Modelling the Common Component Modelling Example in rCOS

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Abstract. This chapter presents the modelling solution to the CoCoME example, the Trading System in rCOS, that is *a Relational Calculus of Object and Component Systems* developed at UNU-IIST. We give a model of requirements based on an abstraction of the use cases described in the description document of the example by the organisers in Chapter 3. We then use the refinement calculus of rCOS to derive design models that are in the same level of abstraction of the informal models given in the description. We make modifications to the models given in description document that we think either they improve the models.
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1 Extra Functionality Analysis

This section provides the specification and analysis of a couple of the extra functionalities given in the description documents. Linking the models used here to the model given in the previous sections is in the obvious way as the specifications share the same logical framework. We also conduct some performance analysis such as the average waiting time per customer and so on.

1.1 Extra-functional Specification

We will specify extra functionality of a method as a property for the time interval for the execution of the method using mathematical logics. We use temporal variables whose value depend on the reference time interval for the execution

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of methods for our specification. Those variables could be $WCET\_m$ which is the duration of the execution of method m, or N_cust which is the number of customers in the referenced observation time interval. Let $T_1 \widehat{=} \mathbf{Intv} \mapsto \mathbb{N}$ and $T_2 \widehat{=} \mathbf{Intv} \mapsto \{tt, ff\}$ be two types of temporal variables. For a formula f on the rigid and temporal variables, for a probability p, $[f]_p$ is a formula saying that f is satisfied with the probability p. As it is well-known in the interval logic, the formulas $\phi;\psi$, which corresponds to the sequential composition of formulas ϕ and ψ, holds for an interval $[a,b]$ iff there is $m \in a..b$ such that ϕ holds for interval $[a,m]$ and ψ holds for interval $[m,b]$; ℓ is a temporal variable denoting the length of the interval it applied to. Intuitively, formula $[0 \leq \mathit{WCET\_ScanItem} < 0.3]_{0.9} \wedge [0.3 \leq \mathit{WCET\_ScanItem} < 1.0]_{0.05} \wedge [1.0 \leq \mathit{WCET\_ScanItem} < 2.0]_{0.04} \wedge [2.0 \leq \mathit{WCET\_ScanItem} \leq 5.0]_{0.01}$ says that the execution time for the operation $ScanItem$ is not more than 5 seconds, and with the probability 0.9 it is less than 0.3 seconds, and with probability 0.05 it is in between 0.3 and 1 second, and etc.

In the following, for saving space, the name of each of temporal variables is self-explainable, each temporal variable of the form 'N_XXX' is of type T_1, and stands for the number of 'XXX'; while each temporal variable of the form 'WCET_XXX' is of type T_1 also, and stands for the worst case execution time for 'XXX'. The temporal variables of type T_2 are to indicate whether or not the predicate indicated in the name is true. For example, *N_CashDesks* stands for the number of cash desks per store; while *WCET_StartNewSale* denotes the worst case execution time for pressing button "Start New Sale"; and *BarcodeScan* means whether or not an item is read by the barcode scanner. Since the arrival and leaving rates are the same (320/3600 arrival per second), and constant, with the exponential distribution we can derive that $[\mathit{N\_Custmoers} = \frac{2}{45}\ell]$ holds for all intervals. The extra functionality specification for usecase UC1 in our notations is as follows.

n0-1 *N_Store* is a temporal variable with type T_1 to denote the number of stores. Thus, n0-1 is expressed as $[\mathit{N\_Stores} = 200]$;

n0-2 Let *N_CashDesks* denote the number of cash desks per store. Then, n0-1 is represented as $[\mathit{N\_CashDesks} = 8]$;

UC1 – [**arr1**] Let *N_Customers* denote the amount of customers. Then arr1 is expressed as $[\mathit{N\_Custmoers} = \frac{4}{45}\ell]$;

– [**n11-1**] Let *N_OpenCashDesks* denote the number of open cash desks per store. So, n11-2 is represented as

$$[0 \leq \mathit{N\_OpenCashDesks} < 2]_{0.1} \wedge [2 \leq \mathit{N\_OpenCashDesks} < 4]_{0.2} \wedge$$
$$[4 \leq \mathit{N\_OpenCashDesks} < 6]_{0.4} \wedge [6 < \mathit{N\_OpenCashDesks} \leq 8]_{0.3} \quad (1)$$

