Kripke Structures

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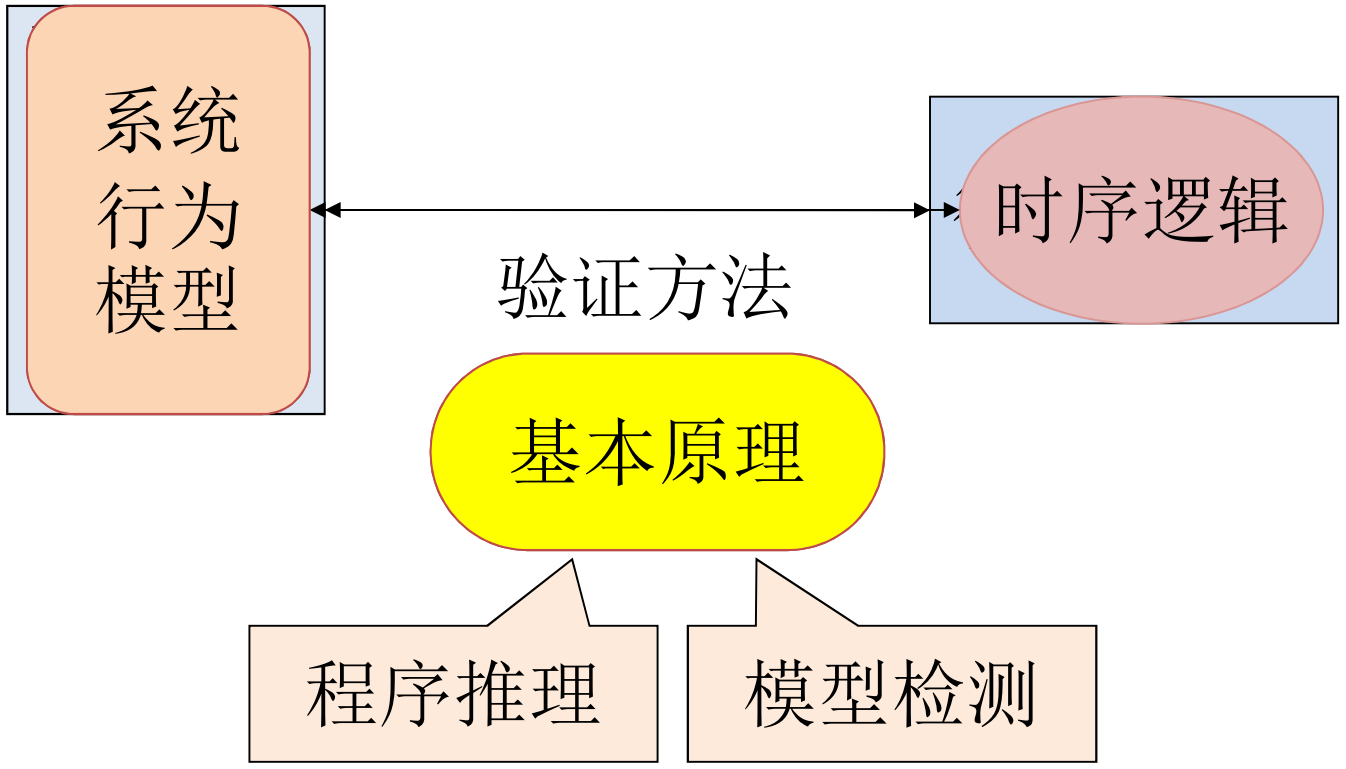
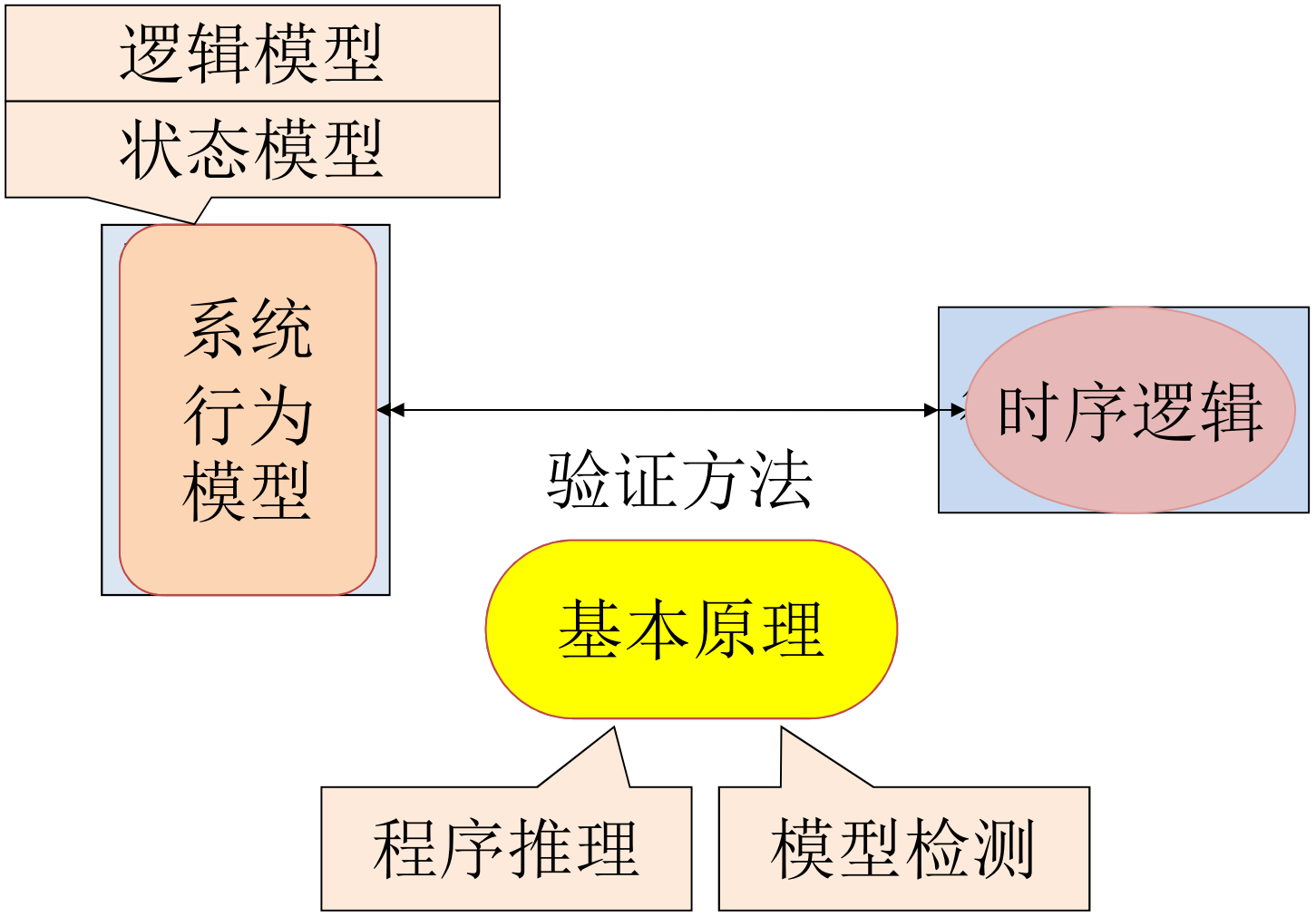
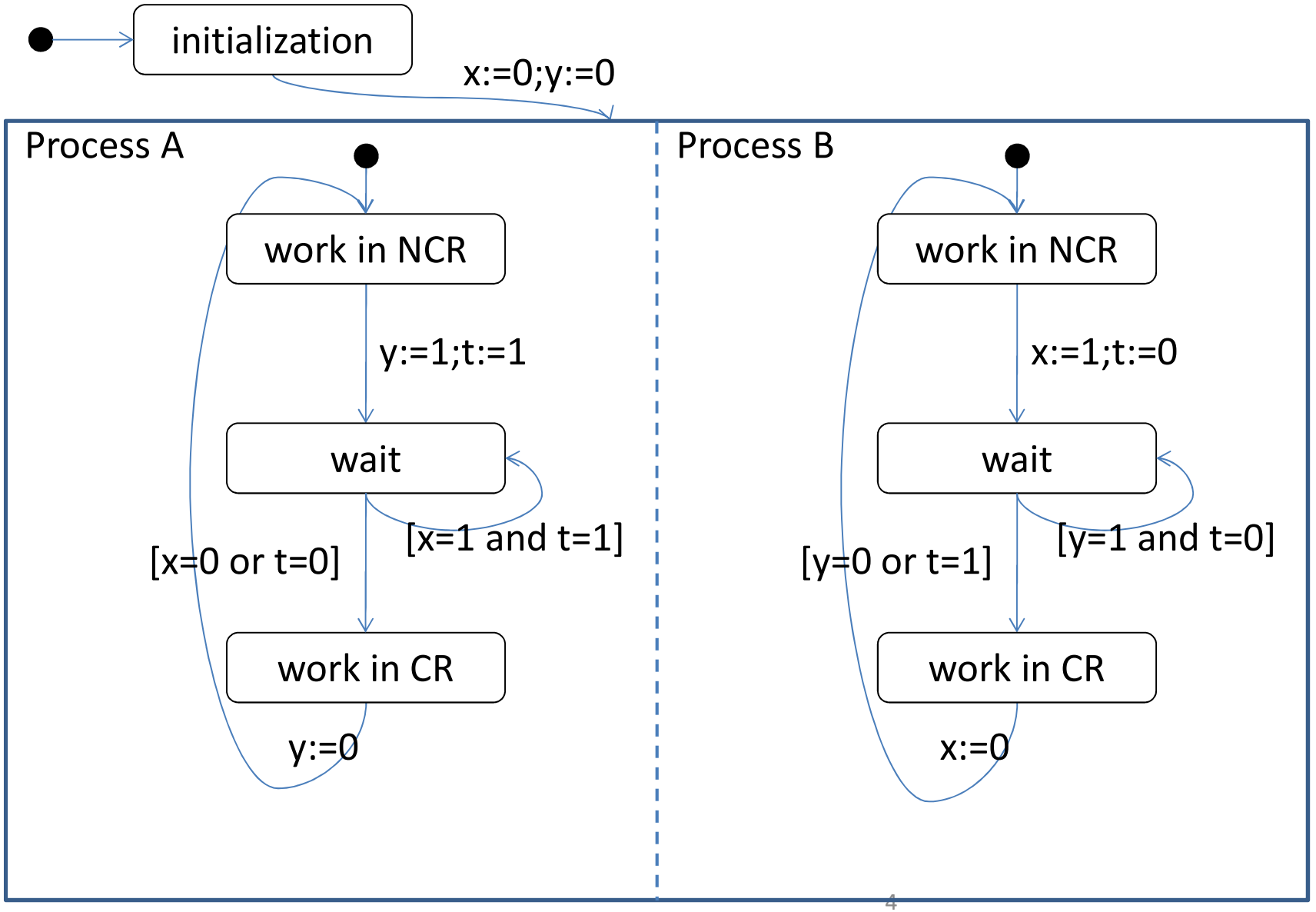
课程内容



课程内容



例子-互斥：状态图(State Diagram)



例子-互斥：算法

VAR: x: 0..1; y: 0..1; t: 0..1;
a: {NCR,wait,CR}; b: {NCR,wait,CR};
INIT: x=0; y=0;
a=NCR; b=NCR;

