Rewarding Probabilistic Hybrid Automata

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joint work with Holger Hermanns

- real-world systems: combination of **digital** controller and **continuous** environment

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- need to formally analyse such **hybrid** systems

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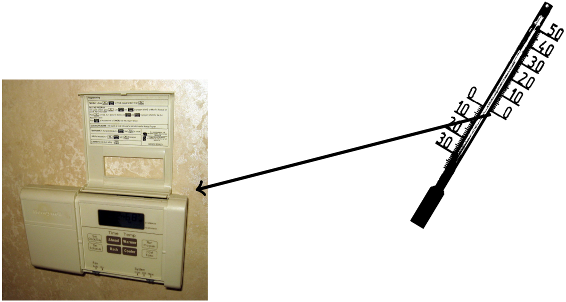
$\longrightarrow$ initial condition $\longrightarrow$ error

here: temperature equal to or above 37° celsius

Probabilities

- probabilistic behaviour in system



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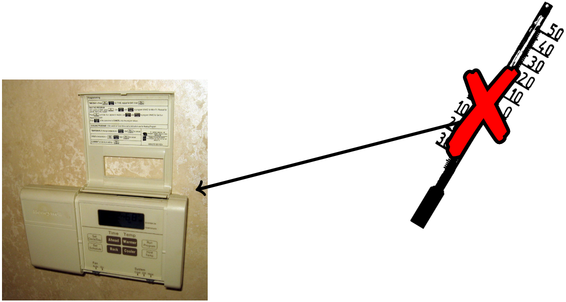

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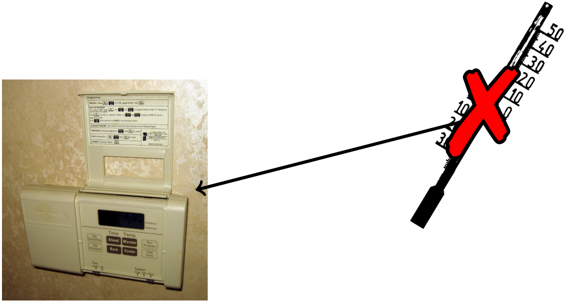

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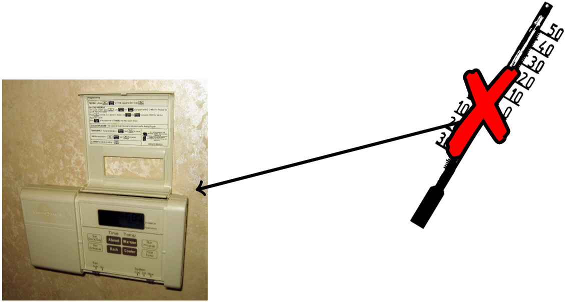

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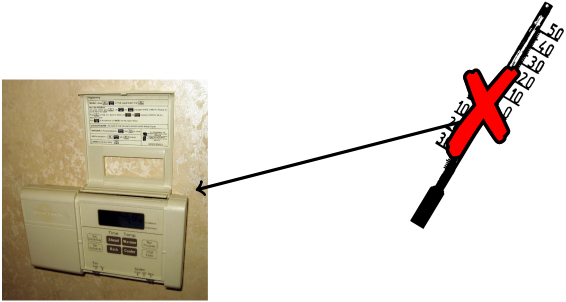

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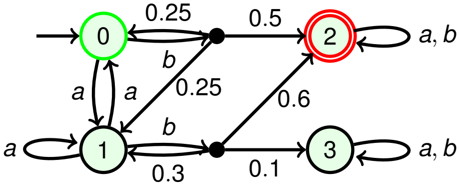
- want **quantitative** bounds on system behaviour
  e.g. "max prob to go above 37° within 20 years: $\leq 10^{-40}$"
- must integrate probabilistic behaviour in system model

Analysis of Stochastic Hybrid Systems

- apply **model checking** to stochastic hybrid systems
  explore all states of the model
  combine with property
  apply analysis method to analyse state-transition system



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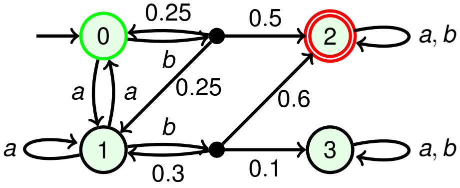

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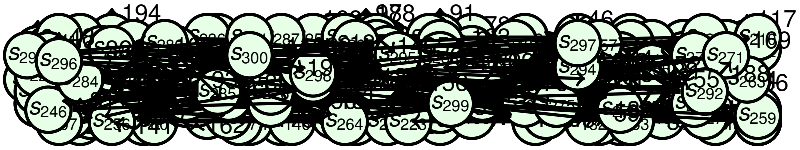

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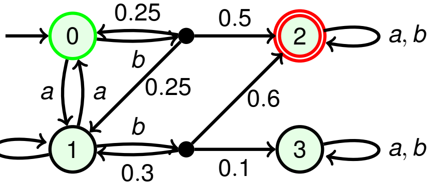

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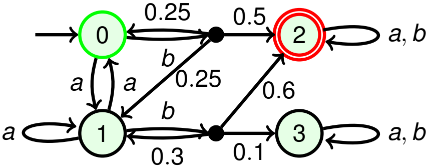

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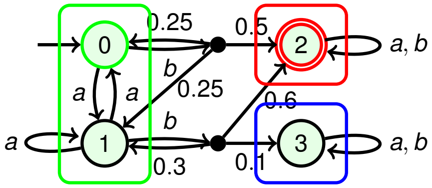

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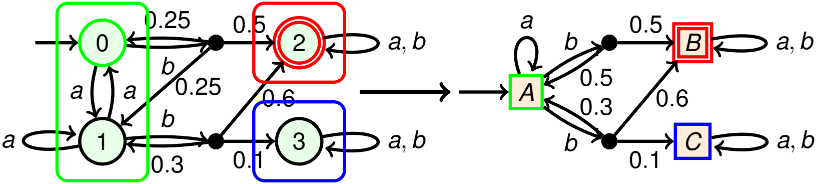

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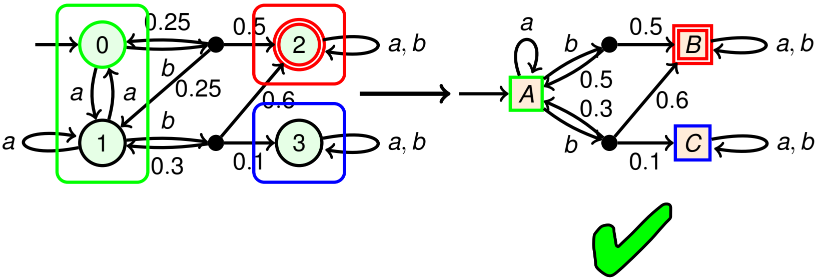

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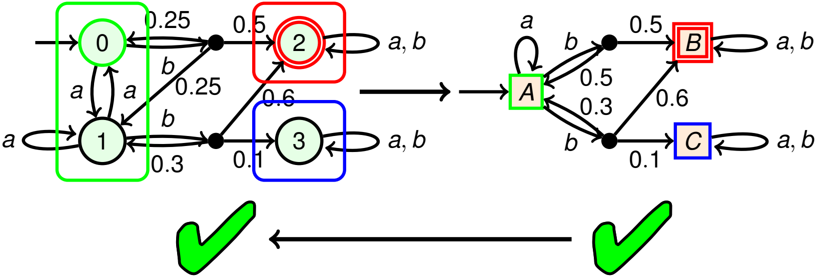

