Bisimulation and Logic
Lecture 6

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Summer School on Model Checking
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- Equivalence checking: is state s equivalent to t ?
- Mostly computing dyadic fixed points e.g. bisimulations to solve it. May need algebraic/combinatorial properties of reachability sets/generators of graph

Active research goal: transfer these techniques to

finite/infinite state systems with binding

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3. $\vdots \quad \vdots \quad \vdots$
4. Application of tree automata to higher-order matching
   [Comon + Jurski 1997, Stirling 2005-9]

Higher Order Schemes

Base type 0: finite/infinite trees with nodes labelled by elements of $\{f_1, \ldots, f_k\}$. Each f_i has an arity ≥ 0.
Scheme is a finite family

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Interpretation of a scheme: tree generated by S

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$$Fbb \quad \rightarrow \quad \begin{tikzcd} & f \arrow[dl, no head] \arrow[dr, no head] & \\ Fbh(b) & & b \end{tikzcd}$$

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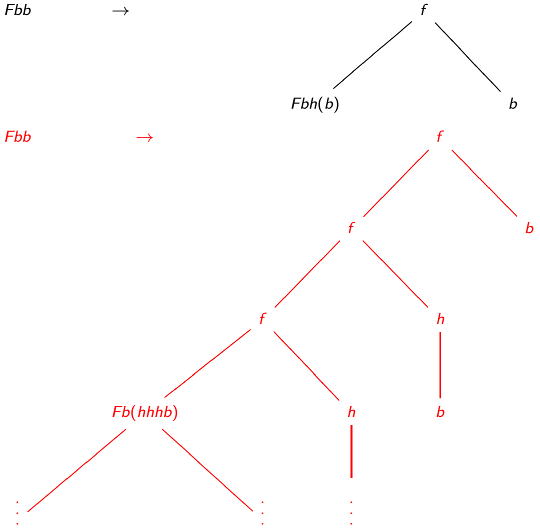

Example: second-order

$$\begin{array}{lcl} F x_1 x_2 x_3 & \stackrel{\text{def}}{=} & f\,(F(G x_1)(H x_2) x_3)\, x_1 (x_2 x_3) \\ G y_1 y_2 & \stackrel{\text{def}}{=} & g(y_1(y_2)) \\ H z_1 z_2 & \stackrel{\text{def}}{=} & h(z_1(z_2)) \\ F g h a & & \text{Start} \end{array}$$

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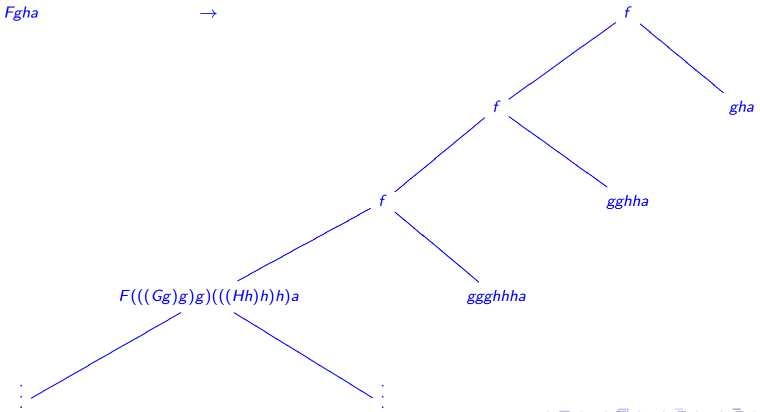

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Proof uses geometry of interaction on infinite λ-terms + higher-order pushdown automata
Extended to all schemes
[Ong 2006, Hague + Murawski + Ong +Serre, 2008]
Proof uses game semantics on infinite lambda terms. Later paper also uses extended higher-order automata

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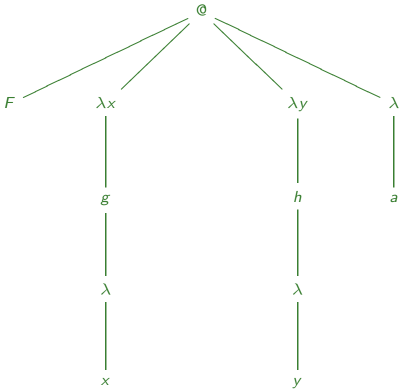
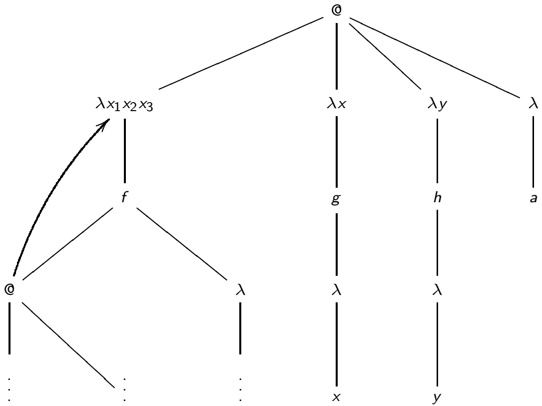


$$F = \lambda x_1 x_2 x_3 . f(@ \ldots)\lambda . x_1(\lambda . x_2 . (\lambda . x_3))$$

Infinite λ-term (can be folded into a finite tree with backedges)

infinite λ-tree



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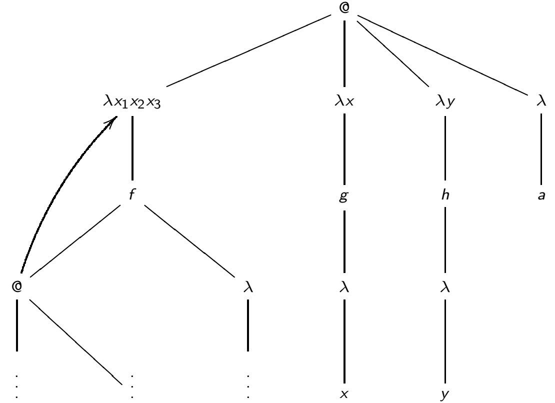

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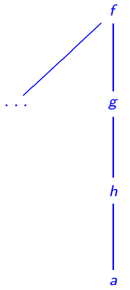

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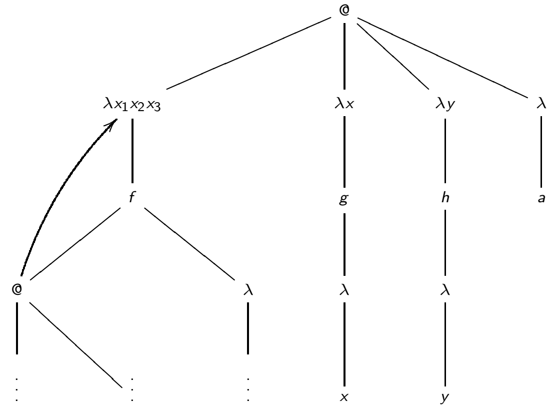

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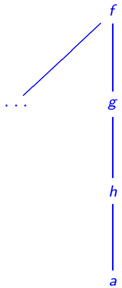

transform parity tree automaton on above tree into one on the infinite λ-tree

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