Late Weak Bisimilarity for Markov Automata

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Markov Automata

Markov Automata

An MA M is a tuple $(S, Act_\tau, \longrightarrow\!\!\!\!\!\bullet\;, \twoheadrightarrow, \bar{s})$ where

- $\bar{s} \in S$ is the initial state,
- S is a finite but non-empty set of states,
- $Act_\tau = Act \,\dot{\cup}\, \{\tau\}$ is a set of actions including the internal action τ,
- $\longrightarrow\!\!\!\!\!\bullet\; \subset S \times Act_\tau \times Dist(S)$ is a finite set of probabilistic transitions,
- $\twoheadrightarrow \; \subset \; S \times \mathbb{R}_{>0} \times S$ is a finite set of Markovian transitions.

Probabilistic Automata

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- $\longrightarrow\!\!\!\!\!\bullet\; \subset S \times Act_\tau \times Dist(S)$ is a finite set of probabilistic transitions,
- $\longrightarrow\!\!\!\!\!\twoheadrightarrow\; = \emptyset$.

Interactive Markov Chain

Interactive Markov Chain

An Interactive Markov Chain M is a tuple $(S, Act_\tau, \longrightarrow\!\!\!\!\!\bullet\;\,, \longrightarrow\!\!\!\!\!\twoheadrightarrow, \bar{s})$ where

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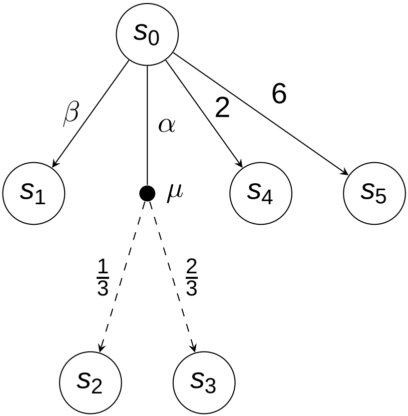
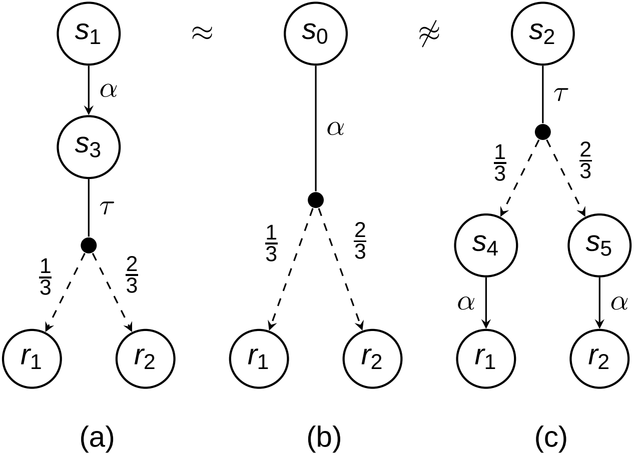
- $Act_\tau = Act \,\dot{\cup}\, \{\tau\}$ is a set of actions including the internal action τ,
- $\longrightarrow\!\!\!\!\!\bullet\; \subset S \times Act_\tau \times S$ is a finite set of transitions,
- $\longrightarrow\!\!\!\!\!\twoheadrightarrow\; \subset S \times \mathbb{R}_{>0} \times S$ is a finite set of Markovian transitions.