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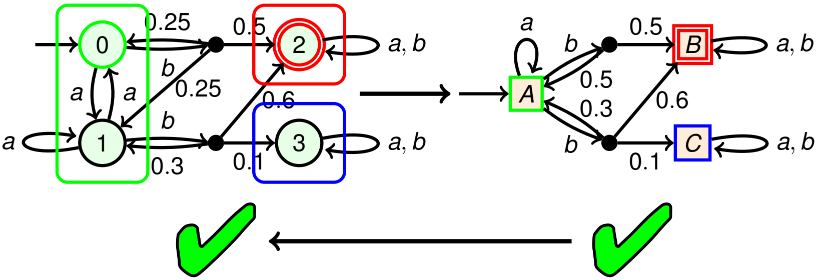

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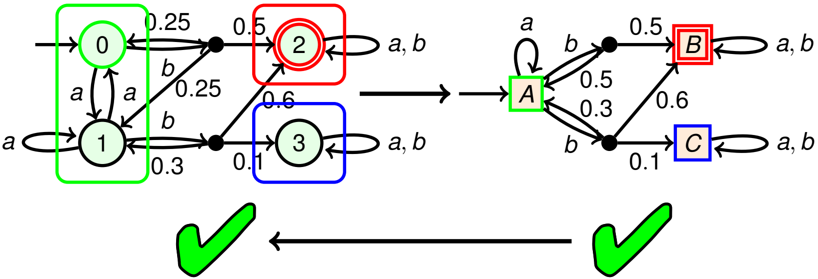

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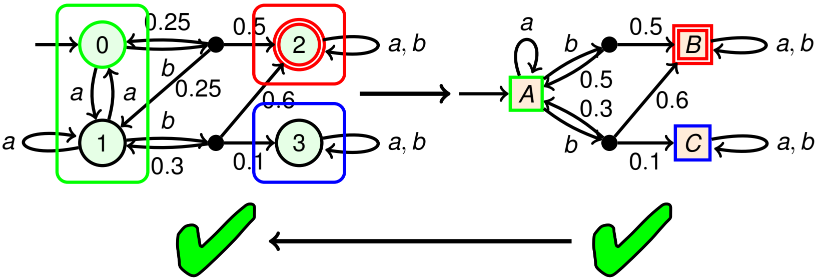

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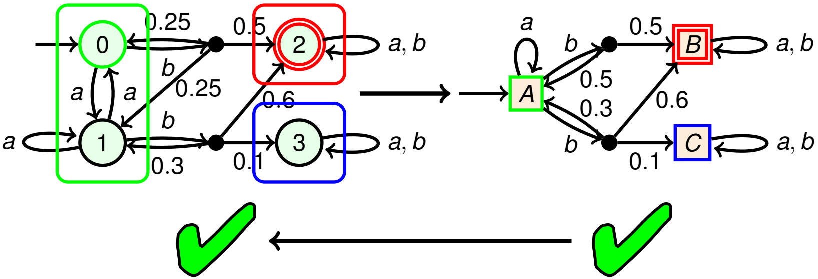

- how to construct such a finite model?
- correctness?
- which models can we handle?
- and which properties?

Contribution

- generic framework for general stochastic hybrid systems

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$\downarrow$

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- builds on classical hybrid solvers (important research area)

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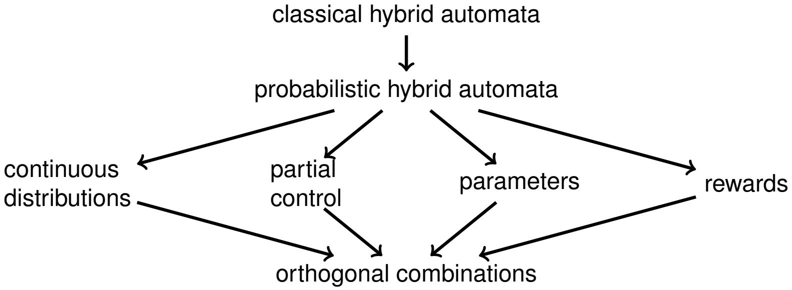


- builds on classical hybrid solvers (important research area)
- applicable to wide area of models and properties

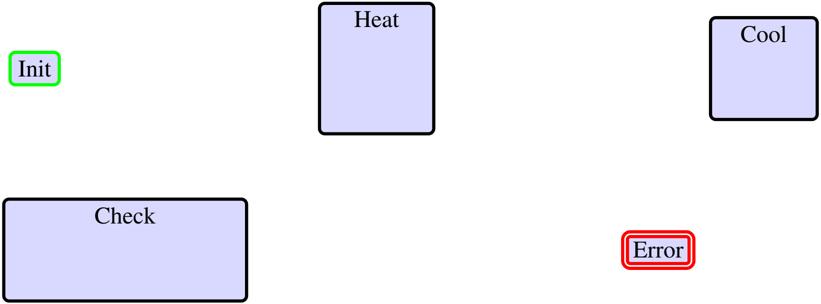
Classical Hybrid Automata

Hybrid Automata (HA)

$\mathcal{H} = (M, \overline{m} \qquad\qquad\qquad\qquad)$

[MalerMP91]

- M: finite set of **modes**
- $\overline{m}$: **initial mode**



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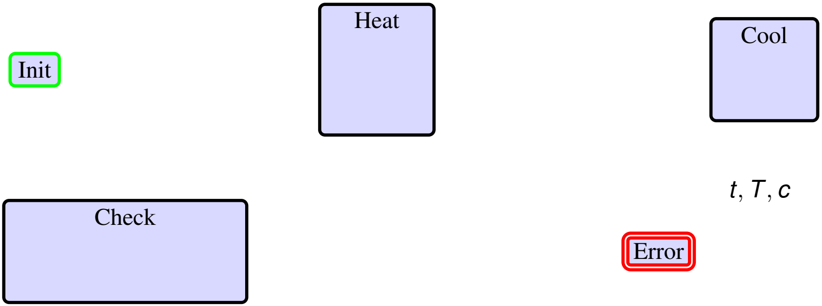

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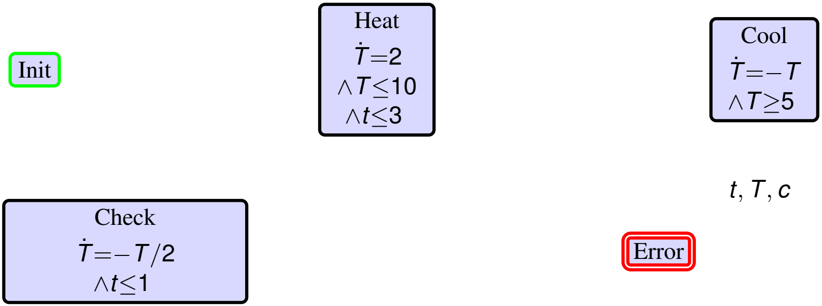