– [**t11-2**] Let *N_GoodsPerCustomer* denote the number of goods per customer. Then t11-2 is specified as

$$[1 \leq \mathit{N\_GoodsPerCustomer} < 8]_{0.3} \wedge [8 \leq \mathit{N\_GoodsPerCustomer} < 15]_{0.1} \wedge$$
$$[15 \leq \mathit{N\_GoodsPerCustomer} < 25]_{0.15} \wedge [25 \leq \mathit{N\_GoodsPerCustomer} < 50]_{0.15} \wedge$$
$$[50 \leq \mathit{N\_GoodsPerCustomer} < 75]_{0.2} \wedge [75 \leq \mathit{N\_GoodsPerCustomer} \leq 100]_{0.1} \quad (2)$$

– [**t12-1**] Let *WCET_StartNewSale* denote the worst case execution time for pressing button “Start New Sale”. Then t12-1 is specified as $[\mathit{WCET\_StartNewSale} = 1]$;
– [**t13-1**] Let *WCET_ScanItem* denote the worst case execution time for scanning an item. Then t13-1 is specified as

$$[0 \leq \mathit{WCET\_ScanItem} < 0.3]_{0.9} \wedge [0.3 \leq \mathit{WCET\_ScanItem} < 1.0]_{0.05} \wedge$$
$$[1.0 \leq \mathit{WCET\_ScanItem} < 2.0]_{0.04} \wedge [2.0 \leq \mathit{WCET\_ScanItem} \leq 5.0]_{0.01} \quad (3)$$

– [**t13-2**] *WCET_ManualEntry* denotes the worst case execution time for manual entry. Then t13-2 is specified as $[\mathit{WCET\_ManualEntry} = 5.0]$
– [**t13-3**] *WCET_SignalError-RejectID* denotes the worst case execution time for signaling error and rejecting an ID. Then t13-3 is specified as $[\mathit{WCET\_SignalError\text{-}RejectID} = 0.01]$
– [**t13-4**] Same as in t13-2.
– [**p13-1&2**] Let *BarcodeScan* and *ManualEntry* be two temporal variables with type T_2. If an item is read by by barcode scanner then *BarcodeScan* holds, otherwise *ManualEntry* is satisfied. So, p13-1&2 can be specified as $[\mathit{BarcodeScan}]_{0.99} \wedge [\mathit{ManualEntry}]_{0.01}$
– [**p13-3&4**] Let *ValidItem* with type T_2 hold if the given item id is valid. Then, p13-3&4 can be specified as $[\mathit{ValidIten}]_{0.999} \wedge [\neg \mathit{ValidItem}]_{0.001}$
– [**p13-5&6**] Let *HumanReadableItem* with type T_2 hold if the item ID is human-readable. Thus, p13-5&6 can be specified as

$$[\mathit{HumanReadableItem}]_{0.9} \wedge [\neg \mathit{HumanReadableItem}]_{0.1}$$

– [**t14-1**] Let *WCET_ShowProduct* with the type T_1 denote the worst case execution time for showing the product description, price, and running total. So, t14-1 is specified as $[\mathit{WCET\_ShowProducts} = 0.01]$
– [**t15-1**] $[\mathit{WCET\_FinishSale} = 1.0]$, where *WCET_FinishSale* denotes the worst case execution time for pressing button “Sale Finished”;
– [**t15a-1**] $[\mathit{WCET\_PrBarPay} = 1.0]$, where *WCET_PrBarPay* denotes the worst case execution time for pressing button “Bar Payment”;
– [**t15a1-1**]

$$[2 \leq \mathit{WCET\_HandOverMoney} < 5]_{0.3} \wedge [5 \leq \mathit{WCET\_HandOverMoney} < 8]_{0.5} \wedge$$
$$[8 \leq \mathit{WCET\_HandOverMoney} \leq 10]_{0.2} \quad (4)$$

where *WCET_HandOverMoney* denotes the worst case execution time for handing over the money;
– [**t15a2-1**] t15a2-1 can be specified as $[\mathit{WCET\_Enter\text{-}ConfirmCash} = 2.0]$, where *WCET_Enter-ConfirmCash* denotes the worst case execution time for entering the cash received and confirming;
– [**p15-1&2**] $[\mathit{PayByCash}]_{0.5} \wedge [\mathit{PayByCard}]_{0.5}$
– [**n15b2-1**]

$$[1 \leq \mathit{N\_TimesEnteringPIN} < 2]_{0.9} \wedge [2 \leq \mathit{N\_TimesEnteringPIN} < 3]_{0.09}$$
$$\wedge [3 \leq \mathit{N\_TimesEnteringPIN} < 4]_{0.01} \quad (5)$$

