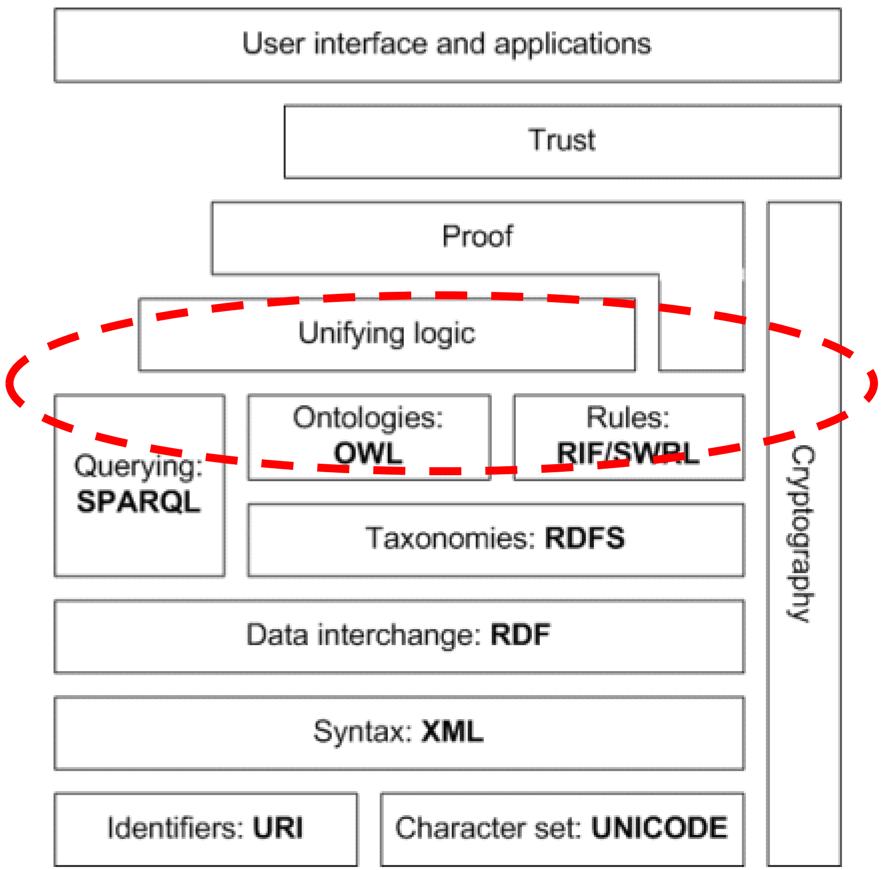
Semantic Web Stack



Integration in the Semantic Web

- **Ontologies** describe terminological knowledge.
- **Rules** model constraints and exceptions over the ontologies.
- They provide complementary descriptions of the same problem domain, so **a unifying logic** is used to
  - integrate the two components, and
  - study the semantic properties of the integrated knowledge base

Three Forms of Integration

- Loose integration
  - Ontologies and rules share no predicate symbols (Eiter et al. 2008, AIJ).
- Tight (or Hybrid) integration
  - Ontologies and rules share some predicate symbols (Rosati 2006, KR; Lukasiewicz 2010, TKDE).
- Full integration
  - Ontologies and rules share the same vocabulary (de Bruijn et al. 2008, KR; Motik and Rosati 2010, JACM).

DL-Programs

- We consider a loose integration, called Description logic programs (or DL-programs) (Eiter et al. 2008, AIJ)
- A DL-program is $KB = (L, R)$
  - L: a DL knowledge base (ontologies).
  - R: an extended logic program under the answer set semantics.

Semantic Issues with DL-Programs

- Weak answer set semantics (Eiter et al. 2008, AIJ)
  - The authors noted that an obvious disadvantage of the semantics is that it may produce counterintuitive answer sets with **circular justifications** by self-supporting loops.
- Strong answer set semantics (Eiter et al. 2008, AIJ)
  - We observed that the problem of **circular justifications** persists in this semantics.
- FLP answer set semantics (Eiter et al. 2005, IJCAI)
  - We observed that the problem of **circular justifications** persists in this semantics.

Semantic Issues with DL-Programs

- Therefore, it presents an interesting yet challenging open problem to develop a new semantics for DL-programs, which produces answer sets **free of circular justifications**.