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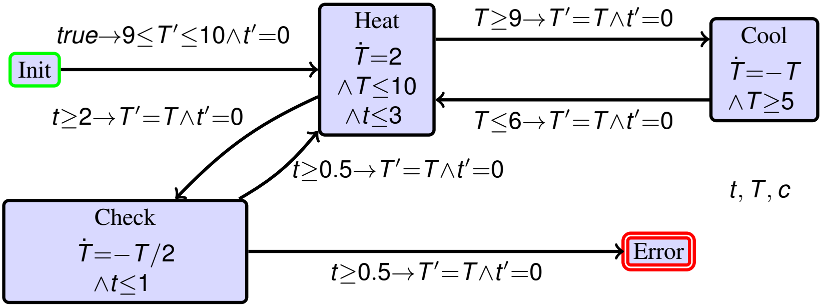
- $Cmds$: finite set of **guarded commands** $g \to u$

  g: **guard**

  u: **update function**



Labelled Transition Systems (LTS)

$\mathcal{M} = (S, \overline{s} \qquad\quad)$

[Keller76]

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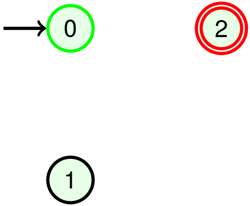

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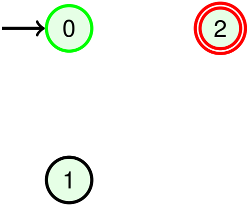

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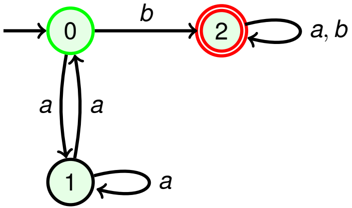

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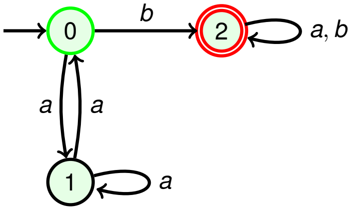
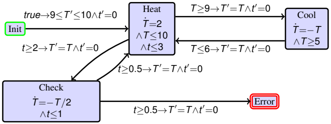


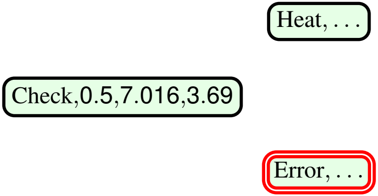
- **path**: state-action sequence legal by $\mathcal{T}$
  e.g. $0 \rightarrow a \rightarrow 1 \rightarrow a \rightarrow 1 \rightarrow a \rightarrow 0 \rightarrow b \rightarrow 2 \rightarrow \ldots$

Semantics of Hybrid Automata

LTS $[\![\mathcal{H}]\!] = (S, \overline{s} \qquad\qquad)$

- $S = M \times \mathbb{R}^k$
- $\overline{s} = (\overline{m}, 0, \ldots, 0)$





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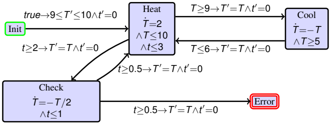

Heat, . . .

Check,0.5,7.016,3.69

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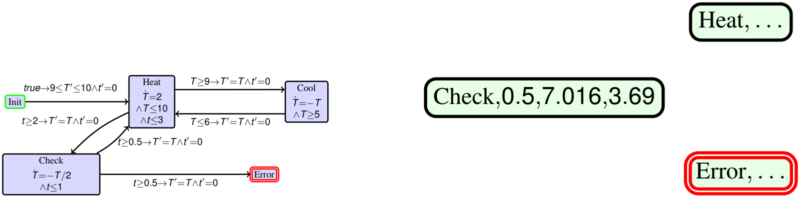

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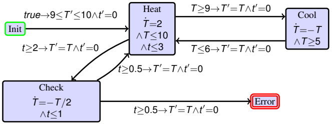

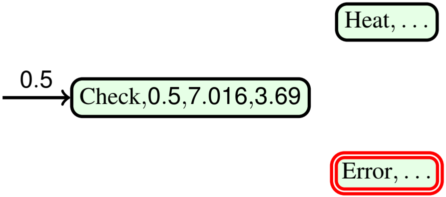

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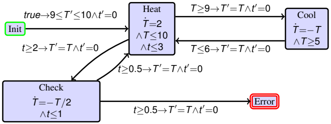
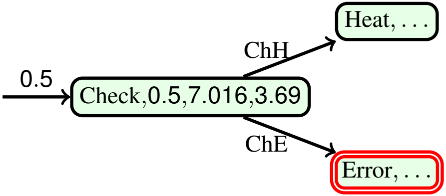
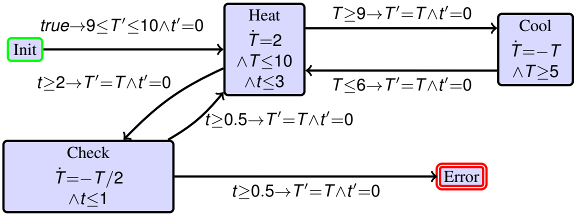
  from time t by $Post_m(s, \mathrm{t})$
  from command $g \to u$ by $u(s)$ if g fulfilled





Reachability

Given $\mathcal{H}$, does there exist a path reaching an unsafe mode?



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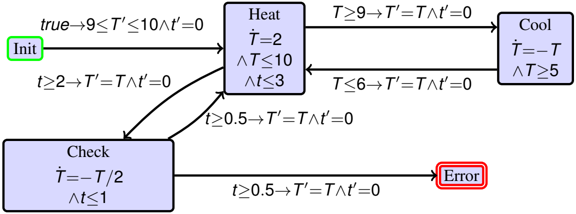
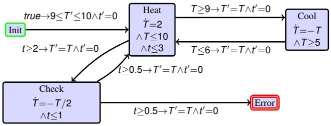


$(\text{Init}, 0, 0, 0) \rightarrow \text{IH} \rightarrow (\text{Heat}, 0, 9, 0) \rightarrow 0.5 \rightarrow (\text{Heat}, 0.5, 10, 0.5) \rightarrow \text{HCo} \rightarrow (\text{Cool}, 0, 10, 0.5) \rightarrow 0.69 \rightarrow (\text{Cool}, 0.69, 5.016, 1.19) \rightarrow \text{CoH} \rightarrow (\text{Heat}, 0, 5.016, 1.19) \rightarrow 2 \rightarrow (\text{Heat}, 2, 9.016, 3.19) \rightarrow \text{HCh} \rightarrow (\text{Check}, 0, 9.016, 3.19) \rightarrow 0.5 \rightarrow (\text{Check}, 0.5, 7.016, 3.69) \rightarrow \text{ChE} \rightarrow (\text{Error}, 0, 7.016, 3.69)$