– [**p15-1&2**] $[\mathit{ValidCCid}]_{0.99} \wedge [\neg \mathit{ValidCCid}]_{0.01}$
– [**t15a3-1**] $[\mathit{WCET\_OpenCashBox} = 1.0]$
– [**t15a4-1**] $[\mathit{WCET\_DisplayBarPayment} = 0.01]$

– [**t15a4-2**]

$$[2 \leq \mathit{WCET\_HandOverChange} < 3]_{0.2} \wedge [3 \leq \mathit{WCET\_HandOverChange} < 4]_{0.6} \wedge [4 \leq \mathit{WCET\_HandOverChange} \leq 5]_{0.2} \quad (6)$$

– [**t15a5-1**] $[\mathit{WCET\_CloseCashBox} = 1.0]$
– [**t15b-1**] $[\mathit{WCET\_PrCardPayment} = 1.0]$
– [**t15b1-1**]

$$[3 \leq \mathit{WCET\_ReceiveCC} < 4]_{0.6} \wedge [4 \leq \mathit{WCET\_ReceiveCC} \leq 5]_{0.4} \quad (7)$$

– [**t15b1-2**] $[\mathit{WCET\_InsertCC} = 2.0]$
– [**t15b2-1**] $[1 \leq \mathit{WCET\_EnterPIN} \leq 5]$
– [**t15b2-2**]

$$[4 \leq \mathit{WCET\_ValidateCC} < 5]_{0.9} \wedge [5 \leq \mathit{WCET\_ValidateCC} \leq 20]_{0.1} \quad (8)$$

– [**t16-1**] $[\mathit{WCET\_SendSaleInf\text{-}UpdateStock} = 0.1]$
– [**t161-1**] $[\mathit{WCET\_WriteLogs} = 2.0]$
– [**p16-1**] $[\mathit{FailureOnInventorySystem}]_{0.001}$
– [**t17-1**] $[\mathit{WCET\_PrintReceipt\text{-}HandOut} = 3.0]$

UC2 – [**arr2**] Let *N_ExpressCheckOuts* be a temporal variable with the type T_1 to denote the number of express checkouts in a given time interval. Then arr2 is expressed as $[\mathit{N\_ExpressCheckouts} = \frac{1}{3600}\ell]$;
– [**p2-1&2**] $[\mathit{NormalMode}]_{0.8} \wedge [\mathit{ExpressMode}]_{0.2} \wedge [\mathit{NormalMode} \Leftrightarrow \neg \mathit{ExpressMode}]$
– [**t21a-1**] $[\mathit{WCET\_SwitchExpressMode} = 0.01]$
– [**t21b-1**] $[\mathit{WCET\_SwitchLightDisplay} = 0.01]$
– [**t21c-1**] $[\mathit{WCET\_DeactivateCardPay} = 0.01]$
– [**t21d-1**] $[\mathit{WCET\_SetMaximalItems} = 0.01]$
– [**t22a-1**] $[\mathit{WCET\_DisableExpressMode} = 1]$
– [**t22b-1**] See t21b-1.
– [**t22c-1**] $[\mathit{WCET\_ReactivateCardPay} = 0.01]$

UC3 – [**arr3**] Let *N_SentOrder* be a temporal variable with the type T_1 to denote the number of the sent orders in a given time interval. Then arr3 is expressed as $[\mathit{N\_Order} = \frac{1}{86400}\ell]$;
– [**n3-1**] *N_AllProducts* denotes the number of all products per store. Then n3-1 is specified as $[\mathit{N\_AllProducts} = 5000]$
– [**n3-2**] Let *N_ProductsRunningOutOfStock* be a temporal variable with the type T_1, to denote the number of products running out of stock. Then n3-2 is specified as

$$[100 \leq \mathit{N\_ProductsRunningOutOfStock} < 200]_{0.25} \wedge [200 \leq \mathit{N\_ProductsRunningOutOfStock} < 300]_{0.25} \wedge [300 \leq \mathit{N\_ProductsRunningOutOfStock} < 400]_{0.25} \wedge [400 \leq \mathit{N\_ProductsRunningOutOfStock} \leq 500]_{0.25} \quad (9)$$

– **[p3-1]** $[\mathit{ReorderProductsOutOfStock}]_{0.98}$
– **[t31-1]** $[\mathit{WCET\_ShowProducts} = 0.01]$
– **[t32-1]** $[\mathit{WCET\_ChooseProducts\text{-}EnterAmount} = 10.0]$
– **[t33-1]** $[\mathit{WCET\_PrOrder} = 1]$
– **[t34-1]** $[\mathit{WCET\_QueryInventoryDataStore} = 0.02]$
– **[t34-2]** $[\mathit{WCET\_CreateNewOrderEntry} = 0.01]$
– **[t34-3]**$[\mathit{WCET\_CreateNewProductOrder} = 0.01]$