Circular Justifications

- A model I of a logic program R is circularly justified if the truth of some $a \in I$ is supported by itself in I.
- **Examples**

1. Consider a logic program $R = \{a \leftarrow b.\ b \leftarrow a\}$ and let $I = \{a, b\}$. $a \in I$ is circularly justified by a self-supporting loop: $\boldsymbol{a \Leftarrow b \Leftarrow a}$
2. Consider a DL-program $KB = (L, R)$ from (Eiter et al. 2008, AIJ), where $L = \emptyset$ and $R = \{p(a) \leftarrow DL[c \uplus p; c](a)\}$. Let $I = \{p(a)\}$. $p(a) \in I$ is circularly justified by a self-supporting loop:

$$\boldsymbol{p(a) \Leftarrow DL[c \uplus p; c](a) \Leftarrow p(a)}$$

Fages' Well-Supportedness Condition

- For normal logic programs, the problem of circular justifications is elegantly handled by Fages' well-supportedness condition (Fages 1994, JMLCS).
- It defines a level mapping, which prevents well-supported models from circular justifications.
- It is **a key property** to characterize the standard answer set semantics (Gelfond and Lifschitz 1991, NJC) :
  - A model of a normal logic program is an answer set under the standard answer set semantics iff it is well-supported (Fages 1994, JMLCS).

Fages' Well-Supportedness Condition

- Can we extend Fages' well-supportedness condition from normal logic programs to DL-programs to overcome circular justifications?
- Our answer is Yes.

Our Contributions

- We solve the semantic problem of circular justifications with DL-programs by
  - extending Fages' well-supportedness condition from normal logic programs to DL-programs, and
  - defining a **well-supported semantics** for DL-programs, which produces answer sets free of circular justifications.

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Notation

- A DL-program is $KB = (L, R)$
- L: a DL knowledge base built over $\Sigma_L = (\mathbf{A} \cup \mathbf{R}, \mathbf{I})$
  - **A, R, I:** atomic concepts, atomic roles, and individuals.
- R: a rule base built over $\Sigma_R = (\boldsymbol{P}, \boldsymbol{C})$
  - **P, C:** predicate symbols, and constants
  - $\mathbf{P} \cap (\mathbf{A} \cup \mathbf{R}) = \emptyset,$ and $\mathbf{C} \subseteq \mathbf{I}$
  - HB_R: Herbrand base of R built over Σ_R
- ground(R): ground instances (relative to HB_R) of all rules in R

Notation

- R consists of rules of the form

$$H \leftarrow A_1, \cdots, Am, not\ B_1, \cdots, not\ B_n$$

where H is an atom, and each A_i and B_i are atoms or dl-atoms

- A dl-atom is an interface between L and R:

$$DL[S_1\ op_1\ p_1, \cdots, S_m\ op_m\ p_m; Q](\boldsymbol{t})$$

  - each S_i is a concept or role built from $\mathbf{A} \cup \mathbf{R}$, each $p_i \in \boldsymbol{P}$ is a predicate symbol, $Q(\boldsymbol{t})$ is a dl-query and $op_i \in \{ \uplus, \cup\!\!\!\!-, \cap\!\!\!\!- \}$

Satisfaction Relation $\models_L$

Definition (Eiter et al. 2008, AIJ) Let $KB = (L, R)$ and I be an interpretation. Define *satisfaction under L*, denoted $\models_L$, as follows:

1. For a ground atom $a \in HB_R, I \models_L a$ if $a \in I$.
2. For a ground dl-atom $A = DL[S_1\, op_1\, p_1, \cdots, S_m\, op_m\, p_m; Q](\boldsymbol{t})$, $I \models_L A$ if $L \cup \bigcup_{i=1}^{m} A_i \models Q(\boldsymbol{t})$, where

$$A_i = \begin{cases} \{S_i(\mathrm{e}) \mid p_i(\mathrm{e}) \in I\}, & \text{if } op_i = \uplus; \\ \{\neg S_i(\mathrm{e}) \mid p_i(\mathrm{e}) \in I\}, & \text{if } op_i = \cupdot; \\ \{\neg S_i(\mathrm{e}) \mid p_i(\mathrm{e}) \notin I\}, & \text{if } op_i = \Cap. \end{cases}$$

*** Any $I \subseteq HBR$ is an interpretation of $KB = (L, R)$. Let $I^- = HB_R \setminus I$ and $\neg I^- = \{\neg a | a \in I^-\}$

Program Transformation Reducts

- Given an interpretation I, FLP reduct $\boldsymbol{fR_L^I}$ is obtained from ground(R) by

  deleting every rule *r* with $I \not\models_L body(r)$.
- Weak transformation reduct $\boldsymbol{wR_L^I}$ is obtained from $\boldsymbol{fR_L^I}$ by

  deleting all negative literals and all dl-atoms.
- Strong transformation reduct $\boldsymbol{sR_L^I}$ is obtained from $\boldsymbol{fR_L^I}$ by

  deleting all negative literals and all nonmonotonic dl-atoms.