Example



Early Weak Bisimilarity



Early Weak Bisimilarity

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A relation $\mathcal{R} \subseteq Dist(S) \times Dist(S)$ is an early weak bisimulation over $\mathcal{M}$ iff $\mu \mathcal{R} \nu$ implies:

- whenever $\mu \xrightarrow{\theta} \mu'$, there exists a $\nu \xRightarrow{\theta} \nu'$ such that $\mu' \mathcal{R} \nu'$;
- whenever $\mu = \sum_{0 \leq i \leq n} p_i \cdot \mu_i$, there exists $\nu \xRightarrow{\tau} \sum_{0 \leq i \leq n} p_i \cdot \nu_i$ such that $\mu_i \mathcal{R} \nu_i$ for each $0 \leq i \leq n$ where $\sum_{0 \leq i \leq n} p_i = 1$;
- symmetrically for ν.

$$s \overset{\bullet}{\approx} r \text{ iff } \delta_s \overset{\bullet}{\approx} \delta_r$$

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$$\mu \xrightarrow{\theta} \mu' \text{ iff } \mu' = \sum_{s \in Supp(\mu) \wedge s \xrightarrow{\theta} \mu_s} \mu(s) \cdot \mu_s$$

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$$\begin{aligned}\{\frac{1}{2} : s_1, \frac{1}{2} : s_2\} &= \frac{1}{2}\delta_{s_1} + \frac{1}{2}\delta_{s_2} \\ &= \frac{2}{3}\{\frac{1}{4} : s_1, \frac{3}{4} : s_2\} + \frac{1}{3}\delta_{s_1}\end{aligned}$$

Properties of $\overset{\bullet}{\approx}$

- Relation on distributions.
- $\overset{\bullet}{\approx}$ is strictly coarser than Weak Probabilistic Bisimulation by Segala.
- $\overset{\bullet}{\approx}$ is compositional.
- $\overset{\bullet}{\approx}$ is the coarsest compositional equivalence preserving trace distribution equivalence.