UC4 – **[arr4]** Let *N_ReceivedOrders* be a temporal variable with the type T_1 to denote the number of the received orders in a given time interval. Then arr4 is expressed as $[\mathit{N\_ReceivedOrders} = \frac{1}{86400}\ell]$;
– **[n4]** Let *N_ArrivingProducts* be a temporal variable with the type T_1, to denote the number of products arriving. Then n4 is specified as

$$[100 \leq \mathit{N\_ArrivingProducts} < 200]_{0.25} \wedge [200 \leq \mathit{N\_ArrivingProducts} < 300]_{0.25} \wedge$$
$$[300 \leq \mathit{N\_ArrivingProducts} < 400]_{0.25} \wedge [400 \leq \mathit{N\_ArrivingProducts} \leq 500]_{0.25} \quad (10)$$

– **[p4-1&2]** $[\mathit{CompleteCorrectOrder}]_{0.99} \wedge [\neg \mathit{CompleteCorrectOrder}]_{0.01}$
– **[t42-1]** $[\mathit{WCET\_CheckCompleteness} = 1800]$
– **[t43-1]** $[\mathit{WCET\_RollReceivedOrder} = 1]$
– **[t44-1]** $[\mathit{WCET\_UpdateInventory} = 0.1]$

UC5 – **[arr5]** Let *N_ReceivedOrders* be a temporal variable with the type T_1 to denote the number of showing stock reports in a given time interval. Then arr5 is expressed as $[\mathit{N\_ShowStockReports} = \frac{1}{10800}\ell]$;
– **[t5-1]** $[\mathit{WCET\_EnterStoreID\text{-}CreateReport} = 1]$
– **[t43-1]** $[\mathit{WCET\_GenerateReport} = 0.5]$

UC6 – **[arr6]** Let *N_ReceivedOrders* be a temporal variable with the type T_1 to denote the number of showing delivery reports in a given time interval. Then arr6 is expressed as $[\mathit{N\_ShowDeliveryReports} = \frac{1}{86400}\ell]$;
– **[t5-1]** $[\mathit{WCET\_EnterStoreID\text{-}CreateDeliveryReport} = 1]$
– **[t43-1]** $[\mathit{WCET\_GenerateDeliveryReport} = 0.5]$

UC7 – **[arr7]** Let *N_ChangePrice* be a temporal variable with the type T_1 to denote the number of changing price in a given time interval. Then arr7 is expressed as $[\mathit{N\_ChangePrice} = \frac{1}{10800}\ell]$;
– **[t71-1]** $[\mathit{WCET\_GenerateOverview} = 10]$
– **[t72-1]** $[\mathit{WCET\_SelectProductItem} = 5]$
– **[t72-2]** $[\mathit{WCET\_ChangePrice} = 5]$
– **[t73-1]** $[\mathit{WCET\_PrEnter} = 1]$

UC8 – **[arr8]** Let *N_ShowStockReports* be a temporal variable with the type T_1 to denote the number of showed stock reports in a given time interval. Then arr8 is expressed as $[\mathit{N\_ShowStockReports} = \frac{1}{86400}\ell]$;
– **[n8-1]**

$$[10 \leq \mathit{N\_StoresNearbyStoreServer} < 20]_{0.7} \wedge$$
$$[20 \leq \mathit{N\_StoresNearbyStoreServer} \leq 30]_{0.3}; \quad (11)$$

– **[p8-1&2]** $[\mathit{FailureOnStoreServer}]_{0.0001} \wedge [\mathit{FailureOnEnterpriseServer}]_{0.0001}$

- **[t82-1]** $[WCET\_StoreServerDetectLowStock = 10]$
- **[t83-1]**

$$[0.0 \leq WCET\_StoreServerQueryEnterpriseServer < 0.5]_{0.5} \wedge$$
$$[0.5 \leq WCET\_StoreServerQueryEnterpriseServer \leq 1.0]_{0.5}; \quad (12)$$

- **[t84-1]**

$$[0.0 \leq WCET\_FlushStoreServer\text{-}Return < 0.5]_{0.5} \wedge$$
$$[0.5 \leq WCET\_FlushStoreServer\text{-}Return \leq 1.0]_{0.5}; \quad (13)$$