Abstraction of Hybrid Automata

LTS $\mathcal{M} = (\mathbf{A}, \bar{\mathbf{z}} \qquad\qquad\qquad)$

- **A**: **covering**
- $\bar{\mathbf{z}}$: contains initial state



$\mathbf{z}_0$ Init

$\mathbf{z}_1$ Heat: $t{\geq}0, c{\geq}0, t{\leq}c, T{\leq}10$

$\mathbf{z}_2$ Check: $t{\geq}0, c{\geq}2, t{\leq}c{-}2, T{\leq}10$

$\mathbf{z}_3$ Error: $c{\leq}5$

$\mathbf{z}_4$ Cool: $t{\geq}0, c{\geq}0, t{\leq}c, T{\leq}10$

$\mathbf{z}_5$ Heat: $t{\geq}0, c{\geq}2.5, t{\leq}c{-}2.5, T{\leq}10$

$\mathbf{z}_6$ Check: $t{\geq}0, c{\geq}4.5, t{\leq}c{-}4.5, T{\leq}10$

$\mathbf{z}_7$ Heat: $t{\geq}0, c{\geq}0, t{\leq}c{-}5, T{\leq}10$

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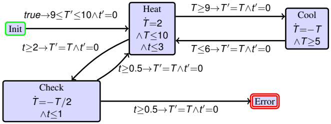
- τ: abstract timed action
- *Cmds*: commands of $\mathcal{H}$



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$\mathbf{z}_5$ Heat: $t \geq 0, c \geq 2.5, t \leq c-2.5, T \leq 10$

$\mathbf{z}_6$ Check: $t \geq 0, c \geq 4.5, t \leq c-4.5, T \leq 10$

$\mathbf{z}_7$ Heat: $t \geq 0, c \geq 0, t \leq c-5, T \leq 10$

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LTS $\mathcal{M} = (\mathbf{A}, \bar{\mathbf{z}}, \{\tau\} \uplus \mathit{Cmds}, \mathcal{T}_{\text{abs}})$

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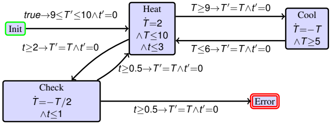

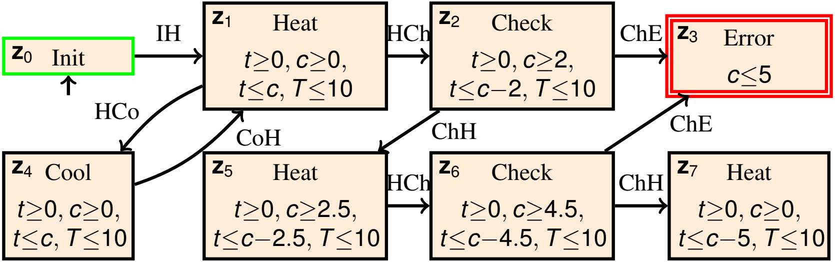

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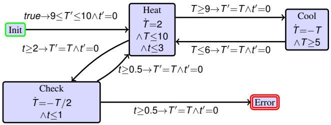

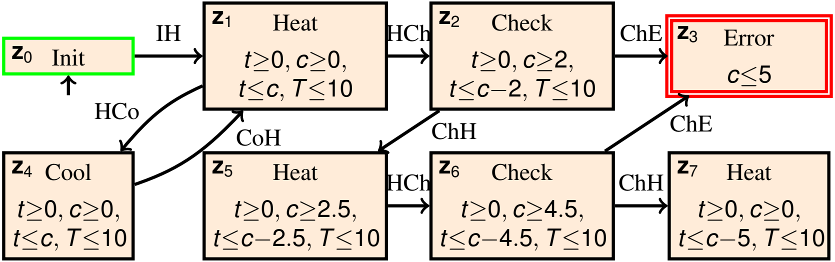

- wide tool support exists (HSolver, PHAVer, SpaceEx, etc.)

Correctness of HA Abstraction

- follows from **simulation relation** semantics $\rightarrow$ abstraction [Milner71]

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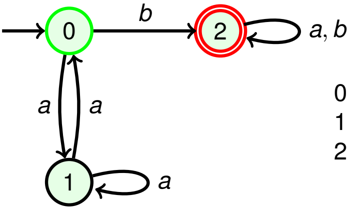

$0 \; R \; A$
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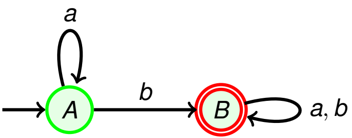

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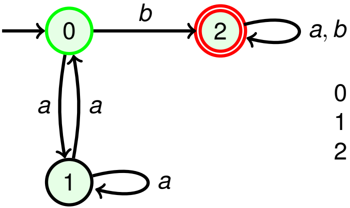

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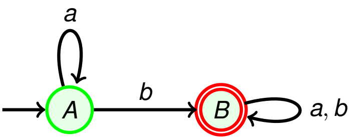


- maintains safeness

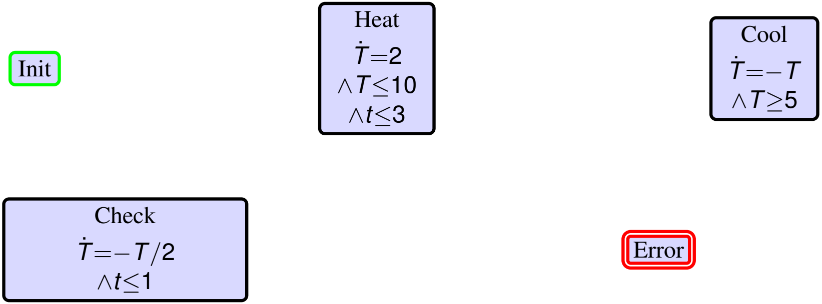
Analysis of Stochastic Hybrid Systems

Probabilistic Hybrid Automata (PHA)

$\mathcal{H} = (M, \overline{m}, k, \langle Post_m \rangle_{m \in M} \qquad)$

- M: finite set of modes
- $\overline{m}$: initial mode
- k: dimension of the automaton
- $Post_m$: timed behaviour

[Sproston00]



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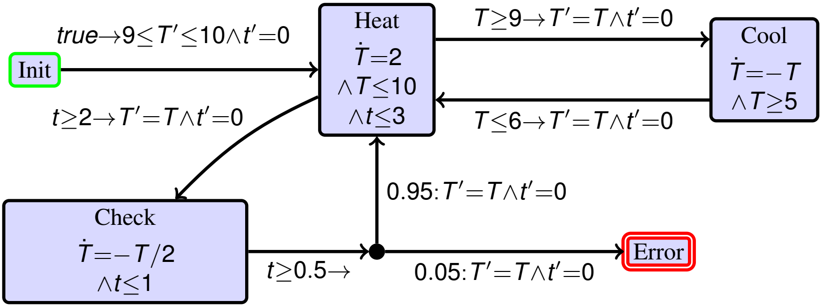
- $Post_m$: timed behaviour
- $Cmds$: finite set of **probabilistic** guarded commands

[Sproston00]

$$g \to p_1{:}u_1 + \ldots + u_n{:}u_n$$

g: guard

u_i: updates p_i: **probabilities**



Probabilistic Automata (PA)

$\mathcal{M} = (S, \overline{s}, Act \quad)$

- S: set of states
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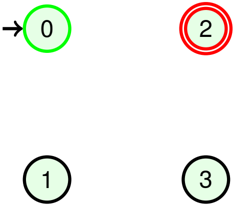
- Act: actions

[SegalaL95]



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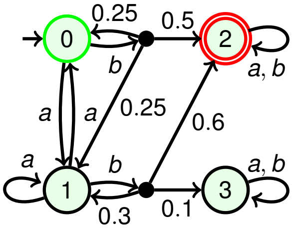