A piece of probabilistic program

```
print("I am going to toss");
r = rand();
if r ≥ 1/2 then
    print("head");
else
    print("tail");
end
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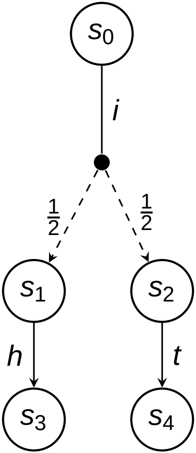
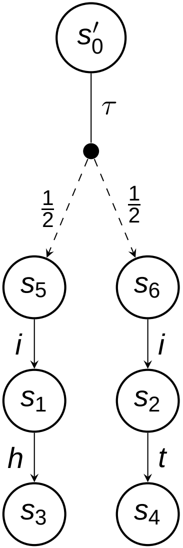


Another piece of probabilistic program

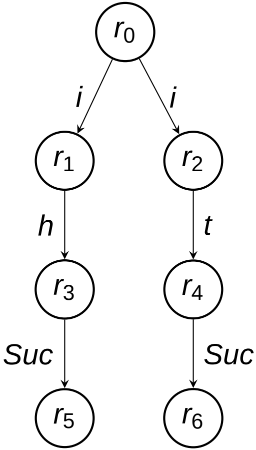
```
r = rand();
if r ≥ 1/2 then
    print("I am going to toss");
    print("head");
else
    print("I am going to toss");
    print("tail");
end
```

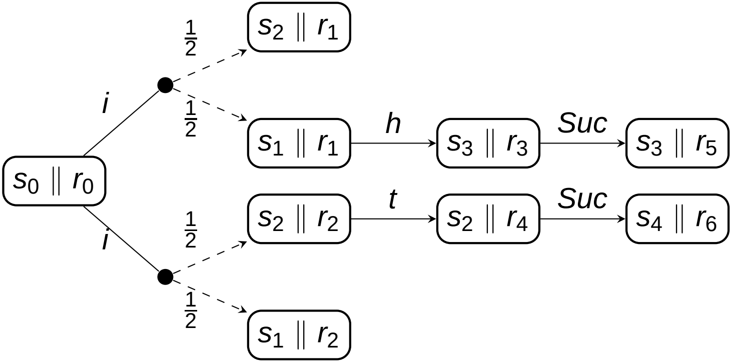
Another piece of probabilistic program

$r = rand();$
if $r \geq \frac{1}{2}$ **then**
  print("I am going to toss");
  print("head");
else
  print("I am going to toss");
  print("tail");
end

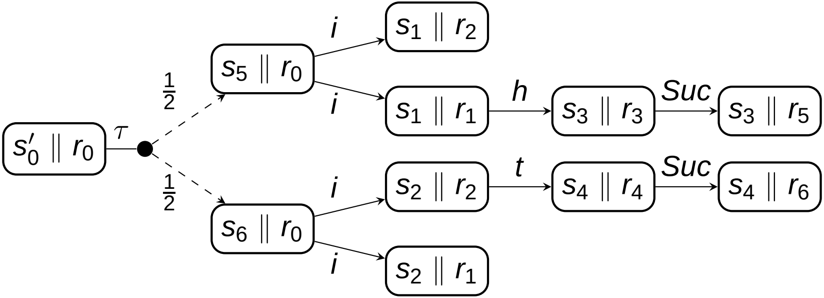


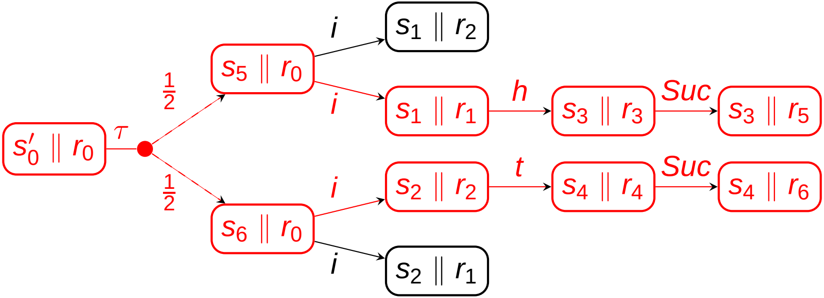
The guesser





s2 ‖ r1
1/2
i
1/2
s1 ‖ r1
h
s3 ‖ r3
Suc
s3 ‖ r5
s0 ‖ r0
t
s2 ‖ r2
s2 ‖ r4
Suc
s4 ‖ r6
i
1/2
1/2
s1 ‖ r2


$s_0' \parallel r_0$
τ
$\frac{1}{2}$
$\frac{1}{2}$
$s_5 \parallel r_0$
$s_6 \parallel r_0$
i
$s_1 \parallel r_2$
$s_1 \parallel r_1$
$s_2 \parallel r_2$
$s_2 \parallel r_1$
h
t
$s_3 \parallel r_3$
$s_4 \parallel r_4$
Suc
$s_3 \parallel r_5$
$s_4 \parallel r_6$


$s'_0 \parallel r_0$
τ
$\frac{1}{2}$
$\frac{1}{2}$
$s_5 \parallel r_0$
$s_6 \parallel r_0$
i
i
i
i
$s_1 \parallel r_2$
$s_1 \parallel r_1$
$s_2 \parallel r_2$
$s_2 \parallel r_1$
h
t
$s_3 \parallel r_3$
$s_4 \parallel r_4$
Suc
Suc
$s_3 \parallel r_5$
$s_4 \parallel r_6$

Two Less Powerful Schedulers