- **[t85-1]** $[WCET\_DatabaseLookupEnterpriseServer = 0.01]$
- **[t86-1]** $[WCET\_DetermineDeliveringStore = 1]$
- **[t87-1]**

$$[0.0 \leq WCET\_ReturnResultToStoreServer < 0.5]_{0.5} \wedge$$
$$[0.5 \leq WCET\_ReturnResultToStoreServer \leq 1.0]_{0.5}; \quad (14)$$

- **[t87-2]** $[WCET\_MarkIncomingGoods = 10]$
- **[t88-1]**

$$[0.0 \leq WCET\_SendDeliveryRequest < 0.5]_{0.5} \wedge$$
$$[0.5 \leq WCET\_SendDeliveryRequest \leq 1.0]_{0.5}; \quad (15)$$

- **[t88-2]** $[WCET\_MarkUnavalableGoods = 10]$

1.2 QoS Analysis

We will use the following reference rules:

$$\textbf{R-Conj:} \quad \frac{[\phi]_{p_1} \wedge [\psi]_{p_2}}{[\phi \wedge \psi]_{p_1 * p_2}}$$
$$\textbf{R-Disj:} \quad \frac{[\phi]_{p_1} \wedge [\psi]_{p_2}}{[\phi \vee \psi]_{p_1 + p_2 - p_1 p_2}}$$
$$\textbf{R-Chop:} \quad \frac{[\phi]_{p_1} \wedge [\psi]_{p_2}}{[\phi ; \psi]_{p_1 p_2}}$$

According to the Specification, in UC1, we have

$$\begin{aligned}
Service &= ReadItems; Pay; Log \\
ReadItems &= StartNewSale; ReadItem^{N_GoodsPerCustomer}; ShowProducts; FinishSale \\
ReadItem &= ScanItem \vee ManualEntry \\
Pay &= PaybyCash \vee PaybyCard \\
PaybyCash &= PrBarPay; HandOverMoney; Enter\text{-}ConfirmCash; OpenCashBox; \\
&\quad DisplayBarPayment; HandOverChange; CloseCashBox \\
PaybyCard &= PrCardPayment; ReceiveCC; InsertCC; ValidateCC \\
Log &= SendSaleInf\text{-}UpdateStock; WriteLogs; PrintReceit\text{-}HandOut
\end{aligned}$$

Therefore, the worst case execution time of service for per customer can be calculated as follows:

$$\begin{aligned}
WCET_Service =\ & WCET_ReadItems + WCET_Pay + WCET_Log && (16)\\
WCET_ReadItems =\ & WCET_StartNewSale + \\
& \quad WCET_ReadItem \times N_GoodsPerCustomer + \\
& \quad WCET_ShowProducts + WCET_FinishSale && (17)\\
WCET_ReadItem =\ & WCET_ScanItem \vee WCET_ReadItem = WCET_ManualEntry && (18)\\
WCET_Pay =\ & WCET_PaybyCash \vee WCET_Pay = WCET_PaybyCard && (19)\\
WCET_PaybyCash =\ & WCET_PrBarPay + WCET_HandOverMoney + \\
& WCET_Enter\text{-}ConfirmCash + WCET_OpenCashBox + \\
& WCET_DisplayBarPayment + WCET_HandOverChange + \\
& WCET_CloseCashBox && (20)\\
WCET_PaybyCard =\ & WCET_PrCardPayment + WCET_ReceiveCC + \\
& WCET_InsertCC + WCET_ValidateCC && (21)\\
WCET_Log =\ & WCET_SendSaleInf\text{-}UpdateStock + WCET_WriteLogs + \\
& WCET_PrintReceipt\text{-}HandOut && (22)
\end{aligned}$$

From the specification for UC1, we have

$$\begin{aligned}
& (\mathbf{t13\text{-}1}) \wedge (\mathbf{t13\text{-}2}) \wedge (\mathbf{p13\text{-}1\&2}) \wedge (18)\\
\Rightarrow & [0 \le WCET_ReadItem < 0.3]_{0.891} \wedge [0.3 \le WCET_ReadItem < 1.0]_{0.0495} \wedge \\
& [1.0 \le WCET_ReadItem < 2.0]_{0.0396} \wedge [2.0 \le WCET_ReadItem \le 5.0]_{0.0099} \wedge \\
& [WCET_ReadItem = 5.0]_{0.01} \qquad \text{(R-Conj)} && (23)
\end{aligned}$$