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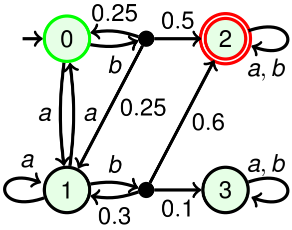

- **path**: sequence state-action-**distribution** e.g.
  $0 \rightarrow b \rightarrow [0 \mapsto 0.25, 1 \mapsto 0.25, 2 \mapsto 0.5] \rightarrow 2 \rightarrow a \rightarrow [2 \mapsto 1] \ldots$

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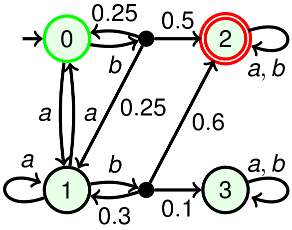

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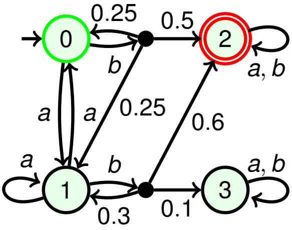

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- induces measure $Pr_{\mathcal{M},\sigma}$ on sets of paths

Semantics of Probabilistic Hybrid Automata

LTS $[\![\mathcal{H}]\!] = (S, \overline{s}, Act \quad)$

- $S = M \times \mathbb{R}^k$
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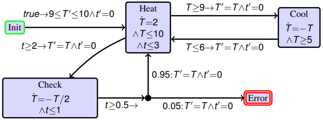

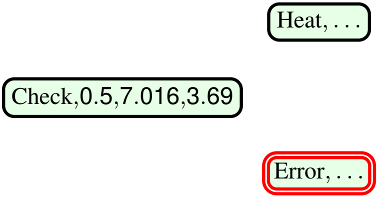

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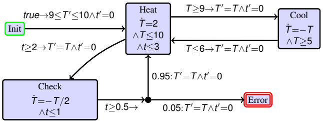
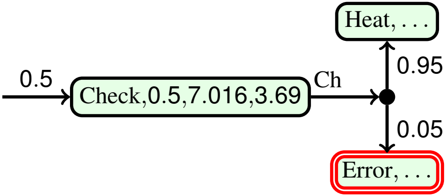
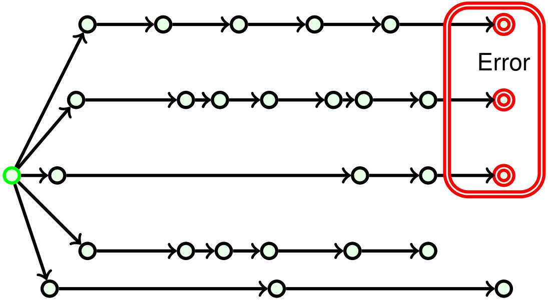
- $Act = \mathbb{R}_{\geq 0} \uplus Cmds$
- $\mathcal{T}$: for $s = \in S$ have transitions
  from command $g \to p_1{:}u_1 + \ldots + u_n{:}u_n$ by $u(s)$ if g fulfilled
  from time t by $Post_m(s, \mathrm{t})$





Probabilistic Reachability

- consideration of single path insufficient



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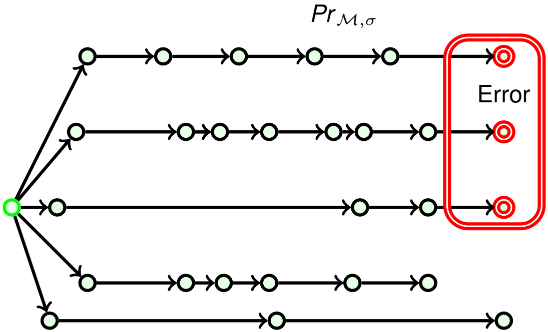

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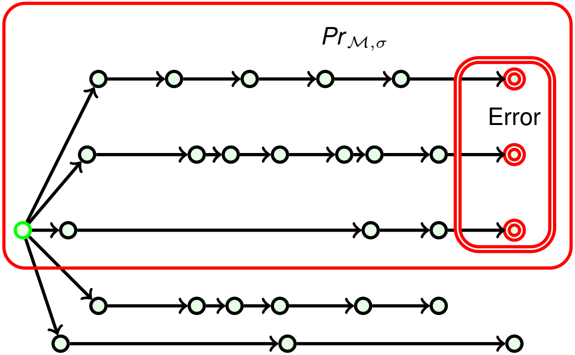

Probabilistic Reachability

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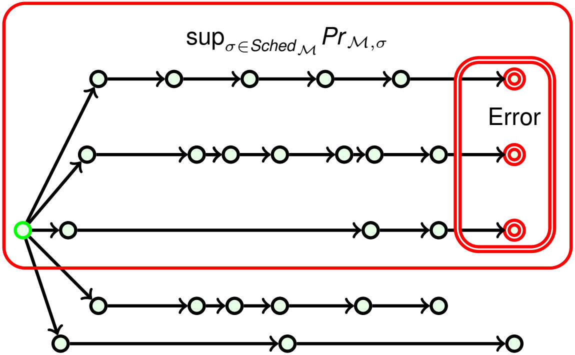
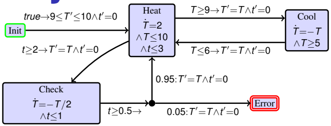
- want probability of paths reaching bad state
- interested in worst case, thus supremum over schedulers

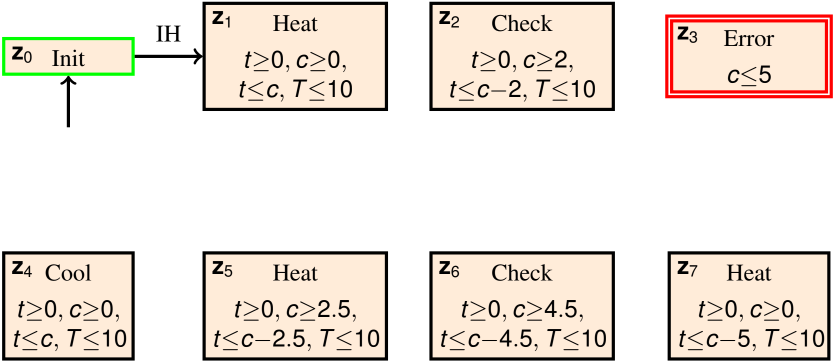


Abstraction of Probabilistic Hybrid Automata



PA $\mathcal{M} = (\mathbf{A}, \bar{\mathbf{z}}, \{\tau\} \uplus \mathit{Cmds}, \mathcal{T}_{\mathrm{abs}})$

- $\mathbf{A}, \bar{\mathbf{z}}, \tau, \mathit{Cmds}$: as before



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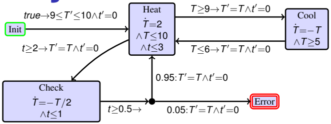