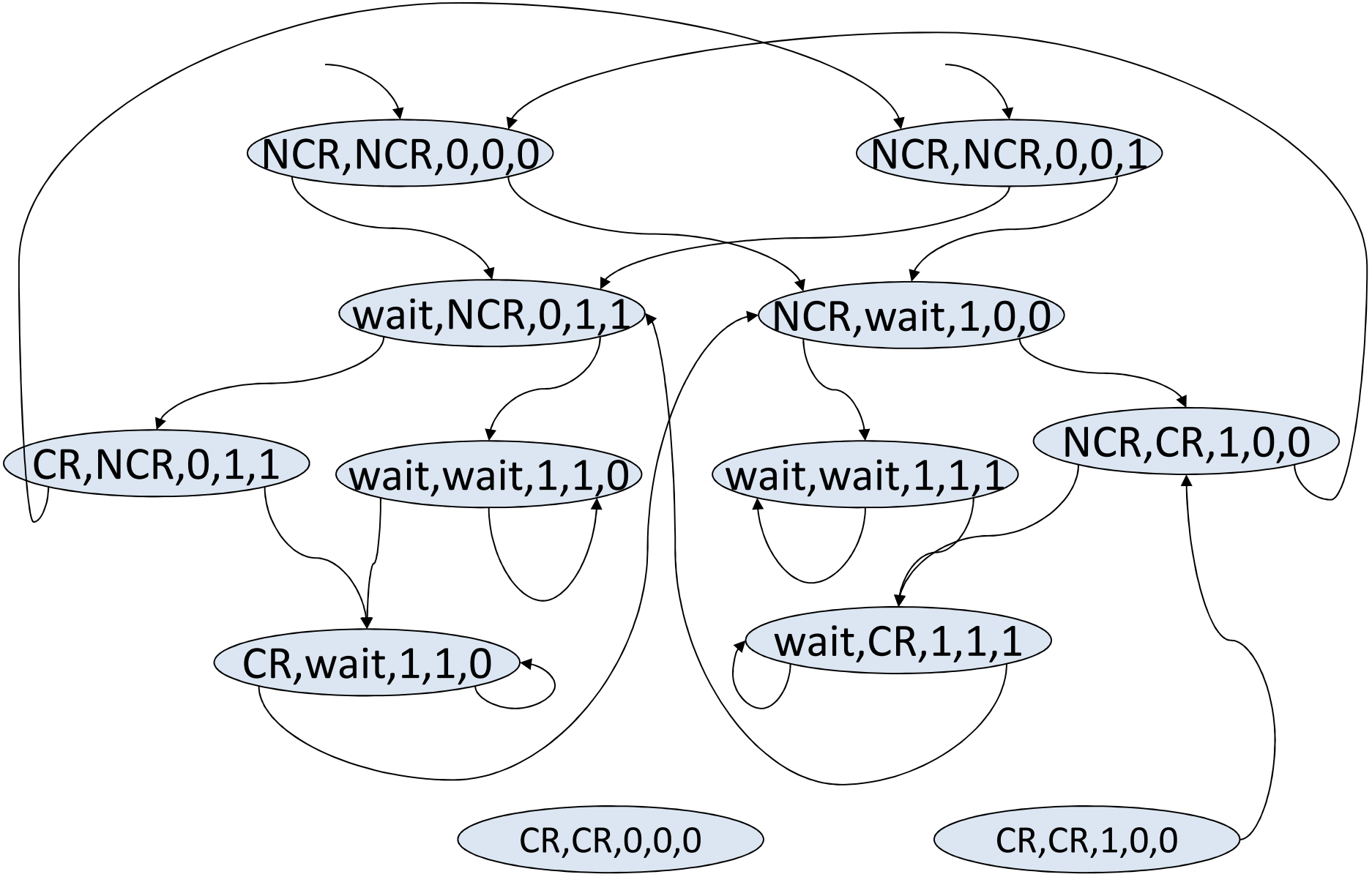
Process A:

a=NCR	→ (a,y,t):=(wait,1,1);
a=wait∧(x=0∨t=0)	→ (a):=(CR);
a=wait∧¬(x=0∨t=0)	→ (a):=(wait);
a=CR	→ (a,y):=(NCR,0);

Process B:

b=NCR	→ (b,x,t):=(wait,1,0);
b=wait∧(y=0∨t=1)	→ (b):=(CR);
b=wait∧¬(y=0∨t=1)	→ (b):=(wait);
b=CR	→ (b,x):=(NCR,0);

例子-互斥：可达状态 + 部分不可达状态



例子-互斥：状态集合

$S = \{ s_0, s_1, \ldots, s_{71} \}$

s_i 对应于五元组(a,b,x,y,t)的一个元素

其中s_i代表 (a,b,x,y,t)
当且仅当　i=a*24+b*8+x*4+y*2+t

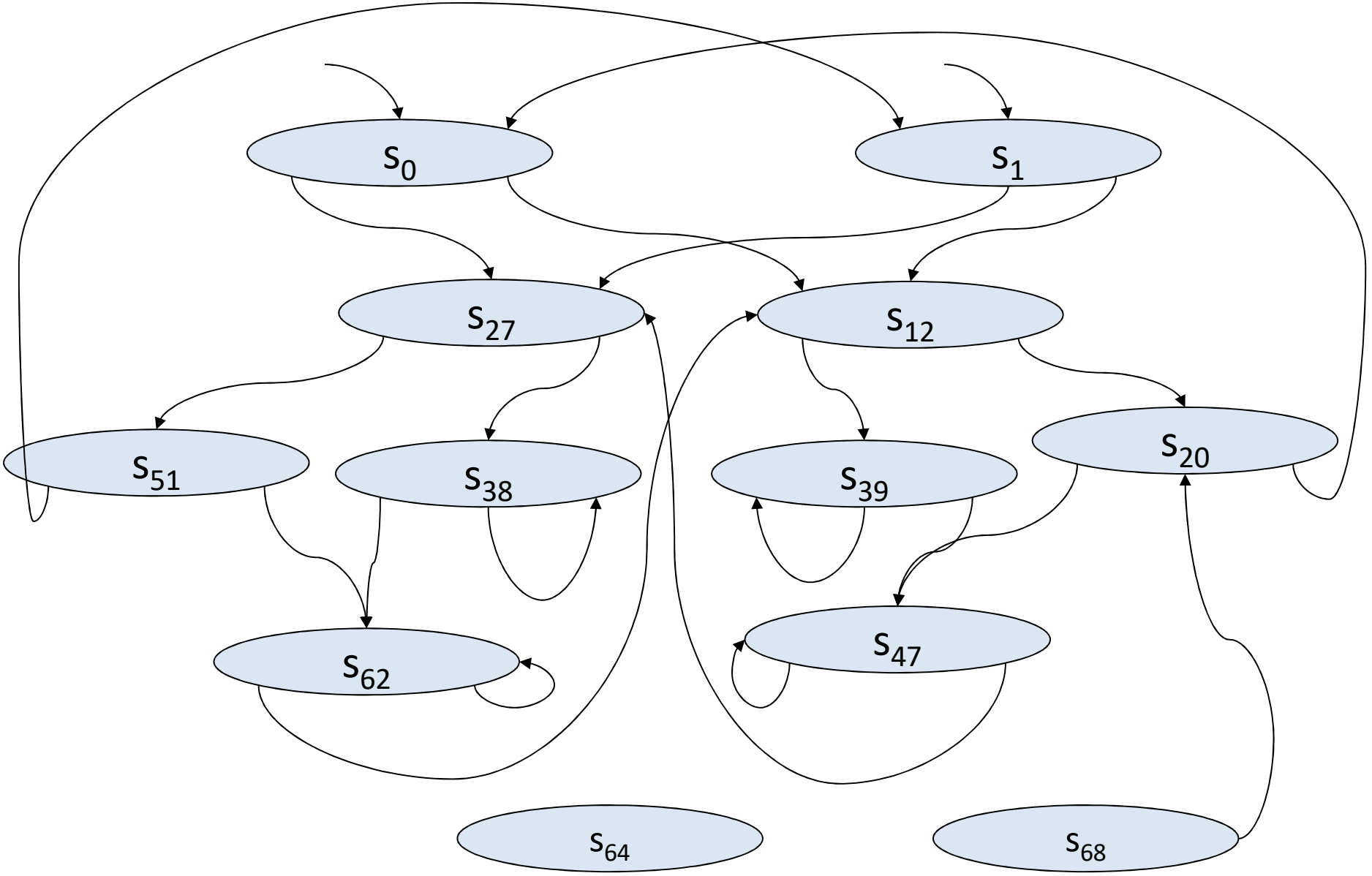
其中NCR代表0, wait代表1, CR代表2

例如:

s_0　　= (0,0,0,0,0)　　= (NCR,NCR,0,0,0)

s_{48}　　= (2,0,0,0,0)　　= (CR,NCR,0,0,0)

例子-互斥：状态迁移图(Kripke结构)



内容

- Kripke结构
- 安全性质相关概念及验证方法
- 必达性质相关概念及验证方法

(I) Kripke Structures

Definition

A Kripke structure is a triple K=<S,R,I>

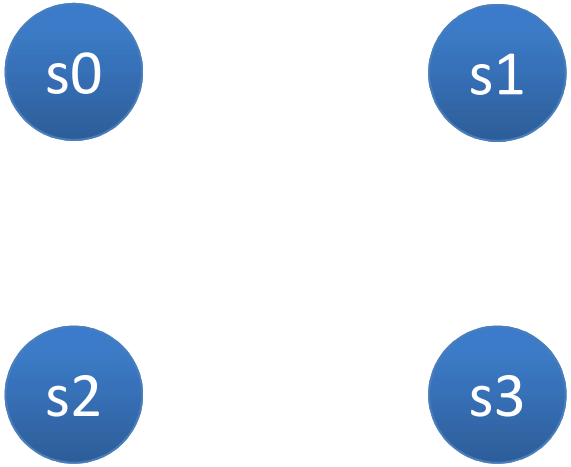
- S : A set of states
- $R \subseteq S \times S$: A total transition relation
- $I \subseteq S$: A set of initial states

R is total, if $\forall s.\exists s'.(s,s') \in R$

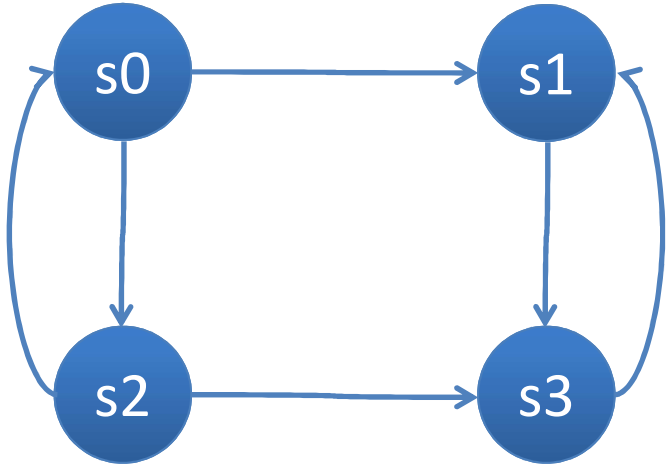
Kripke Structures

- Basic Concepts
  - States, Transition Relation, Initial States
  - Successors, Predecessors
  - Reachable States, Reachability Relation
  - Paths (Finite and Infinite), Computation, Behavior
  - Properties
- Basic System Properties
  - Reachability, Safety
  - Avoidability, Inevitability

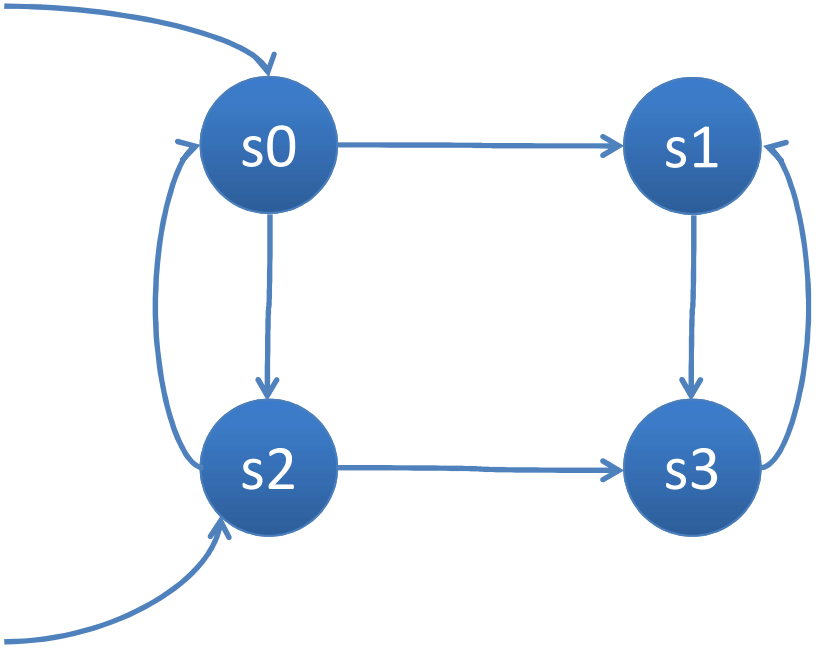
Example: S



Example: R



Example: I



Basic Concepts

1. Successors

s’ is a successor of s, if s → s’, i.e., R(s,s’)