$$\begin{aligned}
& (2) \wedge (23) \wedge (17) \wedge (\mathbf{t12\text{-}1}) \wedge (\mathbf{14\text{-}1}) \wedge (\mathbf{t15\text{-}1})\\
\Rightarrow & [2.01 \le WCET_ReadItems < 3.21]_{0.2673} \wedge [3.21 \le WCET_ReadItems < 6.01]_{0.01485} \wedge \\
& [6.01 \le WCET_ReadItems < 10.01]_{0.01188} \wedge [10.01 \le WCET_ReadItems < 20.01]_{0.00297} \wedge \\
& [2.01 \le WCET_ReadItems < 5.31]_{0.0891} \wedge [5.31 \le WCET_ReadItems < 13.01]_{0.00495} \wedge \\
& [13.01 \le WCET_ReadItems < 24.01]_{0.00396} \wedge [24.01 \le WCET_ReadItems < 57.01]_{0.00099} \wedge \\
& [2.01 \le WCET_ReadItems < 7.86]_{0.13365} \wedge [7.86 \le WCET_ReadItems < 21.51]_{0.007425} \wedge \\
& [21.51 \le WCET_ReadItems < 41.01]_{0.00594} \wedge [41.01 \le WCET_ReadItems < 99.51]_{0.001485} \wedge \\
& [2.01 \le WCET_ReadItems < 13.11]_{0.13365} \wedge [13.11 \le WCET_ReadItems < 39.01]_{0.007425} \wedge \\
& [39.01 \le WCET_ReadItems < 76.01]_{0.00594} \wedge [76.01 \le WCET_ReadItems < 187.01]_{0.001485} \wedge \\
& [2.01 \le WCET_ReadItems < 20.61]_{0.1782} \wedge [20.61 \le WCET_ReadItems < 64.01]_{0.0099} \wedge \\
& [64.01 \le WCET_ReadItems < 126.01]_{0.00792} \wedge [126.01 \le WCET_ReadItems < 312.01]_{0.00198} \wedge \\
& [2.01 \le WCET_ReadItems < 28.26]_{0.0891} \wedge [28.26 \le WCET_ReadItems < 89.51]_{0.00495} \wedge \\
& [89.51 \le WCET_ReadItems < 177.01]_{0.00396} \wedge [177.01 \le WCET_ReadItems < 439.51]_{0.00099} \wedge \\
& [WCET_ReadItems = 22.01]_{0.003} \wedge [WCET_ReadItems = 57.01]_{0.001} \wedge \\
& [WCET_ReadItems = 99.51]_{0.0015} \wedge [WCET_ReadItems = 187.01]_{0.0015} \wedge \\
& [WCET_ReadItems = 312.01]_{0.002} \wedge [WCET_ReadItems = 439.51]_{0.001} && (24)\\
& \qquad \text{(R-Conj,R-Chop,R-Disj)}
\end{aligned}$$

In the above calculating, for simplicity, we use the mean value of *N_GoodsPerCustomer* with the given probability on the reference interval multiplying *WCET_ReadItem* to stand for the possible time on reading the goods either by barcode scanner or by manual entry. On the other hand, the time spending on handling some exceptions is so small that we can ignore it. Analogous remarks will be applied in calculating the possibility of $WCET\_Pay$ below.