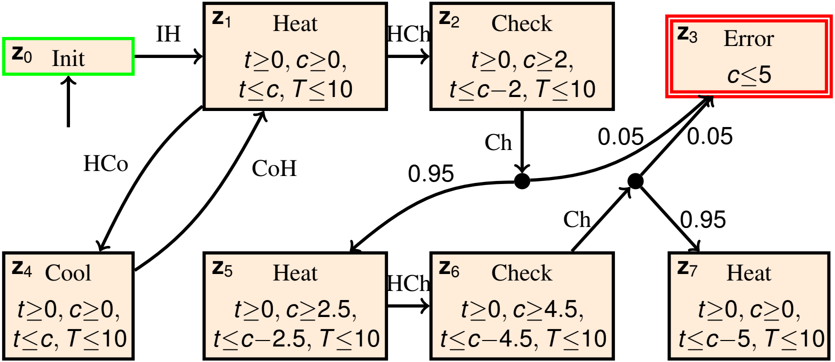

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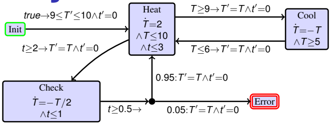
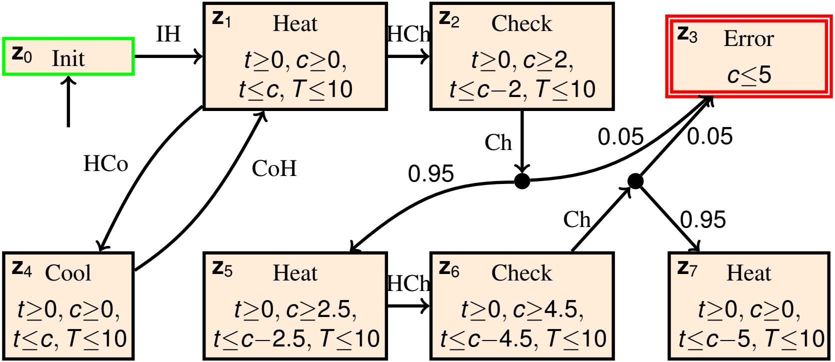
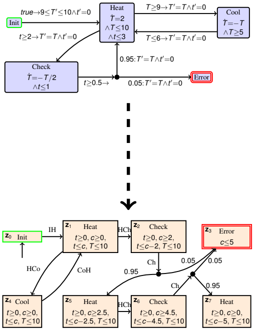




- how to obtain such an abstraction?

Constructing Abstractions of PHAs

- consider probabilistic hybrid automaton $\mathcal{H}$



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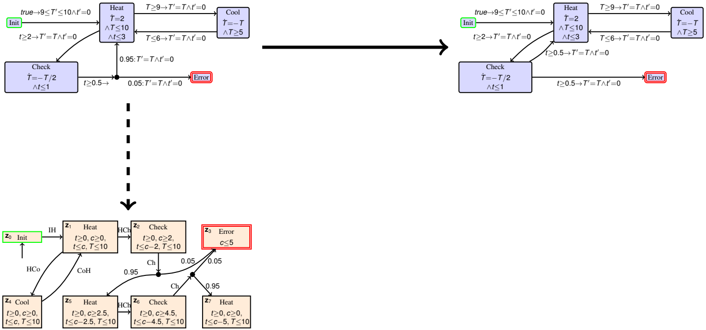

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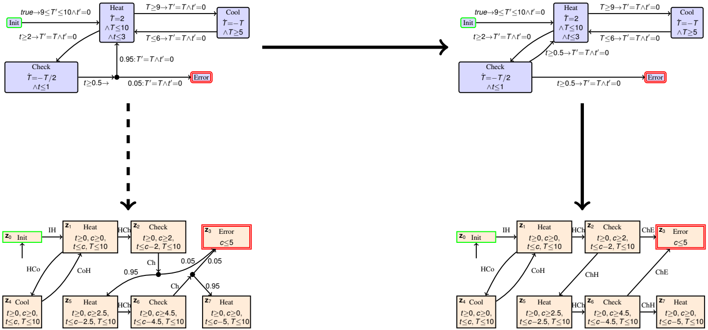

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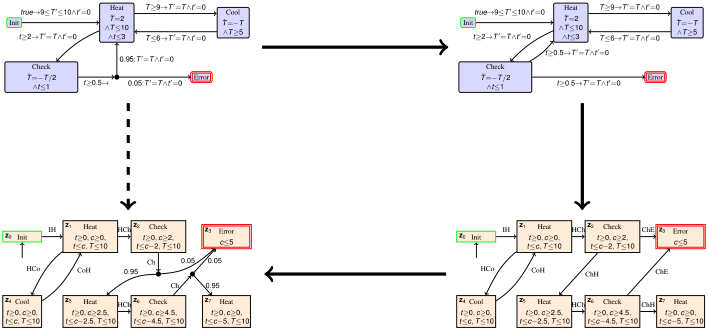
- use $\mathrm{abs}(\mathrm{ind}(\mathcal{H}))$ to compute abstraction $\mathrm{abs}(\mathcal{H})$ of $\mathcal{H}$
  using labellings ℓ_i of $\mathrm{ind}(\mathit{Cmds})$



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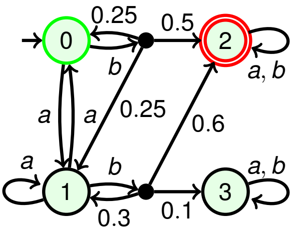

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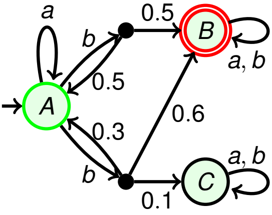

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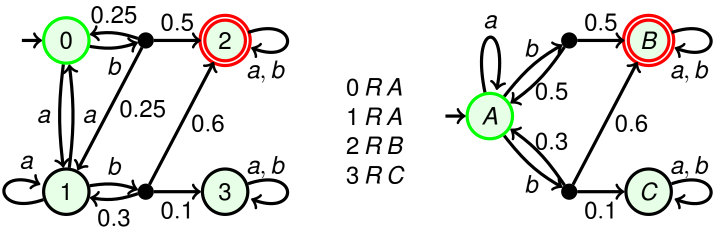

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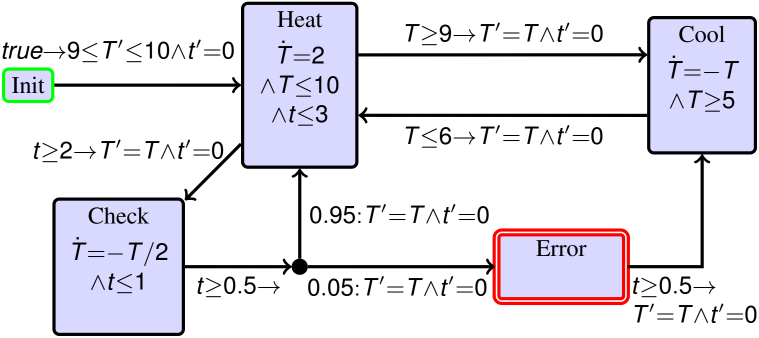
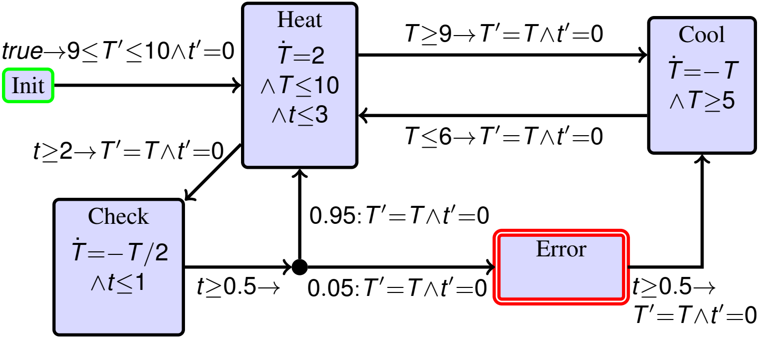
Rewards