Partial Information Schedulers

L. De Alfaro. The verification of probabilistic systems under memoryless partial-information policies is hard.
Technical report, DTIC Document, 1999.

Two Less Powerful Schedulers

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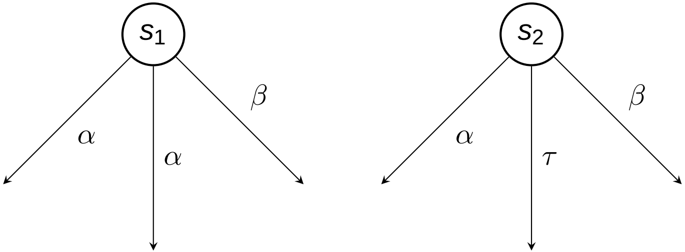
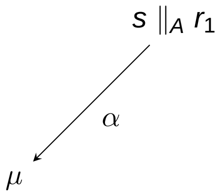
Distributed Schedulers

Sergio Giro and Pedro R. D'Argenio. Quantitative model checking revisited: neither decidable nor approximable.
In *FORMATS*, pages 179–194, Berlin, Heidelberg, 2007. Springer-Verlag.

Partial Information Schedulers



Distributed Schedulers



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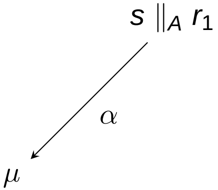

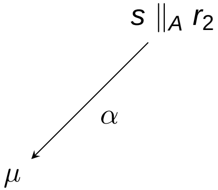

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where $\overrightarrow{\mu}$ if all states in μ have the same observable actions.

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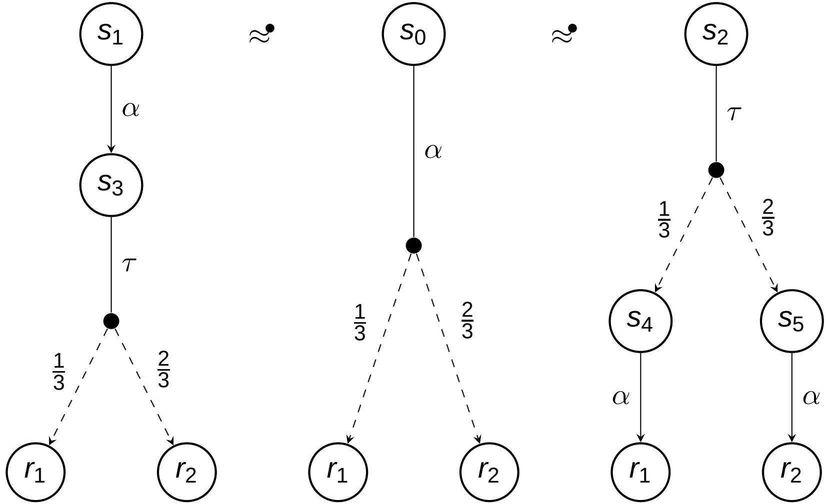
Examples


s_1
$\approx^{\bullet}$
s_0
$\approx^{\bullet}$
s_2
α
s_3
τ
$\frac{1}{3}$
$\frac{2}{3}$
α
τ
$\frac{1}{3}$
$\frac{2}{3}$
s_4
s_5
α
α
$\frac{1}{3}$
$\frac{2}{3}$
r_1
r_2
r_1
r_2
r_1
r_2

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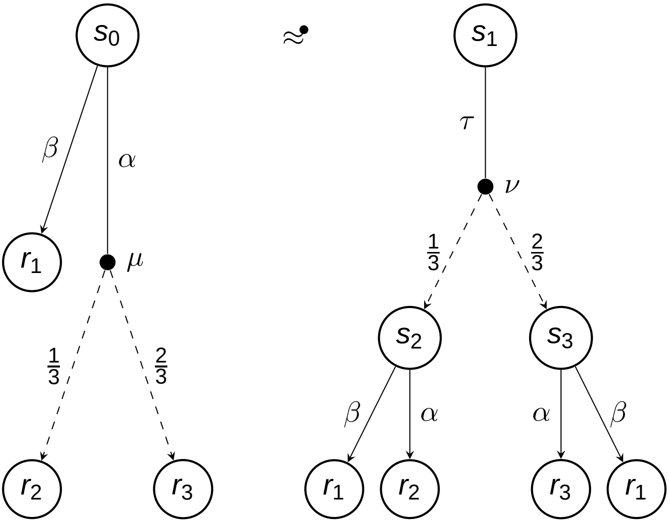

Properties of $\approx^{\bullet}$

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Conclusion and Future Work

- Efficient Decision Algorithm (Currently exponential).
- Logical Characterization.
- . . ..

Thank You

Q& A