$$
\begin{aligned}
&(\mathbf{t15a\text{-}1}) \wedge (4) \wedge (\mathbf{t15a2\text{-}1}) \wedge (\mathbf{t15a3\text{-}1}) \wedge (\mathbf{t15a4\text{-}1}) \wedge (6) \wedge (\mathbf{t15a\text{-}1}) \wedge (20)\\
\Rightarrow &[9.01 \leq WCET\_PaybyCash < 13.01]_{0.06} \wedge [10.01 \leq WCET\_BaybyCash < 14.01]_{0.18} \wedge\\
&[11.01 \leq WCET\_PaybyCash < 15.01]_{0.06} \wedge [12.01 \leq WCET\_PaybyCash < 16.01]_{0.1} \wedge\\
&[13.01 \leq WCET\_PaybyCash < 17.01]_{0.3} \wedge [14.01 \leq WCET\_PaybyCash < 18.01]_{0.1} \wedge\\
&[15.01 \leq WCET\_PaybyCash < 18.01]_{0.04} \wedge [16.01 \leq WCET\_PaybyCash < 19.01]_{0.12} \wedge\\
&[17.01 \leq WCET\_PaybyCash < 20.01]_{0.04} \qquad \text{(R-Chop, R-Conj)} \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{n15b2\text{-}1}) \wedge (\mathbf{15b\text{-}1}) \wedge (\mathbf{15b1\text{-}1}) \wedge (\mathbf{15b\text{-}2}) \wedge (\mathbf{15b2\text{-}1}) \wedge (\mathbf{15b2\text{-}2}) \wedge (21)\\
\Rightarrow &[11 \leq WCET\_PaybyCard < 17]_{0.486} \wedge [12 \leq WCET\_PaybyCard < 22]_{0.0486} \wedge\\
&[13.5 \leq WCET\_PaybyCard < 28.5]_{0.0054} \wedge [12 \leq WCET\_PaybyCard < 18]_{0.324} \wedge\\
&[13 \leq WCET\_PaybyCard < 23]_{0.0324} \wedge [14.5 \leq WCET\_PaybyCard < 29.5]_{0.0036} \wedge\\
&[12 \leq WCET\_PaybyCard < 32]_{0.054} \wedge [13 \leq WCET\_PaybyCard < 37]_{0.0054} \wedge\\
&[14.5 \leq WCET\_PaybyCard < 43.5]_{0.0006} \wedge [13 \leq WCET\_PaybyCard < 33]_{0.036} \wedge\\
&[14 \leq WCET\_PaybyCard < 38]_{0.0036} \wedge [15.5 \leq WCET\_PaybyCard < 44.5]_{0.0004} \qquad (26)\\
&\text{(R-Chop, R-Conj)}
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{p15\text{-}1\&2}) \wedge (19) \wedge (25) \wedge (26)\\
\Rightarrow &[11 \leq WCET\_Pay < 17]_{0.243} \wedge [12 \leq WCET\_Pay < 22]_{0.0243} \wedge\\
&[13.5 \leq WCET\_Pay < 28.5]_{0.0027} \wedge [12 \leq WCET\_Pay < 18]_{0.162} \wedge\\
&[13 \leq WCET\_Pay < 23]_{0.0162} \wedge [14.5 \leq WCET\_Pay < 29.5]_{0.0018} \wedge\\
&[12 \leq WCET\_Pay < 32]_{0.027} \wedge [13 \leq WCET\_Pay < 37]_{0.0027} \wedge\\
&[14.5 \leq WCET\_Pay < 43.5]_{0.0003} \wedge [13 \leq WCET\_Pay < 33]_{0.018} \wedge\\
&[14 \leq WCET\_Pay < 38]_{0.0018} \wedge [15.5 \leq WCET\_Pay < 44.5]_{0.0002} \wedge\\
&[9.01 \leq WCET\_Pay < 13.01]_{0.03} \wedge [10.01 \leq WCET\_Bay < 14.01]_{0.09} \wedge\\
&[11.01 \leq WCET\_Pay < 15.01]_{0.03} \wedge [12.01 \leq WCET\_Pay < 16.01]_{0.05} \wedge\\
&[13.01 \leq WCET\_Pay < 17.01]_{0.15} \wedge [14.01 \leq WCET\_Pay < 18.01]_{0.05} \wedge\\
&[15.01 \leq WCET\_Pay < 18.01]_{0.02} \wedge [16.01 \leq WCET\_Pay < 19.01]_{0.06} \wedge\\
&[17.01 \leq WCET\_Pay < 20.01]_{0.02} \qquad \text{(R-Chop, R-Conj,R-Disj)} \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{t16\text{-}1}) \wedge (\mathbf{t161\text{-}1}) \wedge (\mathbf{t17\text{-}1}) \wedge (22)\\
\Rightarrow &[WCET\_Log = 5.1] \qquad \text{(R-Chop)} \qquad (28)
\end{aligned}
$$

According to (24),(27),(28) and (16), it is easy to calculate the possibility of $WCET\_Service$. For simplicity, here we just calculate the mean value of $WCET\_Service$, i.e. $WCET\_Service = 32.075$. Therefore, the rate of service is $\mu = 1/WCET\_Service = 1/32.075 = 0.0311$. For convenience, we denote $WCET\_Service$ by D in the later. From **arr1**, we have the rate of customers arriving is $\lambda = N\_Customers/\ell = 4/45$. According to (**n11-2**), the average number c of open cash desks is $c = (0+1)/2 \times 0.1 + (2+3)/2 * 0.2 + (4+5)/2 * 0.4 + (6+8)/2 * 0.3 = 4.45$. Let P_i denote the possibility that there are i customers in the store. So, from the knowledge from queueing theory [?], we have

$$\rho = \lambda/(c\mu) \tag{29}$$
$$[P_1 = 4 \times \rho \times P_0] \tag{30}$$
$$[P_2 = \frac{4^2}{2!} \times \rho^2 \times P_0] \tag{31}$$
$$[P_3 = \frac{4^3}{3!} \times \rho^3 \times P_0] \tag{32}$$
$$[P_4 = \frac{4^4}{4!} \times \rho^4 \times P_0] \tag{33}$$
$$[P_i = \frac{4^4}{4!} \times \rho^i \times P_0] \tag{34}$$
$$[1 = \sum_{i=0} P_i] \tag{35}$$
$$[l_q = \frac{5^5}{5!} \times \rho^6/(5! \times (1-\rho)^2)] \tag{36}$$
$$[w_q = \frac{l_q}{\lambda}] \tag{37}$$