- interest in properties other than reachability



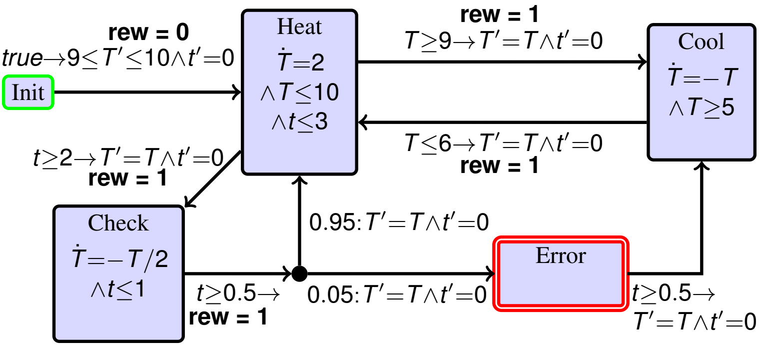
Rewards

- interest in properties other than reachability
- attach **rewards**



Rewards

- interest in properties other than reachability
- attach **rewards**
- obtained per command execution



Rewards

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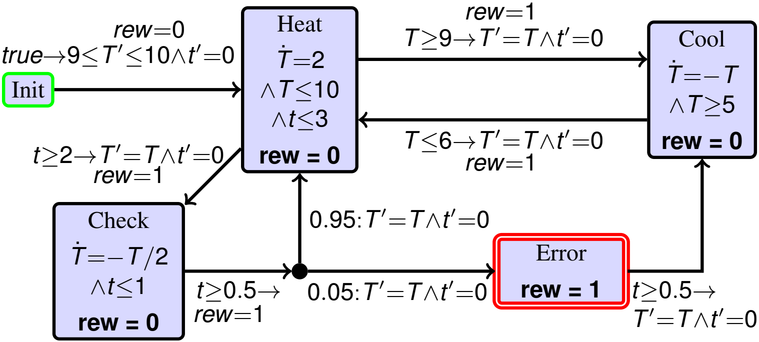
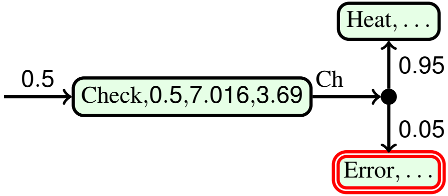
- attach **rewards**
- obtained per command execution
- or per time unit



Rewards Semantics

- induce two reward structures in PA semantics



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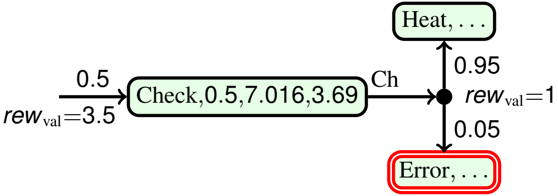

Rewards Semantics

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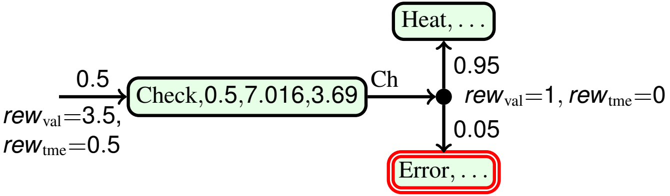

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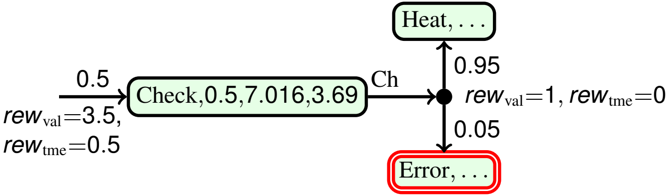

- can now express properties
  accumulated: $\mathrm{val}^{\sigma}_{\mathcal{M},\mathit{rew},\mathrm{acc}} \stackrel{\mathrm{def}}{=} E_{\mathcal{M},\sigma}\left[\lim_{n\to\infty} \sum_{i=0}^{n} \mathit{rew}_{\mathrm{val}}\right]$
  long-run: $\mathrm{val}^{\sigma}_{\mathcal{M},\mathit{rew},\mathrm{lra}} \stackrel{\mathrm{def}}{=} E_{\mathcal{M},\sigma}\left[\lim_{n\to\infty} \frac{\sum_{i=0}^{n} \mathit{rew}_{\mathrm{val}}}{\sum_{i=0}^{n} \mathit{rew}_{\mathrm{tme}}}\right]$

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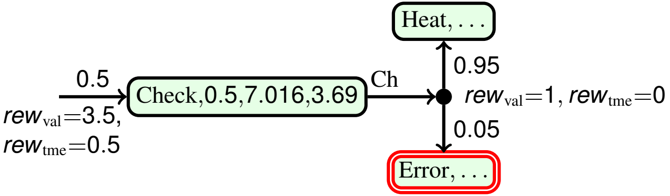

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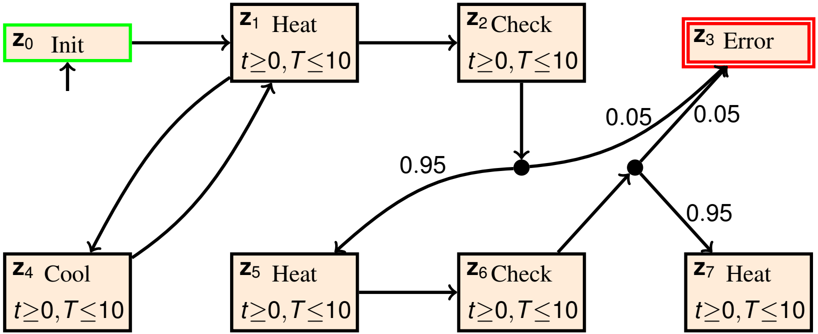
- interested in min/max of values