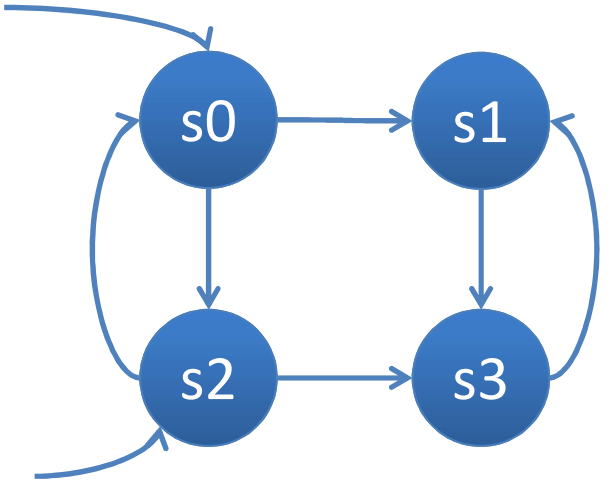
The set of successors of s:

R(s)

The set of successors of X:

$R(X) = \cup_{s \in X} R(s)$

R is total if $R(s) \neq \emptyset$ for all $s \in S$



Example:

R(s0) = { s1,s2}

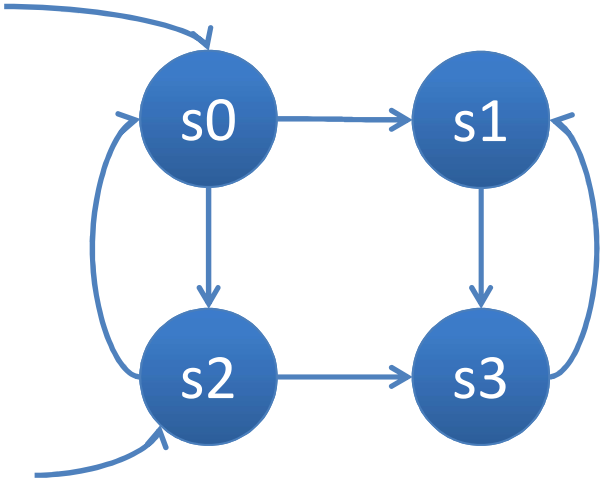
R({s0,s1}) = { s1,s2, s3 }

1a. Predecessors

s is a predecessor of s', if s → s', i.e.,
$R(s,s')$, $R^{-1}(s',s)$

The set of predecessors of s:
$R^{-1}(s)$

The set of predecessors of X :
$R^{-1}(X) = \cup_{s \in X} R^{-1}(s)$



Example:

$R^{-1}(s0) = \{ s2\}$

$R^{-1}(\{s0,s1\}) = \{ s0,s2, s3 \}$

2. Reachable States (from s)

$\rightarrow^*$:

the reflexive and transitive closure of $\rightarrow$

s’ is reachable from s, if

$s \rightarrow^* s'$

The set of states reachable from s is $\{ s' \mid s \rightarrow^* s' \}$:

$R^*(s)$.

2. Reachable States (from A,K)

s’ is reachable from A, if $s \rightarrow^* s'$ for some $s \in A$

The set of states reachable from A is $\{ s' \mid s \rightarrow^* s', s \in A \}$: $R^*(A)$.

The set of states reachable in K is $R^*(I)=\{ s' \mid s \rightarrow^* s', s \in I \}$: $Rh(K)$.

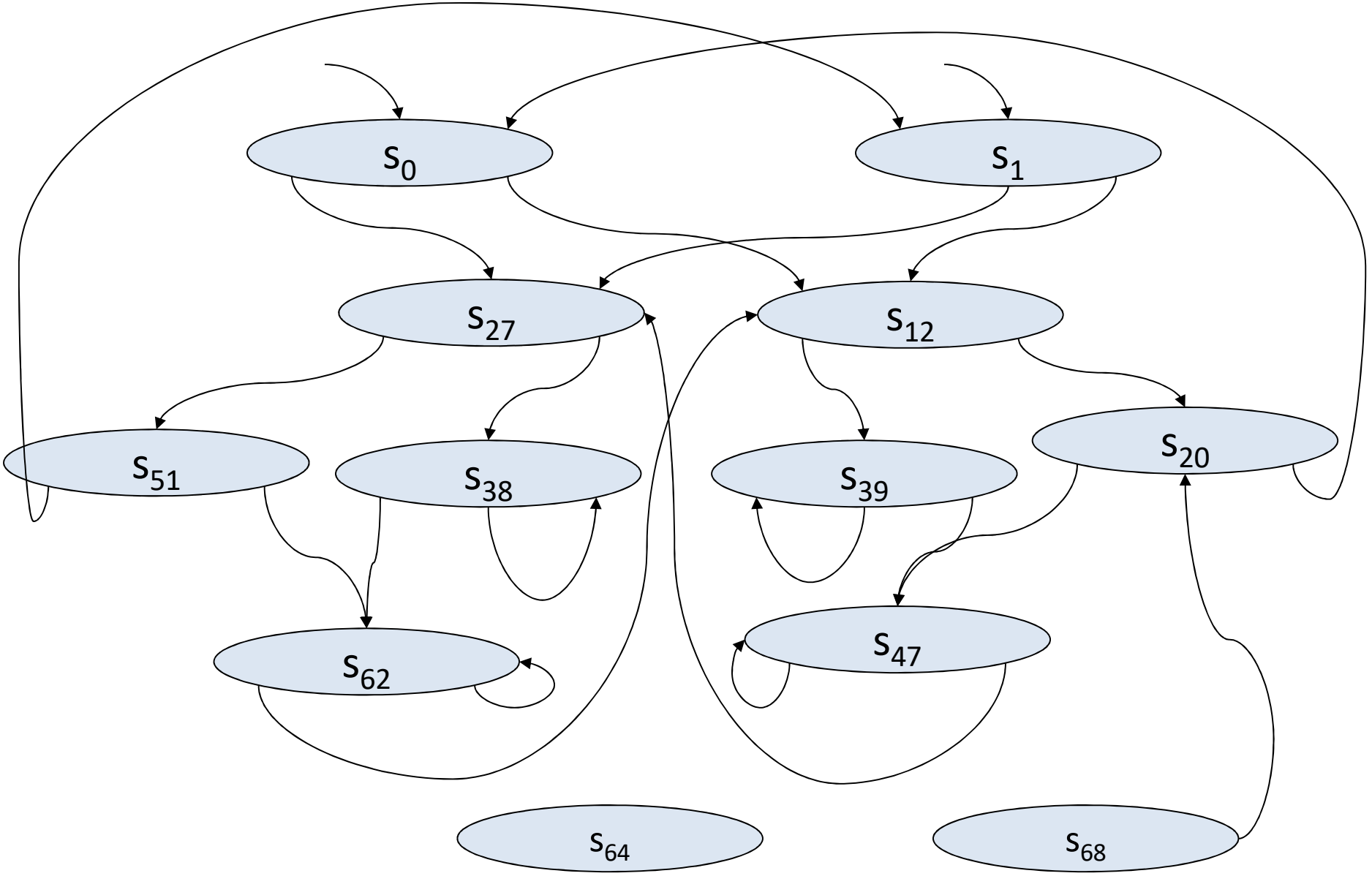
Reachable States

Reachable states of s: $R^*(s) = \{ s' \mid s \rightarrow^* s' \}$

Reachable states of X: $R^*(X) = \cup_{s \in X} R^*(s)$

Reachable states of K: $Rh(K) = R^*(I)$

例子-互斥：状态迁移图(Kripke结构)



2a. Reachability Relation

A set Y is reachable from s,

if there is a state s' of Y,

such that $s \rightarrow^* s'$

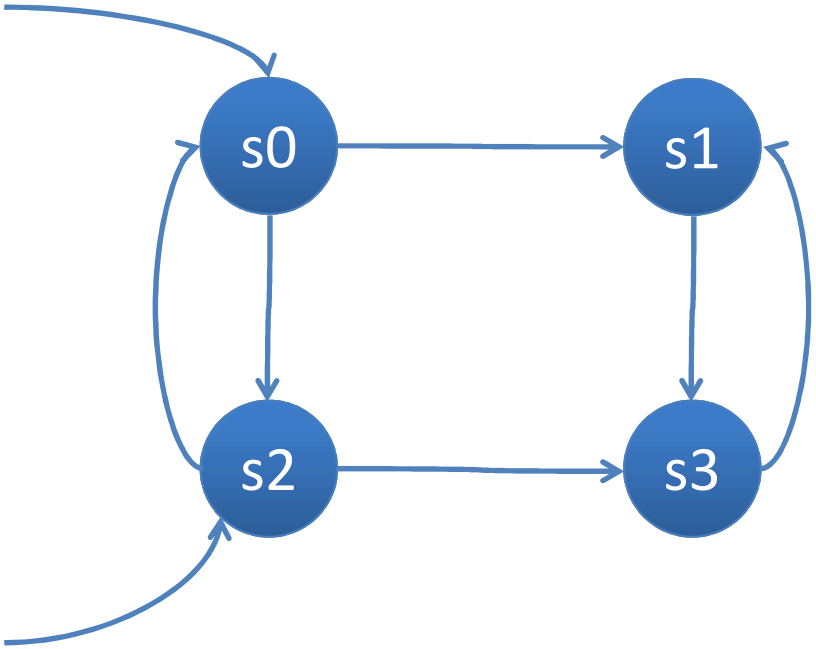
A set Y is reachable from X,

if there is a state s' of Y and a state s of X,

such that $s \rightarrow^* s'$

$X \rightarrow^* Y$

Example:



The set of reachable states (Y=R(X)):

X = { s1 }, Y = { s1, s3 }; X = { s0,s1 }, Y = { s0,s1,s2,s3 }

Reachability relation (X →* Y):

X = { s1 }, Y = { s2, s3 }; X = { s0,s1,s2 }, Y = { s3 }

3. Path

Definition

An infinite path is an infinite sequence of S:

$s_0 s_1 s_2 \ldots$

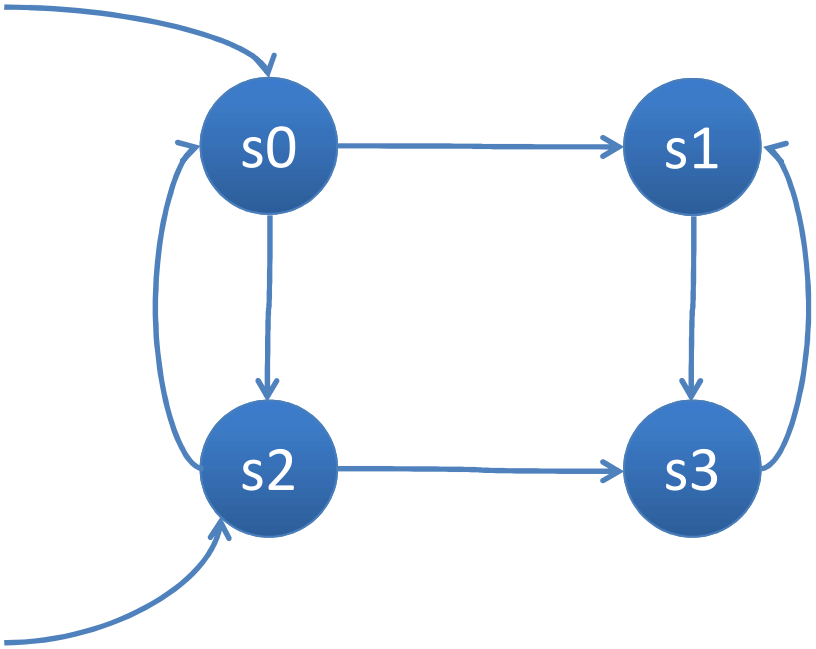
such that $s_i \rightarrow s_{i+1}$ for all $i \geq 0$

Definition

A finite path is a finite prefix of an infinite path:

$s_0 \ldots s_n$

Example: Paths



Infinite paths:

s0s1s3s1s3s1…

$s0(s1s3)^{\omega}$

s1s3s1s3s1s3s…

Finite paths:

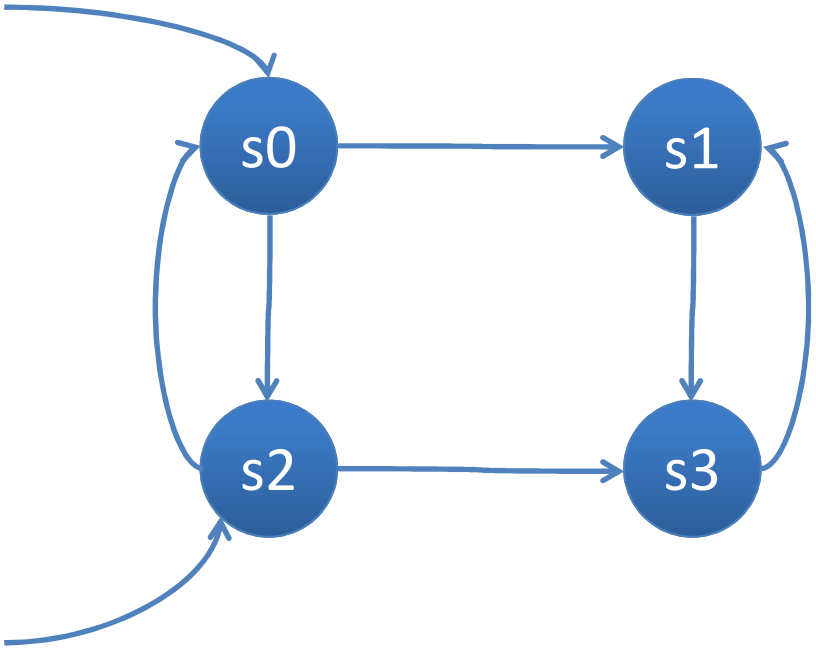
s0s1s3s1

s1s3s1s3s1

3a. Computation

A **computation** of K is an infinite path $s_0\, s_1\, s_2 \ldots$
such that $s_0 \in I$

Example: Computations



$s0(s1s3)^{\omega}$

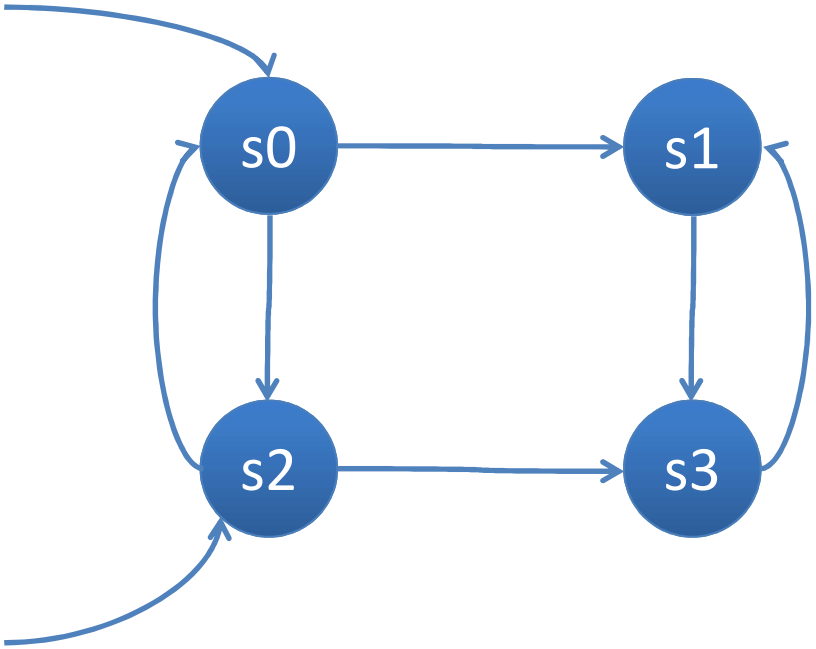
$s2(s3s1)^{\omega}$

$s2(s0s2)^{*}(s1s3)^{\omega}$

3b. Behavior

The behavior of K is the set of computations of K, denoted [[K]].

Example: Behavior



$(s0s2)^{\omega}$

$s0(s2s0)^{*}(s1s3)^{\omega}$

$s0s2(s0s2)^{*}(s3s1)^{\omega}$

$(s2s0)^{\omega}$

$s2(s0s2)^{*}(s3s1)^{\omega}$

$s2s0(s2s0)^{*}(s1s3)^{\omega}$

4. Properties

A property is represented by a set of states

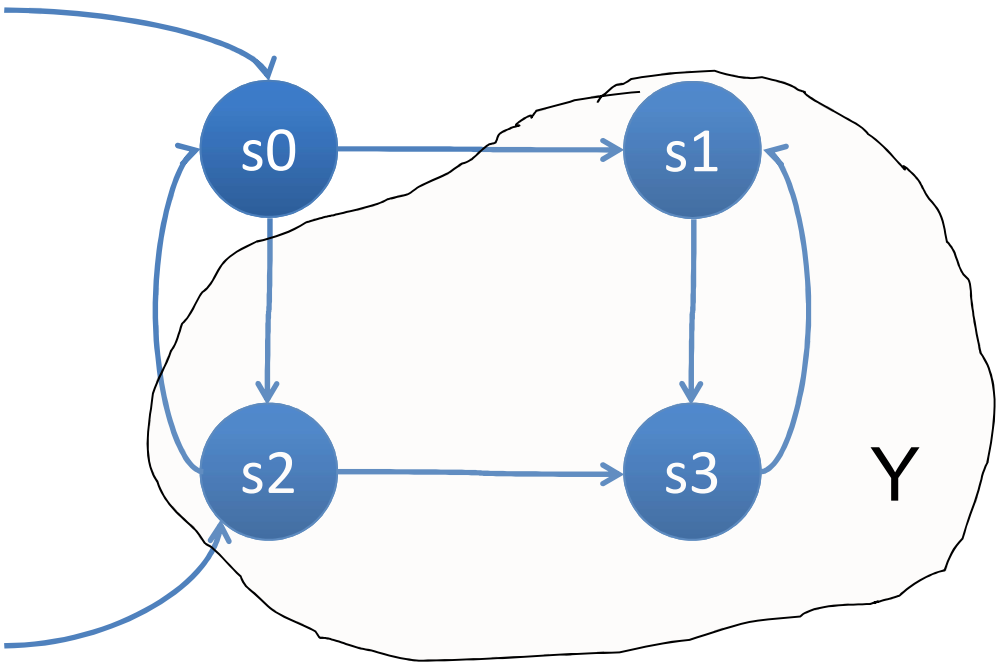
- X
- Y

DEF: s satisfies Y, if $s \in Y$.

DEF: X satisfies Y, if for all $s \in X$, s satisfies Y, i.e., $X \subseteq Y$

s is called a Y-state, if s satisfies Y.

Example: Y-States



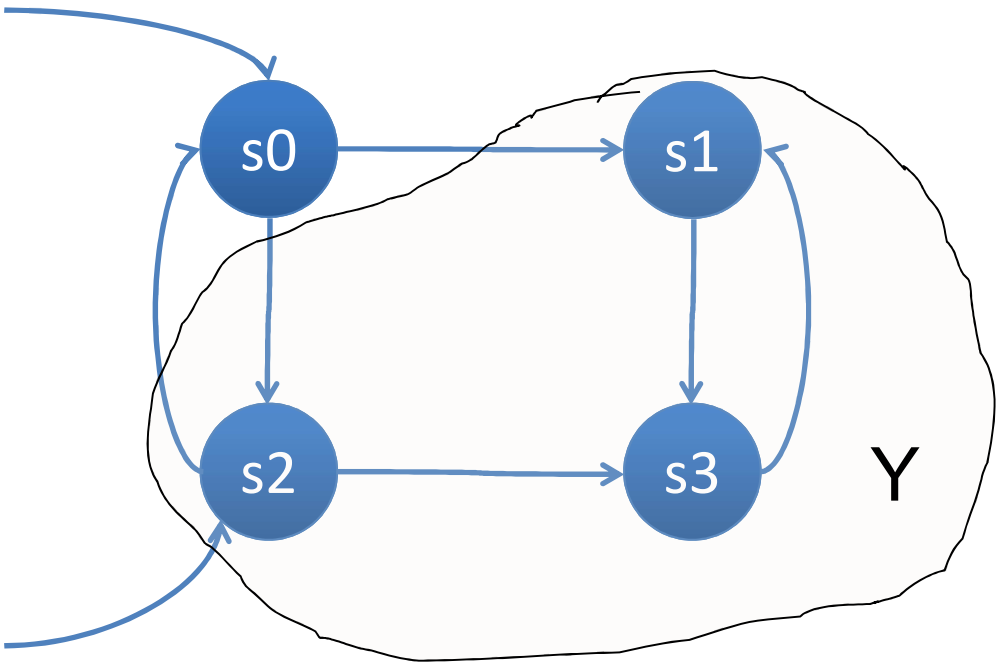
s1, s2 and s3 are Y-states

4a. Path Properties

A path (computation) π
is called a Y-path (Y-computation),
if all states appear on π satisfy Y.

A path (computation) π reaches (passes) Y-states,
if some state appears on π satisfies Y.

Example: Y-Paths, Y-Computations



(s1s3) is a finite Y-path
$(s1s3)^{\omega}$ is a Y-path
$s2(s1s3)^{\omega}$ is a Y-computation

Complementary Sets - Negation

Y-States, Y-Paths, Y-Computations

Notation: $\neg Y \equiv (S \backslash Y)$

$\neg$Y-States, $\neg$Y-Paths, $\neg$Y-Computations

Reachability

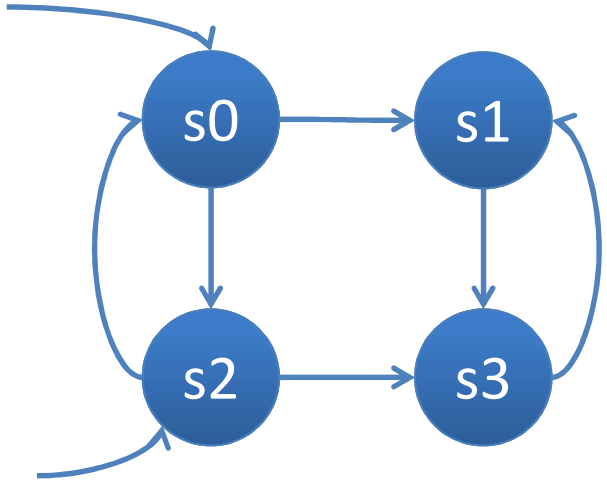
Y is a reachable, if

there is a computation of K that reaches a Y-state.

Reachability Problem:

Given a set Y.

Is Y reachable?



Avoidability

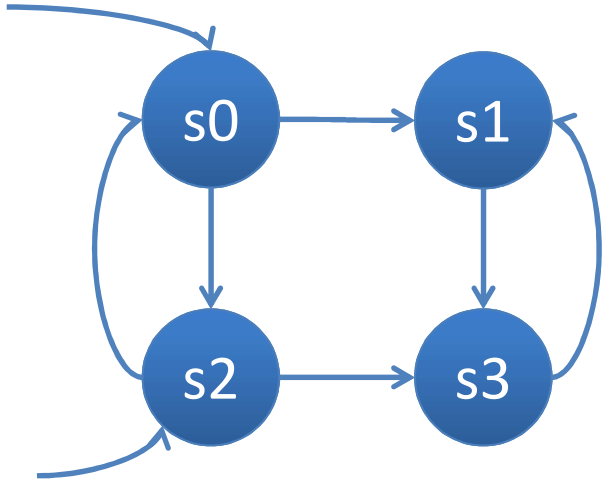
Y is avoidable, if

there a computation that does not reach any Y-state.

Avoidability Problem:

Given a set Y.

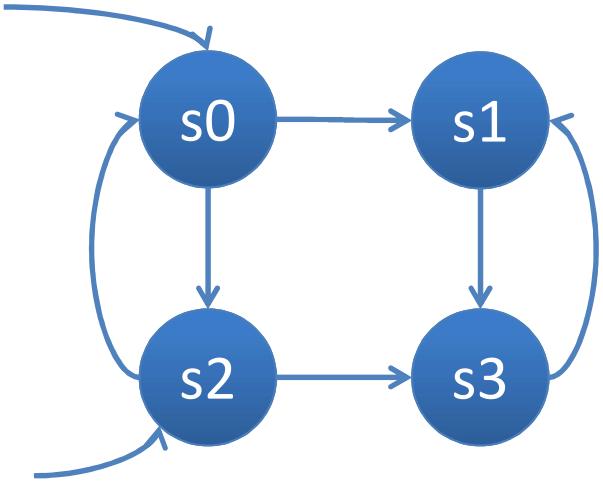
Is Y avoidable?