In order to estimate the average waiting time per customer, we assume that each store adopts the following strategy to schedule its cash desks:

$$\begin{aligned} &([0 \leq N\_Customers \leq n_1] \Rightarrow [0 \leq N\_OpenCashDesks < 2]) \land \\ &([n_1 < N\_Customers \leq n_2] \Rightarrow [2 \leq N\_OpenCashDesks < 3]) \land \\ &([n_2 \leq N\_Customers \leq n_3] \Rightarrow [4 \leq N\_OpenCashDesks < 6]) \land \\ &([n_3 \leq N\_Customers] \Rightarrow [6 \leq N\_OpenCashDesks < 8]) \end{aligned} \tag{38}$$

That is, if the number of customers is less than or equal to n_1, then at most one cash desk is open; if the number of customers is between n_1 and n_2, then open 2 or 3 cash desks; if the number of customers is between n_2 and n_3, then open 4 or 5 cash desks; otherwise, open all eight cash desks. Let P_i denote the possibility that there are i customers in the store. So, from (**n11-1**) and some knowledge from queueing theory [**?**], we have

$$[P_1 = 2\lambda D P_0] \tag{39}$$

$$[P_i = (2\lambda D)^i P_0] \qquad 1 \leq i \leq n_1 \tag{40}$$

$$[\sum_{i=0}^{n_1} P_i = \frac{(2\lambda D)^{n_1+1} - 2\lambda D}{2\lambda D - 1} P_0 = 0.1] \tag{41}$$

$$[P_{n_1+1} = \frac{2\lambda D}{5} P_{n_1}] \tag{42}$$

$$[P_i = (\frac{2\lambda D}{5})^{i-n_1} P_{n_1}] \qquad n_1 + 1 \leq i \leq n_2 \tag{43}$$

$$[\sum_{i=n_1+1}^{n_2} P_i = \frac{(\frac{2\lambda D}{5})^{n_2-n_1+1} - \frac{2\lambda D}{5}}{\frac{2\lambda D}{5} - 1} P_{n_1} = \frac{(\frac{2\lambda D}{5})^{n_2-n_1+1} - \frac{2\lambda D}{5}}{\frac{2\lambda D}{5} - 1} (2\lambda D)^{n_1} P_0 = 0.2] \tag{44}$$

$$[P_{n_2+1} = \frac{2\lambda D}{9} P_{n_2}] \tag{45}$$

$$[P_i = (\frac{2\lambda D}{9})^{i-n_2} P_{n_2}] \qquad n_2 + 1 \leq i \leq n_3 \tag{46}$$

$$\begin{aligned}[\sum_{i=n_2+1}^{n_3} P_i &= \frac{(\frac{2\lambda D}{9})^{n_3-n_2+1} - \frac{2\lambda D}{9}}{\frac{2\lambda D}{9} - 1} P_{n_2} \\ &= \frac{(\frac{2\lambda D}{9})^{n_3-n_2+1} - \frac{2\lambda D}{9}}{\frac{2\lambda D}{9} - 1} (\frac{2\lambda \mu}{5})^{n_2-n_1} (2\lambda D)^{n_1} P_0 = 0.4]\end{aligned} \tag{47}$$

$$[P_i = (\frac{\lambda D}{7})^{i-n_3} P_{n_3}] \qquad n_3 < i \tag{48}$$

$$[\sum_{i=n_3+1}^{\infty} P_i = \frac{\frac{\lambda D}{7}}{1 - \frac{\lambda D}{7}} P_{n_3} = \frac{\frac{\lambda D}{7}}{1 - \frac{\lambda D}{7}} (\frac{2\lambda D}{9})^{n_3-n_2} (\frac{2\lambda D}{5})^{n_2-n_1} (2\lambda D)^{n_1} P_0 = 0.3] \tag{49}$$

2 Conclusion

This is to provide a summary and evaluation of the solution.