Abstraction for Rewards

- transform timed rewards → command rewards



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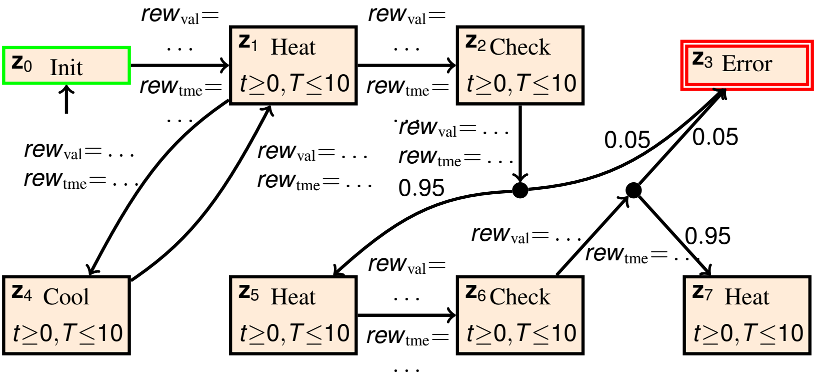
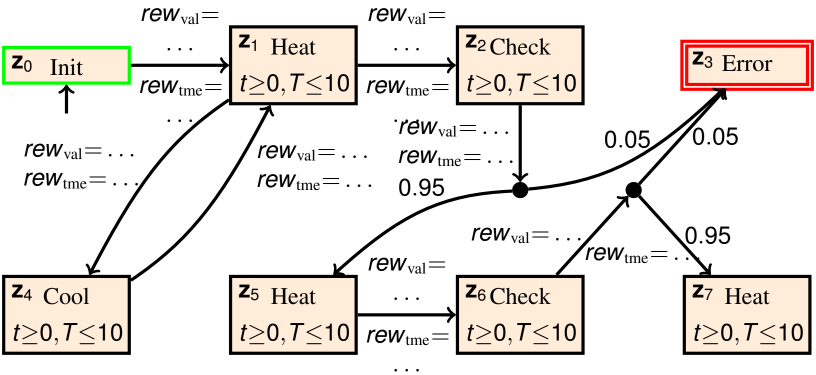
- transform timed rewards → command rewards
- compute reward structures for abstraction



Abstraction for Rewards

- transform timed rewards $\rightarrow$ command rewards
- compute reward structures for abstraction
- compute overapproximation of values

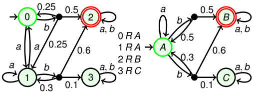
$\sigma :=$ initial scheduler
repeat
  compute reachability probability v under σ
  forall the z $\in$ **A do**
    improve $\sigma(\mathbf{z})$ if possible
until no further improvement possible
return (v, σ)

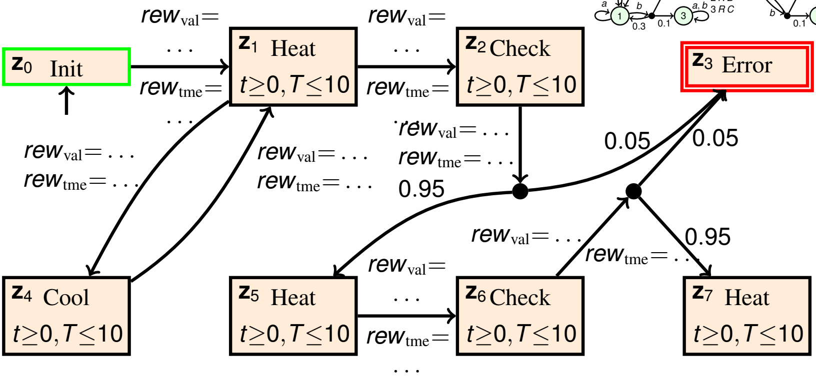


Abstraction for Rewards

- transform timed rewards → command rewards
- compute reward structures for abstraction
- compute overapproximation of values
- correctness: extended simulation relation

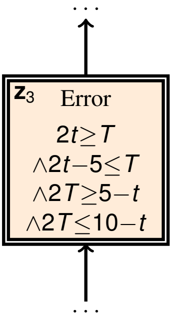
```
σ := initial scheduler
repeat
    compute reachability probability v under σ
    forall the z ∈ A do
        improve σ(z) if possible
until no further improvement possible
return (v, σ)
```





Computing Reward Structures

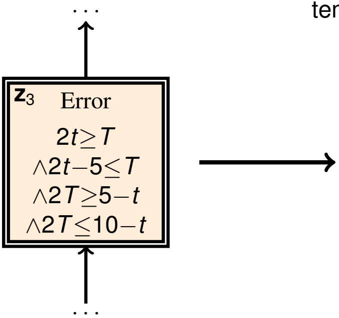
- depends on hybrid automata tool



max time stay in error?

Computing Reward Structures

- depends on hybrid automata tool
- for polyhedra: maps to linear programming



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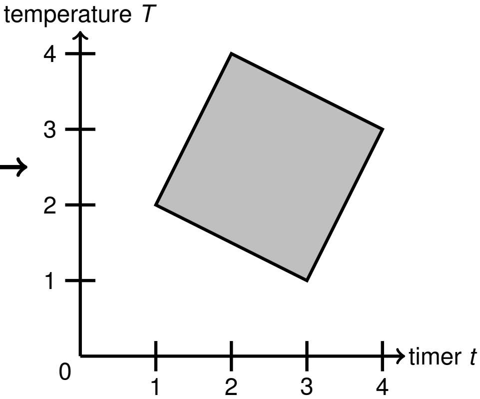

Computing Reward Structures

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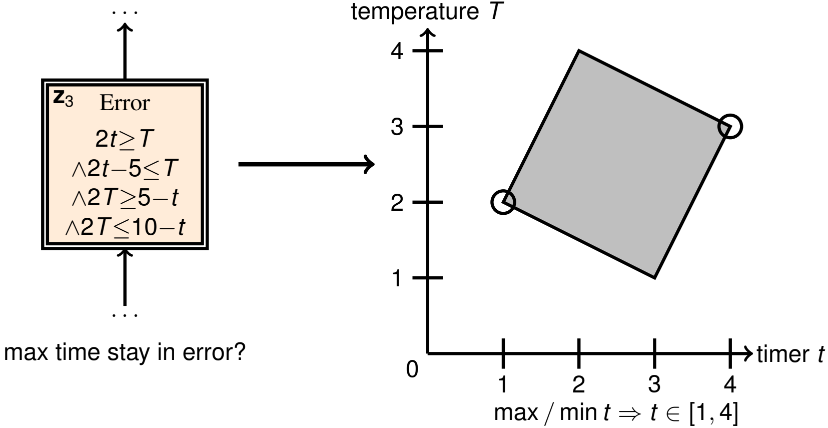

Computing Reward Structures

- depends on hybrid automata tool
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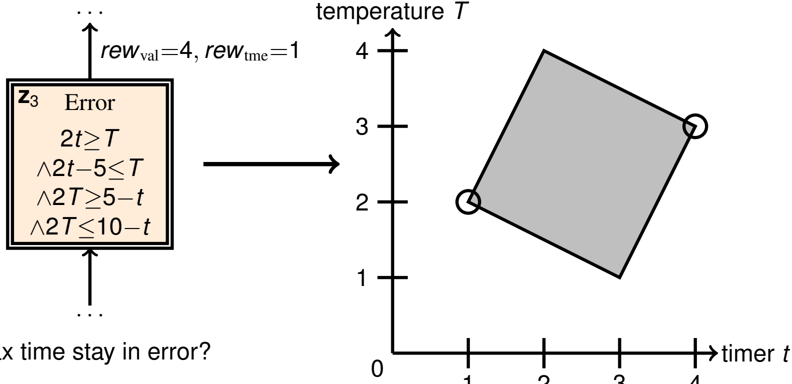

max time stay in error?

max / min $t \Rightarrow t \in [1, 4]$

Thesis Statement

Abstraction enables automatic verification of a very general class of properties of generic stochastic hybrid automata, by extending existing established methods.