Basic System Properties

(1) Reachability, Safety

(2) Avoidability, Inevitability

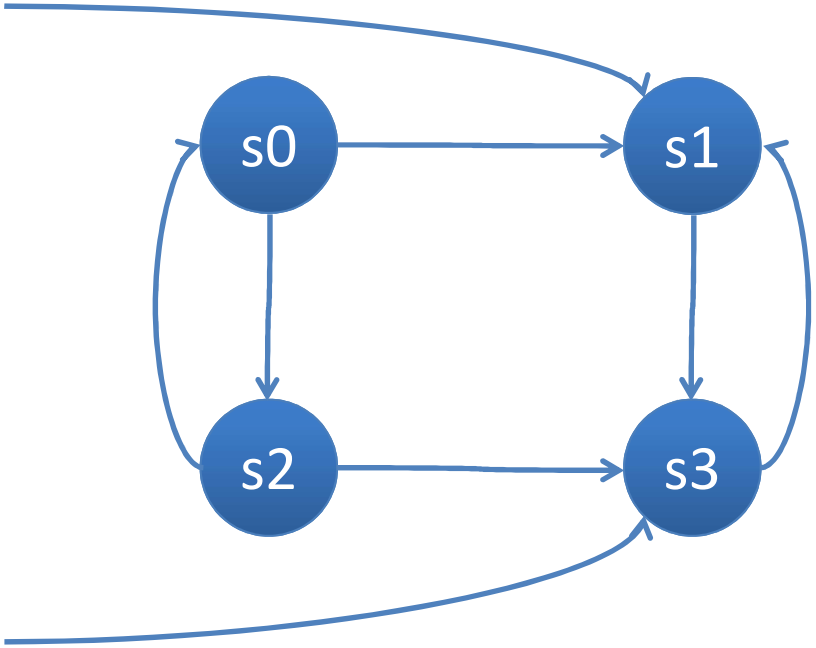


(II) Reachability Property - Possibility

Let Y be a set of states.

Y is a reachability property, if
there is a computation of K that reaches a Y-state.

Example: {s0,s2}, {s0,s1,s2}



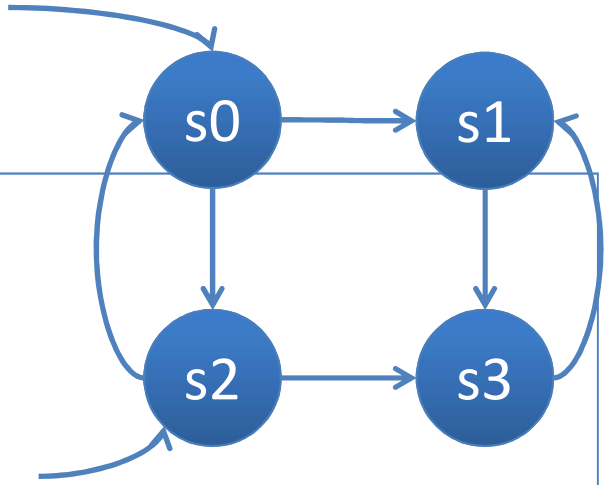
Reachability Problem

Given a set $A \subseteq S$.

Is A a reachability property?

Reachability Analysis

Let K=<S,R,I> and A $\subseteq$S.




```
BOOL ReachabilityAnalysis(K,A)
  {     w:=I;
        repeat until w={};
                s:=w.getElement(); if (s in A) return true;
                visited[s]:=true;
                for each (s' in R(s)),
                        if (visited[s']=false) w.putElement(s');
                w.removeElement(s);
        return false;
  }
```

Reachability Analysis

Let K=<S,R,I> be a Kripke structure, and $A \subseteq S$.

Proposition:

ReachabilityAnalysis(K,A) =true

Iff

A is a reachability property

Reachability Analysis (Alternatively)

Let K=<S,R,I> be a Kripke structure, and $A \subseteq S$.

Proposition:

A is a reachability property, iff $Rh(K) \cap A \neq \emptyset$

➔ Reachability analysis based on fixpoint computation

Fixpoints

- Let f: A → A be a function.
- $a \in A$ is a fixpoint of f, if f(a)=a
- Questions:

  Existence of a fixpoint?

  Computation of a fixpoint?

Fixpoints

- Let S be a finite set and $(2^S, \subseteq)$ be a lattice.
- Let f: $2^S \rightarrow 2^S$ be a monotonic function.

- The least fixpoint of f denoted μf (or $\mu Z.f(Z)$) is:

  $\mu f = \cup \{ f^k(\emptyset) \mid k=0,1,2,\ldots \}$

- The greatest fixpoint of f denoted νf (or $\nu Z.f(Z)$) is:

  $\nu f = \cap \{ f^k(S) \mid k=0,1,2,\ldots \}$

Reachability Analysis (FP)

Let K=<S,R,I> be a Kripke structure, and $A \subseteq S$.

Let $f: 2^S \rightarrow 2^S$ be defined by $f(w) = I \cup R(w)$

THEN

$Rh(K) = \mu f$

A is a reachability property, iff $\mu f \cap A \neq \emptyset$

Reachability Analysis (FP)

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

```
SET leastfixpoint(f,K)
{
    w={};
    repeat
          w’:=w;
          w :=f(w,K);
    until w’=w;
    return w;
}
```

Reachability Analysis (FP)

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

```
SET leastfixpoint(f,K)
{
    w={};
    repeat
        w':=w;
        w :=f(w,K);
    until w'=w;
    return w;
}
```

```
SET f(w,K)
{
    return I ∪ R(w);
}

BOOL ReachabilityAnalysisFP(K,A)
{
    w=leastfixpoint(f,K);
    return ( w ∩ A ≠ ∅ ) ;
}
```

Reachability Analysis (FP)

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

Proposition:

ReachabilityAnalysisFP(K,A) =true

Iff

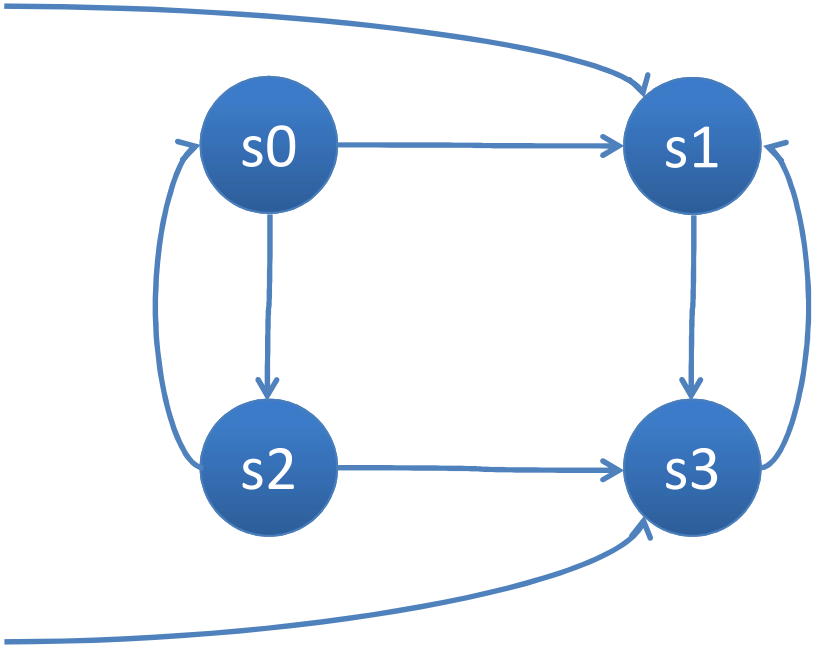
A is a reachability property

Safety Property - Universality

Let Y be a set of states.

Y is a safety property, if
every computation is a Y-computation.

Example: {s2,s3}, {s1,s2,s3}



Reachability & Safety

Safety is a dual property of reachability.

Proposition
A system K=<S,R,I> is safe with respect to Y, iff
$Rh(K) \subseteq Y$.

Corollary [Duality]
A system K=<S,R,I> is safe with respect to Y, iff
$\neg Y$ is not reachable in K.

Safety Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

```
BOOL SafetyAnalysis(K,A)
 {   w:=I;
     repeat until w={};
          s:=w.getElement(); if (s not in A) return false;
               visited[s]:=true;
               for each (s' in R(s)),
                    if (visited[s']=false) w.putElement(s');
               w.removeElement(s);
       return true;
   }
```

Safety Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

Proposition:

SafetyAnalysis(K,A) =true

Iff

A is a safety property

Safety Analysis (FP)

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

```
SET leastfixpoint(f,K)
{
    w={};
    repeat
        w':=w;
        w :=f(w,K);
    until w'=w;
    return w;
}
```

```
SET f(w,K)
{
    return I ∪ R(w);
}

BOOL SafetyAnalysisFP(K,A)
{
    w=leastfixpoint(f,K);
    return ( w ∩ (¬A) = ∅ ) ;
}
```

Safety Analysis (FP)

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

Proposition:

SafetyAnalysisFP(K,A) =true

Iff

A is a safety property

Reachability & Safety

SafetyAnalysis(K,A) =true $\Leftrightarrow$
ReachabilityAnalysis(K,$\neg$A) =false

Deductive Safety Analysis

- Transition invariant
- System invariant
- Inductive invariant

Transition Invariant

K=(S,R,I)

Definition

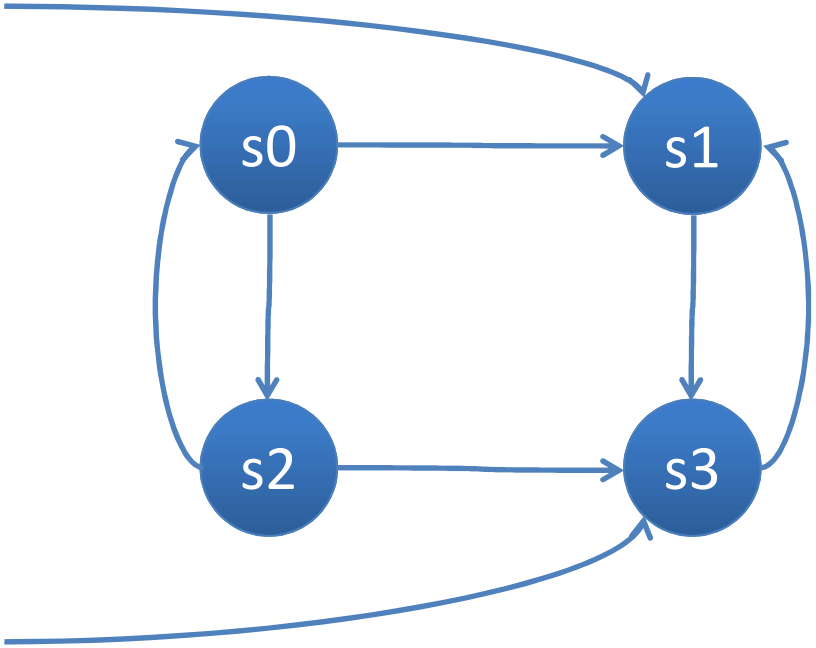
X is a transition invariant, if

for every s $\in$X, if s$\rightarrow$s', then s' $\in$X.

Lemma

If R(X) $\subseteq$X, then X is a transition invariant.