*** A ground dl-atom A is monotonic

if for any $I \subseteq J \subseteq HB_R$, $I \models_L A$ implies $J \models_L A$.

Three Semantics of DL-Programs

- Weak/strong/FLP answer set semantics

  A model I of $KB = (L, R)$ is a weak (resp. strong and FLP) answer set if I is a minimal model of $\boldsymbol{wR_L^I}$ (resp. $\boldsymbol{sR_L^I}$ and $\boldsymbol{fR_L^I}$) (Eiter et al. 2008, AIJ; Eiter et al. 2005, IJCAI).

- FLP answer sets are minimal models, but weak/strong answer sets may not.

Circular Justification Problem

- The three answer set semantics suffer from the problem of **circular justifications**.

- **Example** Consider a DL-program $KB = (L, R)$, where $L = \emptyset$ and

$$R: \quad p(a) \leftarrow q(a)$$
$$q(a) \leftarrow DL[c \uplus p, b \cap\!\!\!- q; c \sqcup \neg b](a)$$

$I = \{p(a), q(b)\}$ is the only model of KB. It is also a weak, a strong, and an FLP answer set. $p(a) \in I$ is circularly justified by a self-supporting loop:

$$p(a) \Leftarrow q(a) \Leftarrow DL[c \uplus p, b \cap\!\!\!- q; c \sqcup \neg b](a) \Leftarrow p(a) \vee \neg q(a) \Leftarrow p(a)$$

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Fages' Well-Supportedness

- Fages' well-supportedness condition (Fages 1994, JMLCS):

A model *I* of a normal logic program is well-supported if there is a level mapping on *I* such that for every $a \in I$, there is a rule

$$a \leftarrow A_1, \cdots, Am, not\ B_1, \cdots, not\ Bn$$

where *I* satisfies the rule body and the level of each A_i is below the level of a.

- This well-supportedness condition does not apply to DL-programs, due to occurrences of dl-atoms.

up to Satisfaction $(E, I) \vDash_L A$

- To handle dl-atoms, we introduce up to satisfaction.
- Informally, for $E \subseteq I \subseteq HB_R$,

  $(E, I) \vDash_L \alpha$ if for every F with $E \subseteq F \subseteq I, F \vDash_L \alpha$.
- $(E, I) \vDash_L \alpha$ implies that the truth of α depends only on E and I^-, and is independent of $I \backslash \mathrm{E}$.
- For instance, if $E = \{a\}$, $I = \{a, b, c\}$ and $\alpha = a \wedge \neg d$, then for every F with $E \subseteq F \subseteq I, F \vDash_L \alpha$. Therefore, $(E, I) \vDash_L \alpha$.

up to Satisfaction $(E, I) \vDash_L A$

Definition Let $KB = (L, R)$ and $E \subseteq I \subseteq HB_R$. For any ground literal A, define $E\ up\ to\ I\ satisfies\ A\ under\ L$, denoted $(E, I) \vDash_L A$, as follows:

1. For a ground atom $a \in HB_R$,

   $(E, I) \vDash_L a$ if $a \in E$; $(E, I) \vDash_L not\ a$ if $a \notin I$.

2. For a ground dl-atom A,

   $(E, I) \vDash_L A$ if for every F with $E \subseteq F \subseteq I, F \vDash_L A$;

   $(E, I) \vDash_L not\ A$ if for no F with $E \subseteq F \subseteq I,\ F \vDash_L A$.

Monotonicity of $(E, I) \models_L A$

- **Proposition** Let A be a ground atom or dl-atom. For any $E_1 \subseteq E_2 \subseteq I$,
  - if $(E_1,\ I) \models_L A$ then $(E_2,\ I) \models_L A$;
  - and if $(E_1,\ I) \models_L \mathit{not}\ A$ then $(E_2,\ I) \models_L \mathit{not}\ A$.

- We use this up to satisfaction to extend Fages' well-supportedness condition and define well-supported models for DL-programs.

Well-Supported Models

- Informally, a model *I* of a DL-program is strongly well-supported if there is a level mapping on *I* such that for every $a \in I$, there is $E \subset I$ and a rule $a \leftarrow body(r)$, where $(E, I) \models_L body(r)$ and the level of each element in E is below the level of a.
- Put another way,
  - $a \in I$ is supported by $body(r)$,
  - while the truth of $body(r)$ is determined by E and I^-,
  - where no $b \in E$ is circularly dependent on *a*.
- This guarantees that strongly well-supported models are free of circular justifications.