Example: {s1,s2,s3}×, {s0,s2,s3}×, {s1,s3}√



System Invariant

Definition

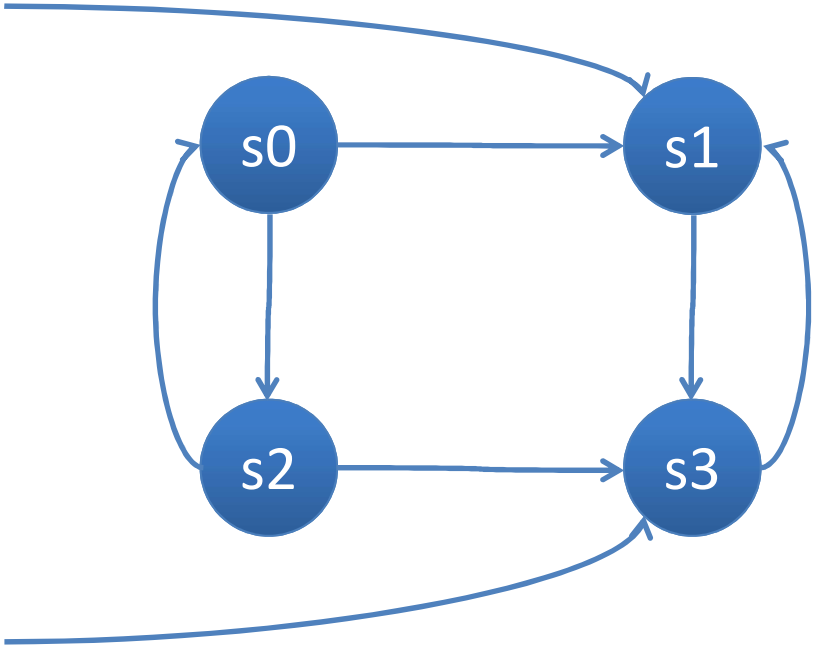
X is a system invariant, if

$I \subseteq X$ and $X \cap Rh(K)$ is a transition invariant.

Lemma

X is a system invariant iff X is safety property.

Example: {s1,s2,s3}√, {s0,s2,s3}×, {s1,s3}√



Inductive Invariant

Definition

X is an inductive invariant, if

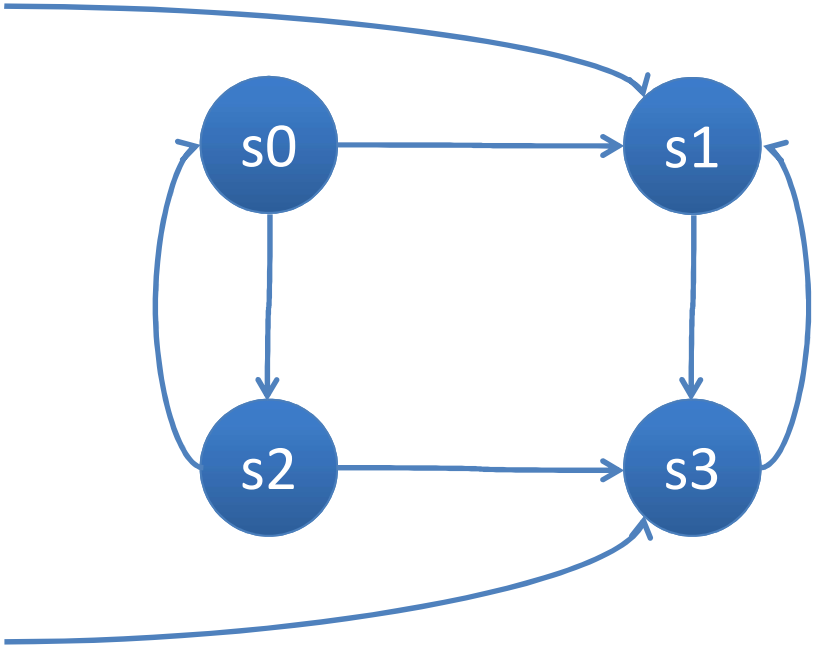
I $\subseteq$ X and X is a transition invariant.

Lemma

X is an inductive invariant iff

X is a system invariant and a transition invariant.

Example: {s1,s2,s3}×, {s0,s2,s3}×, {s1,s3}√



Comparison

Rh(K) is an inductive Invariant；

an inductive invariant is a system invariant；

X is a system invariant iff X is a safety property.

Proof of Safety

Given X.

Question: is X a safety property of K?

Proof of Safety (1)

Given X.

Question: is X a safety property of K?

Lemma

If X is an inductive invariant,

then K is safe w.r.t. X.

Proof of Safety (1)

Given X.

Question: is X a safety property of K?

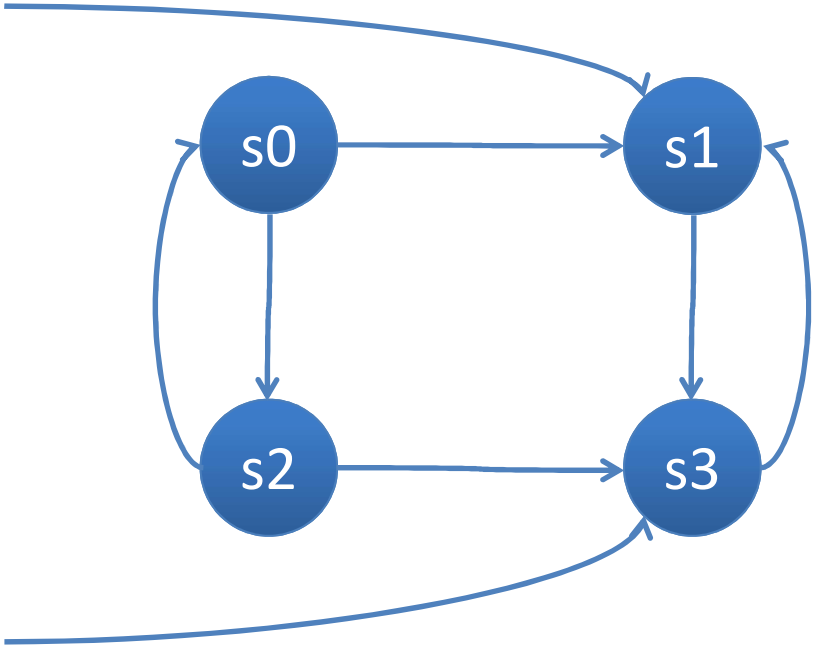
$I \subseteq X$

$R(X) \subseteq X$

X is a safety property

(not always applicable)

Example: {s1,s3}, {s1,s2,s3}



Proof of Safety (2)

Given X.

Question: is X a safety property of K?

Lemma

If there is an inductive invariant Y such that $Y \subseteq X$,

then K is safe w.r.t. X.

Completeness

If the conclusion holds, then such a Y exists.

Proof of Safety (2)

Given X.

Question: is X a safety property of K?

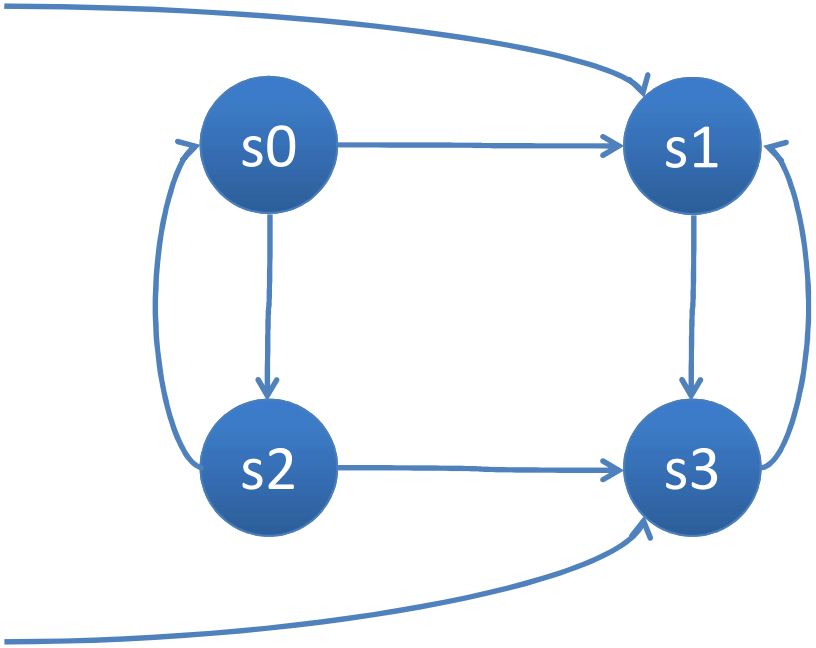
$I \subseteq Y$

$R(Y) \subseteq Y$

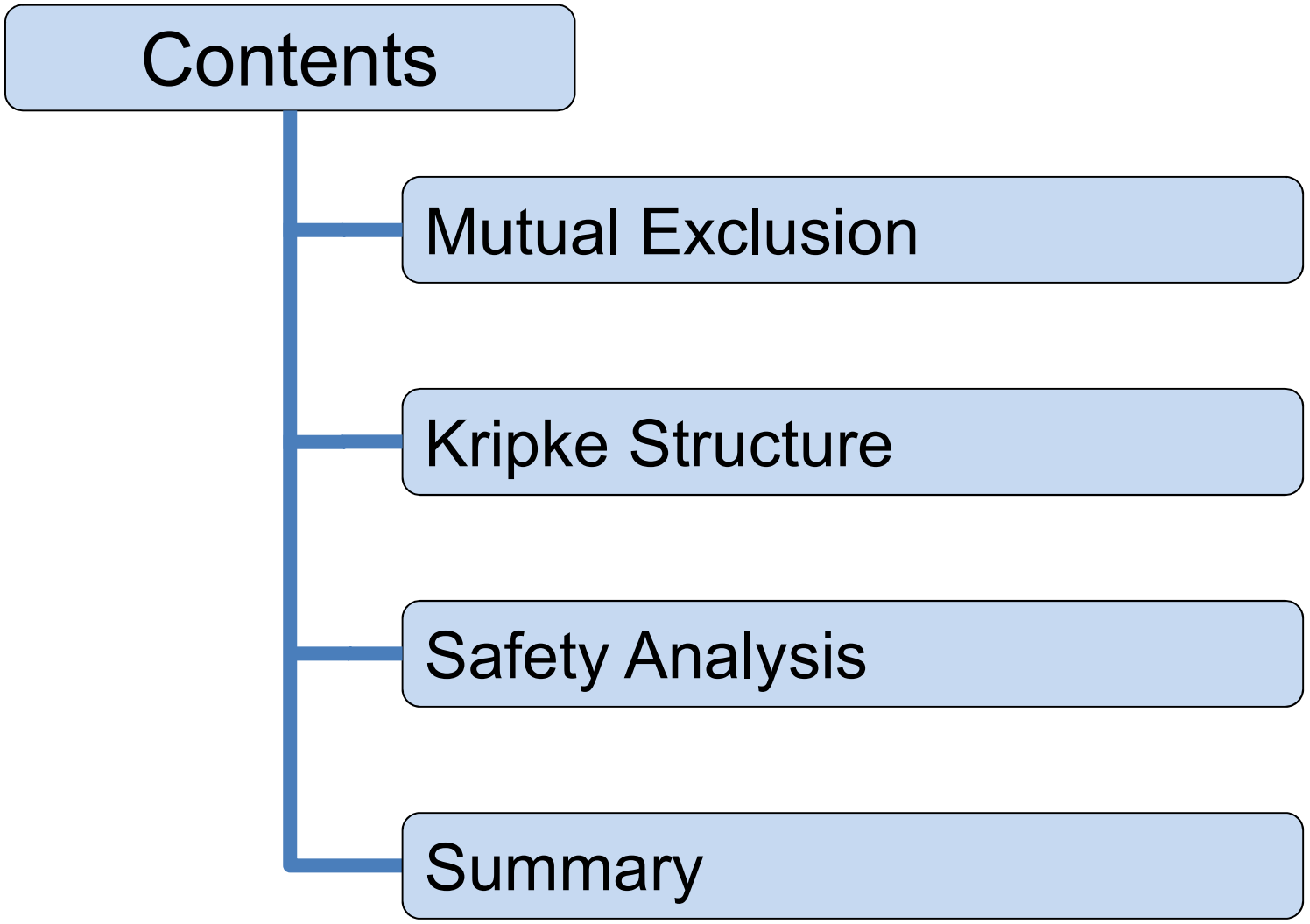
$Y \subseteq X$

X is a safety property

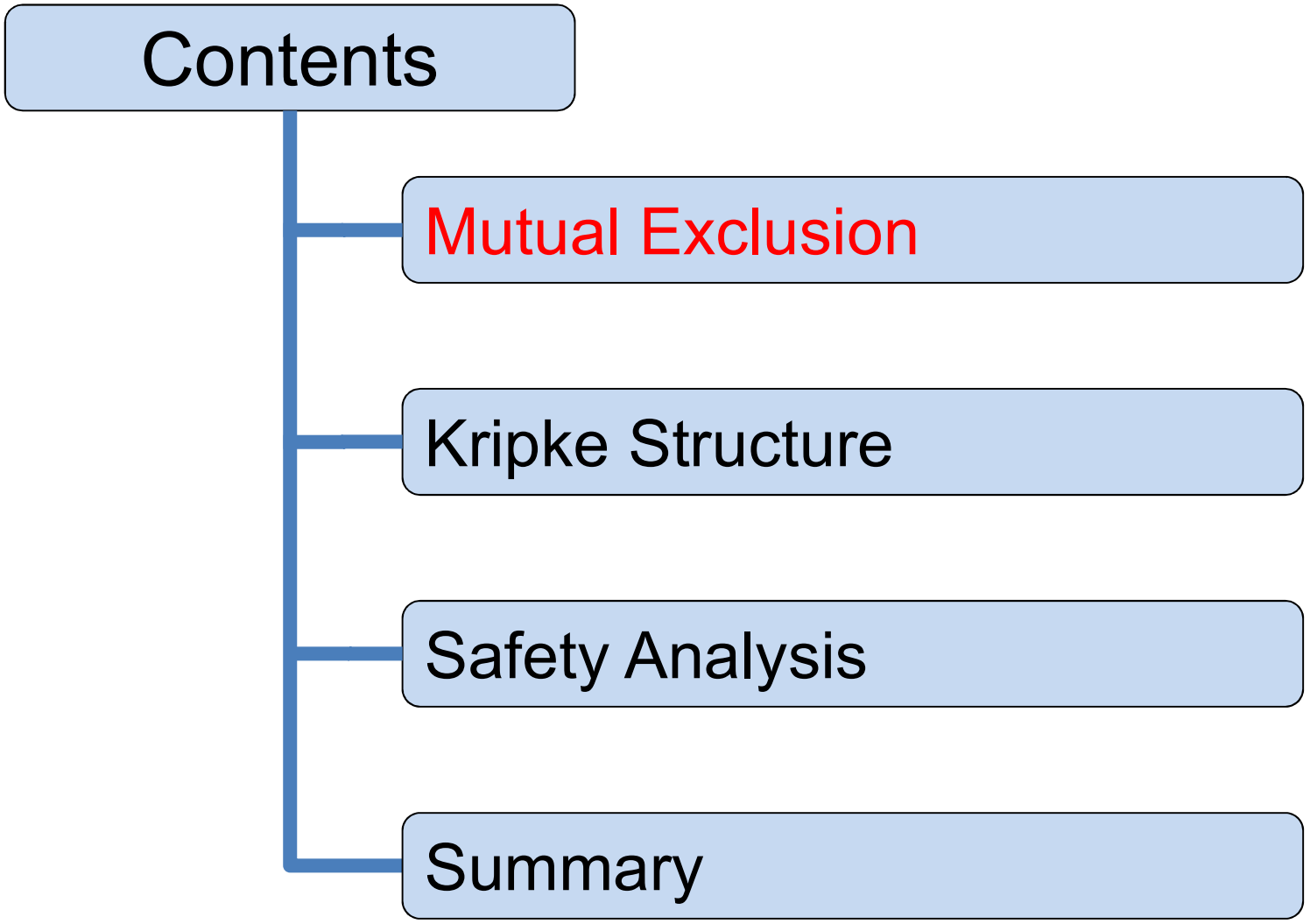
Example: {s1,s3}, {s1,s2,s3}



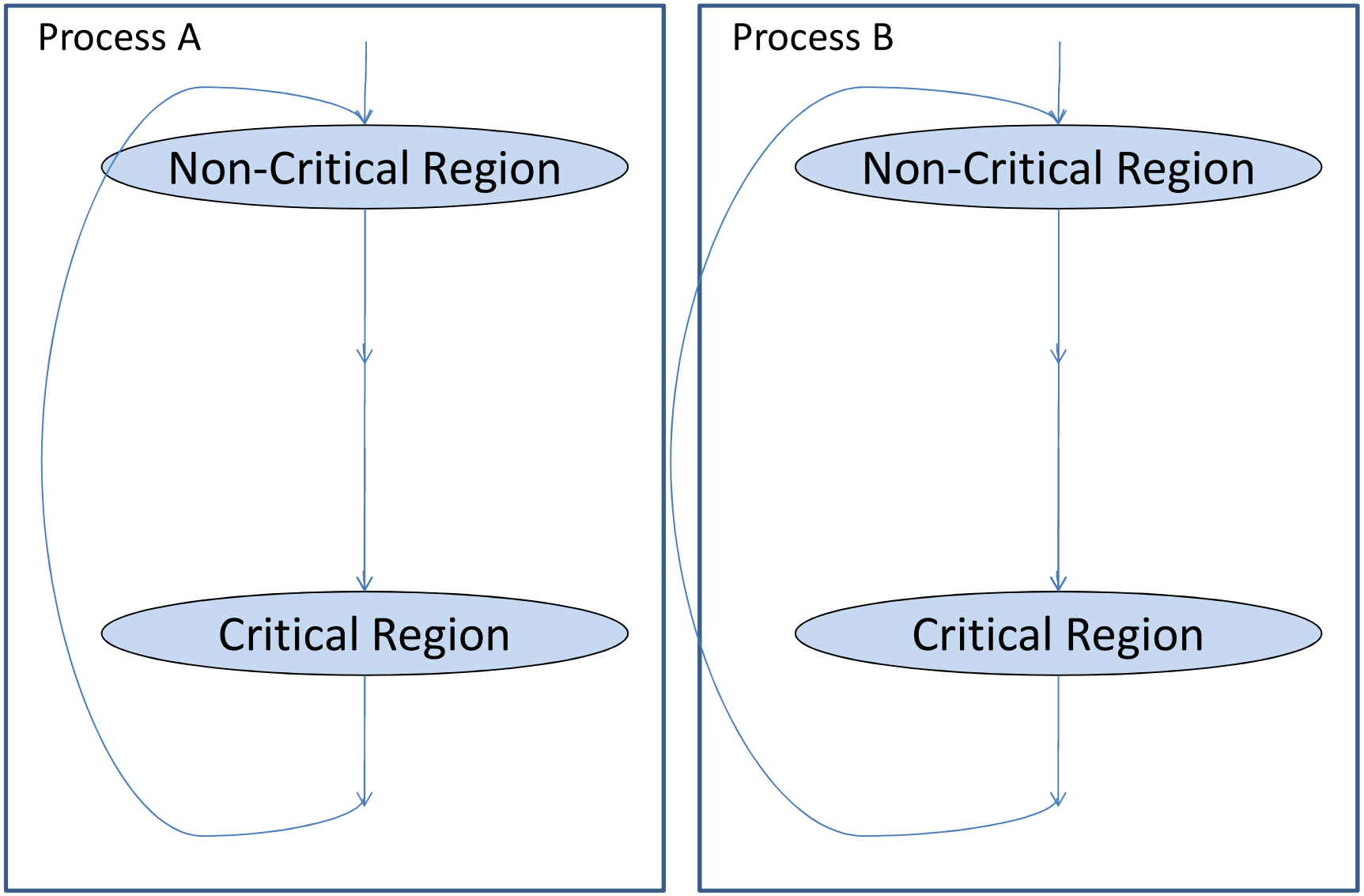
Safety Analysis – An Example



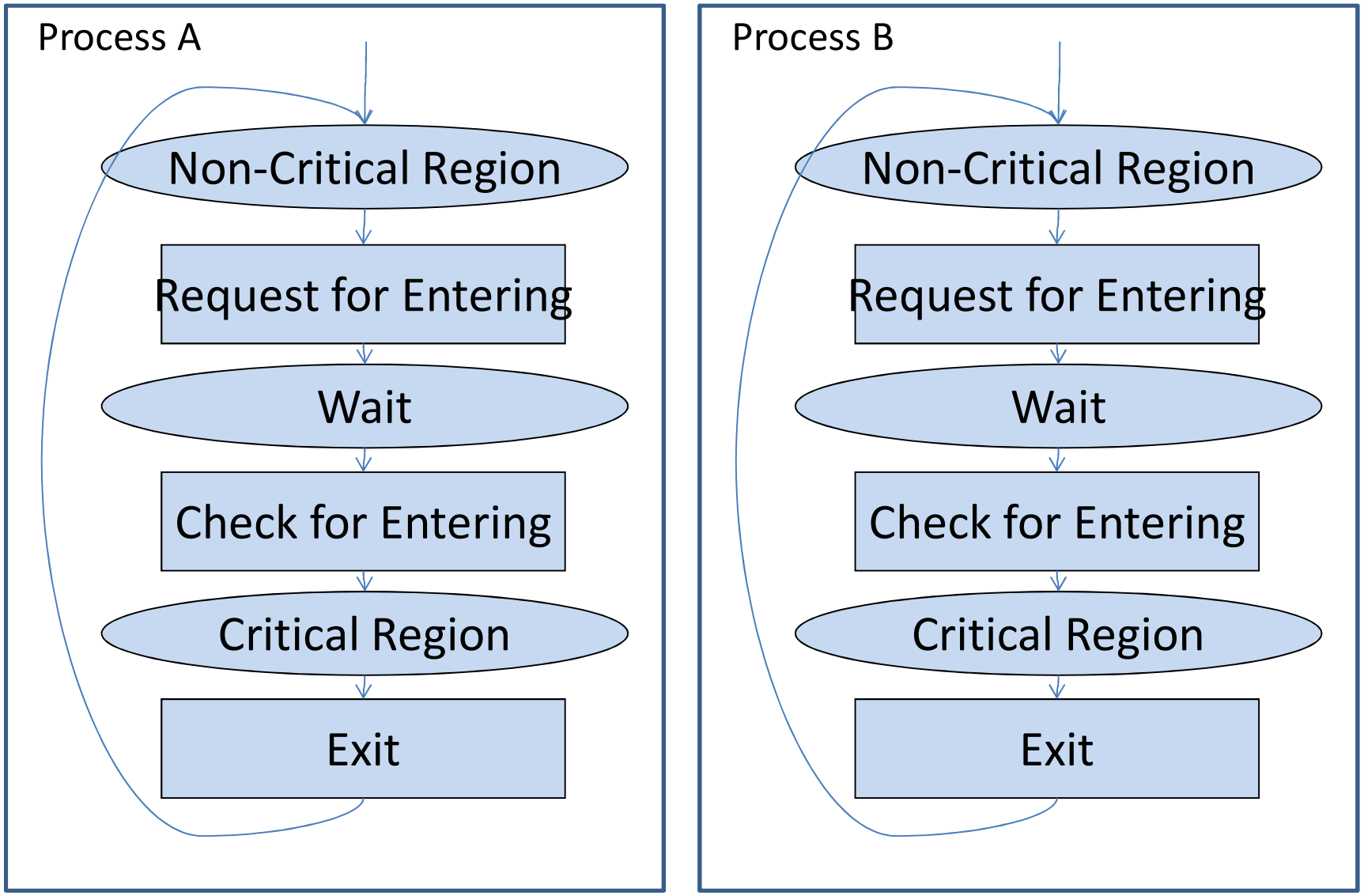
Safety Analysis – An Example



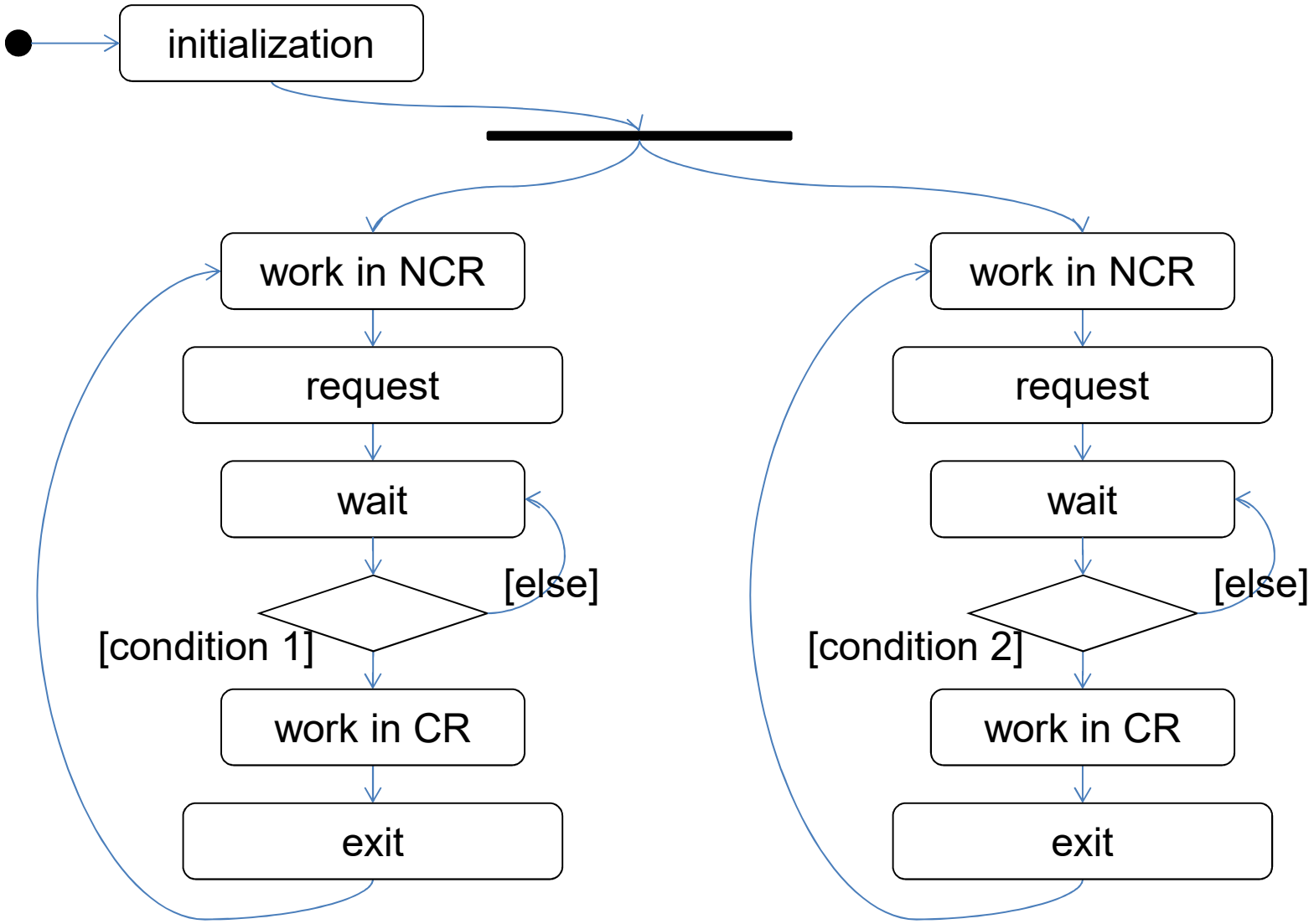
Example: Mutual Exclusion



Example: Mutual Exclusion



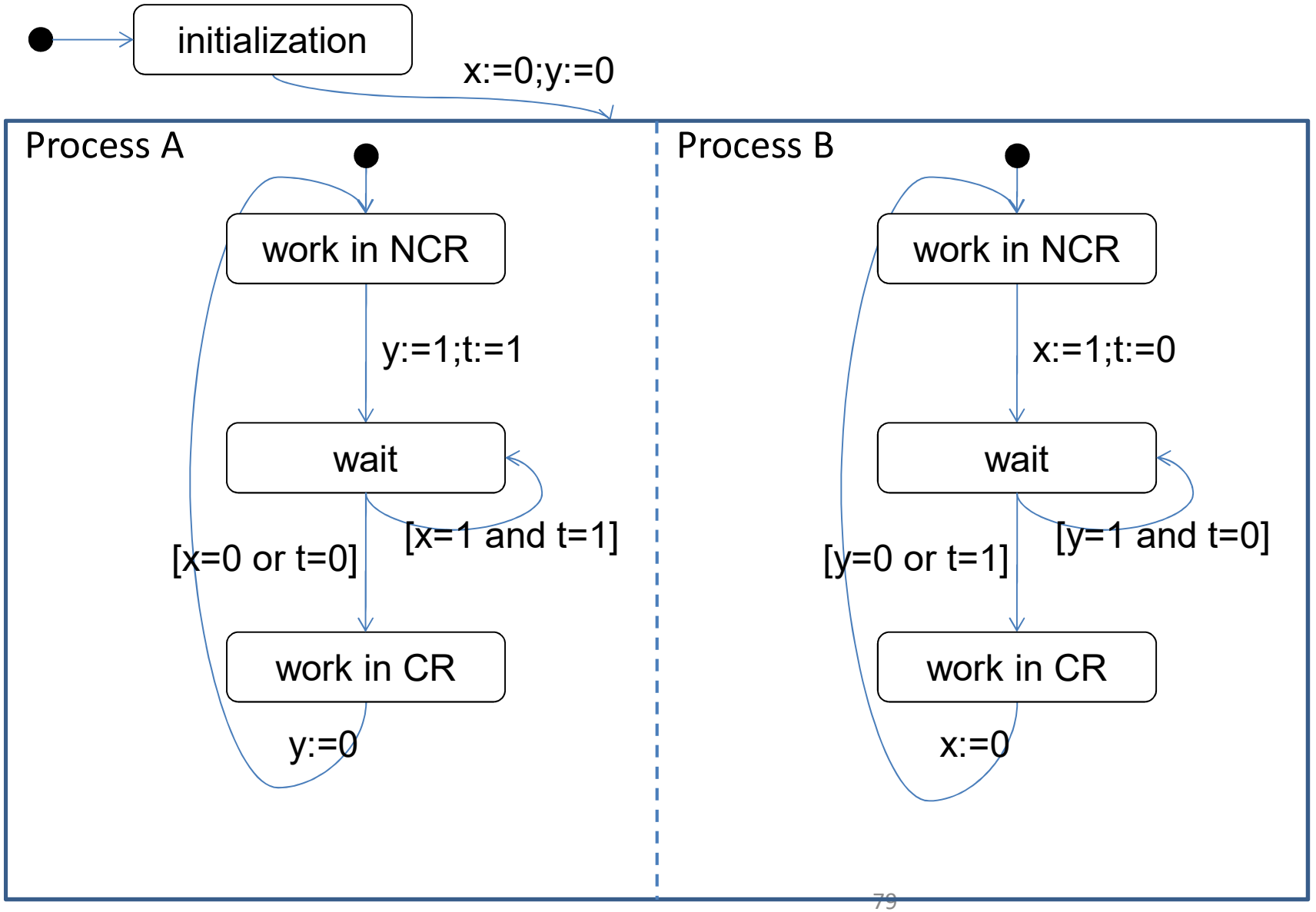
Design of Mutual Exclusion (Activity)



Design of Mutual Exclusion

- Purpose:
  - ensure that not both processes are working in the critical region (CR)
- Mechanism:
  - use shared variables
  - y=1: the first process is applying for entering CR or it is in CR
  - x=1: the second process is applying for entering CR or it is in CR
  - t=(i-1): the i-th process has priority for entering CR

Design of Mutual Exclusion (State)



Correctness of the Design

- How do we know that the design is correct?

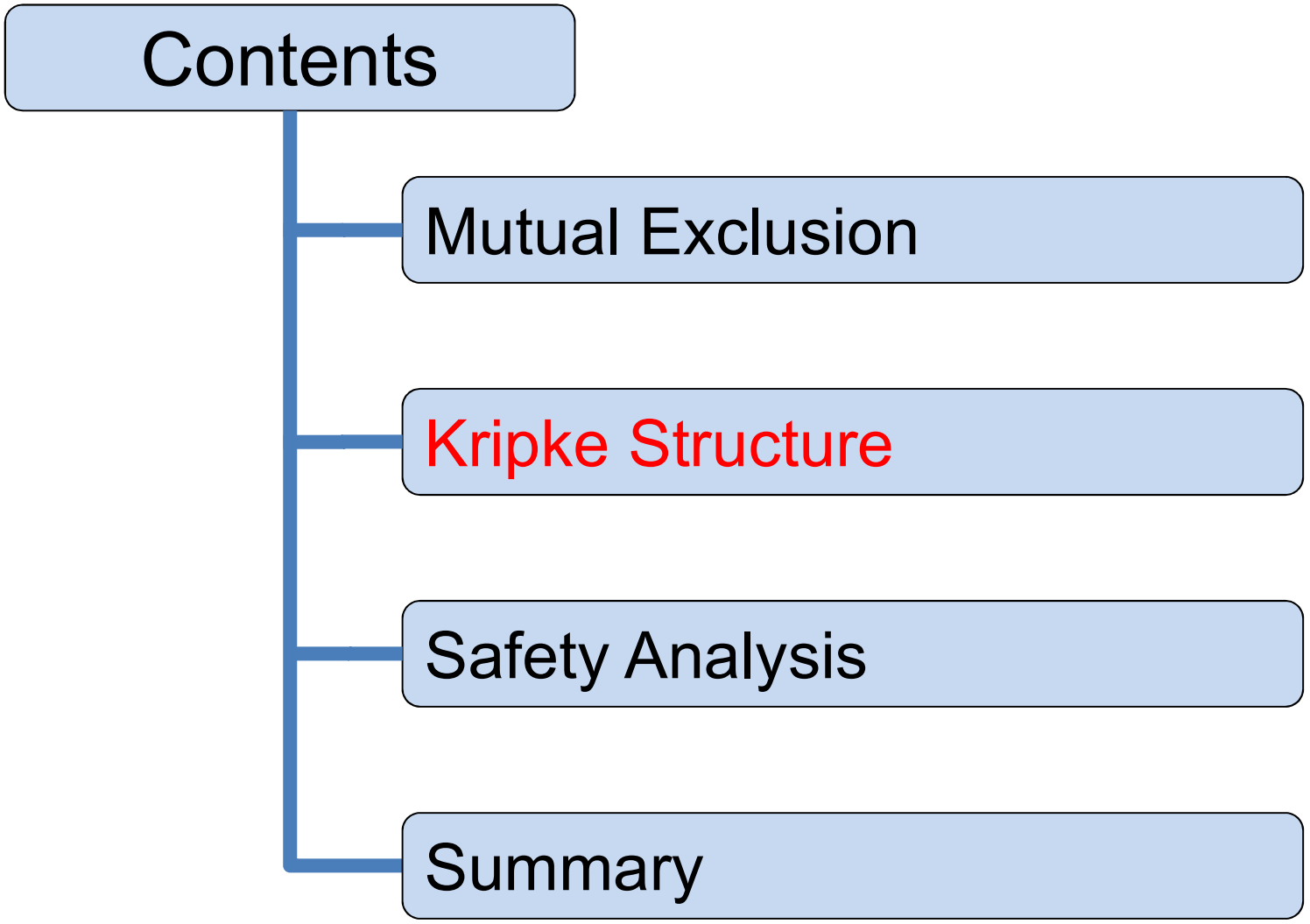
Combined States of the Two Processes

Process A	Process B	x,y,t	Remark
NCR	NCR		
NCR	wait		
NCR	CR		
wait	NCR		
wait	wait		