Well-Supported Models

Definition A model *I* of a DL-program $KB = (L, R)$ is strongly well-supported if there exists a strict well-founded partial order $\prec$ on *I* such that for every $a \in I$, there is $E \subset I$ and a rule $a \leftarrow body(r)$ in $ground(R)$ such that

$(E, I) \vDash_L body(r)$ and for every $b \in E$, $b \prec a$.

Well-Supported Models

Example Consider a DL-program $KB = (L, R)$, where $L = \emptyset$ and

$$R:\quad p(a) \leftarrow q(a)$$

$$q(a) \leftarrow DL[c \uplus p, b \cap\!\!\!\!- q; c \sqcup \neg b](a)$$

$I = \{p(a), q(b)\}$ is the only model of KB. It is also a weak, a strong, and an FLP answer set. However, I is not a strongly well-supported model, since for $p(a) \in I$ there is no $E \subset I$ satisfying the well-supportedness condition.

Well-Supported Models

- **Theorem** Let $KB = (L, R)$ be a DL-program, where $L = \emptyset$ and R is a normal logic program. A model I is a strongly well-supported model of KB iff I is a well-supported model of R under Fages' definition.

- As a result, Fages' well-supportedness condition is extended to DL-programs.

Outline

Consequence Operator $T_{KB}(E, I)$

- **Definition** Let $KB = (L, R)$ and $E \subseteq I \subseteq HB_R$. Define

  $T_{KB}(E, I) = \{a | a \leftarrow body(r) \in ground(R)$ and $(E, I) \vDash_L body(r)\}$

- Monotonicity property of $T_{KB}(E, I)$

Theorem Let I be a model of KB. For any $E_1 \subseteq E_2 \subseteq I$,

$T_{KB}(E_1,\ I) \subseteq T_{KB}(E_2,\ I) \subseteq I$.

Fixpoint $T_{KB}^{\alpha}(\emptyset, I)$

- $T_{KB}^{\alpha}(\emptyset, I)$: a fixpoint from the monotone sequence $\langle T_{KB}^{i}(\emptyset, I)\rangle_{i=0}^{\infty}$ with $T_{KB}^{0}(\emptyset, I) = \emptyset$ and $T_{KB}^{i+1}(\emptyset, I) = T_{KB}\left(T_{KB}^{i}(\emptyset, I), I\right)$

- **Theorem**_ Let I be a model of $KB = (L, R)$. If $I = T_{KB}^{\alpha}(\emptyset, I)$ then I is a minimal model of KB.

Well-Supported Semantics

- **Definition** Let I be a model of a DL-program $KB = (L, R)$. I is an answer set of KB if $I = T^{\alpha}_{KB}(\emptyset, I)$.

- Answer sets are exactly strongly well-supported models

Theorem I is an answer set of KB iff I is a strongly well-supported model of KB.

- Therefore, we call such answer sets well-supported answer sets, which are **free of circular justifications.**

Well-Supported Semantics

Theorem If I is a well-supported answer set of KB, then

1. I is a minimal model of KB.
2. I is a strong answer set of KB that is also a weak answer set of KB.
3. I is an FLP answer set of KB.

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Related Work

1. Weak answer set semantics (Eiter et al. 2008, AIJ)
   - There are circular justifications by self-supporting loops.
2. Strong answer set semantics (Eiter et al. 2008, AIJ)
   - The problem of circular justifications persists.
3. FLP answer set semantics (Eiter et al. 2005, IJCAI)
   - Weak/strong answer sets may not be minimal models.
   - FLP answer sets are minimal models.
   - The problem of circular justifications persists.
4. Loop formula based semantics (Wang et al. 2010, TPLP)
   - The problem of circular justifications persists.

Related Work

- FLP answer set semantics is based on FLP-reduct, a concept introduced in (Faber et al. 2004, JELIA) to define answer set semantics for logic programs with aggregates.
- Our up to satisfaction relation is inspired by conditional satisfaction, a concept introduced in (Son et al. 2007, JAIR) to define answer set semantics for logic programs with aggregates.
- DL-programs and logic programs with aggregates are closely related. Exploiting the deep connection presents an interesting future work.

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Summary and Future Work

- **Summary:**

To resolve the semantic problem of circular justifications with DL-programs, we

- extended Fages' well-supportedness condition from normal logic programs to DL-programs, and
- presented a well-supported semantics for DL-programs, which produces answer sets free of circular justifications.

Summary and Future Work

- **Future work:**
  - Extend the work to DL-programs with disjunctive rule heads.
  - Study the complexity properties.
  - Exploit the connection between DL-programs and logic programs with aggregates.

Thanks !

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