wait	CR		
CR	NCR		
CR	wait		
CR	CR		Bad state

Correctness of the Design

- How do we know that the design is correct?
  - We have to be sure that the bad state is not reachable in all possible executions of the algorithm
  - We may use state exploration (model checking) techniques or deductive proof methods

Safety Analysis – An Example



Kripke Structures

A Kripke structure is a triple K=<S,R,I>

- S : A finite set of states
- $R \subseteq S \times S$: A total transition relation
- $I \subseteq S$: A set of initial states

Domains of Process and Variable States

Process A	Process B	x	y	t
NCR	NCR	1	1	1
wait	wait	0	0	0
CR	CR			

(a,b,x,y,t)

The Set of States: S

$\{(a,b,x,y,t) \mid a,b \in \{NCR,wait,CR\} \text{ and } x,y,t \in \{0,1\}\}$

Transition Relation: R

(NCR,b,x,y,t)	→ (wait,b,x,1,1)
(wait,b,0,y,t)	→ (CR,b,0,y,t)
(wait,b,x,y,0)	→ (CR,b,x,y,0)
(wait,b,1,y,1)	→ (wait,b,1,y,1)
(CR,b,x,y,t)	→ (NCR,b,x,0,t)
(a,NCR,x,y,t)	→ (a,wait,1,y,0)
(a,wait,x,0,t)	→ (a,CR,x,0,t)
(a,wait,x,y,1)	→ (a,CR,x,y,1)
(a,wait,x,1,0)	→ (a,wait,x,1,0)
(a,CR,x,y,t)	→ (a,NCR,0,y,t)

The Set of Initial States: I

{ (NCR,NCR,0,0,0), (NCR,NCR,0,0,1) }

Safe States

(NCR,b,x,y,t)

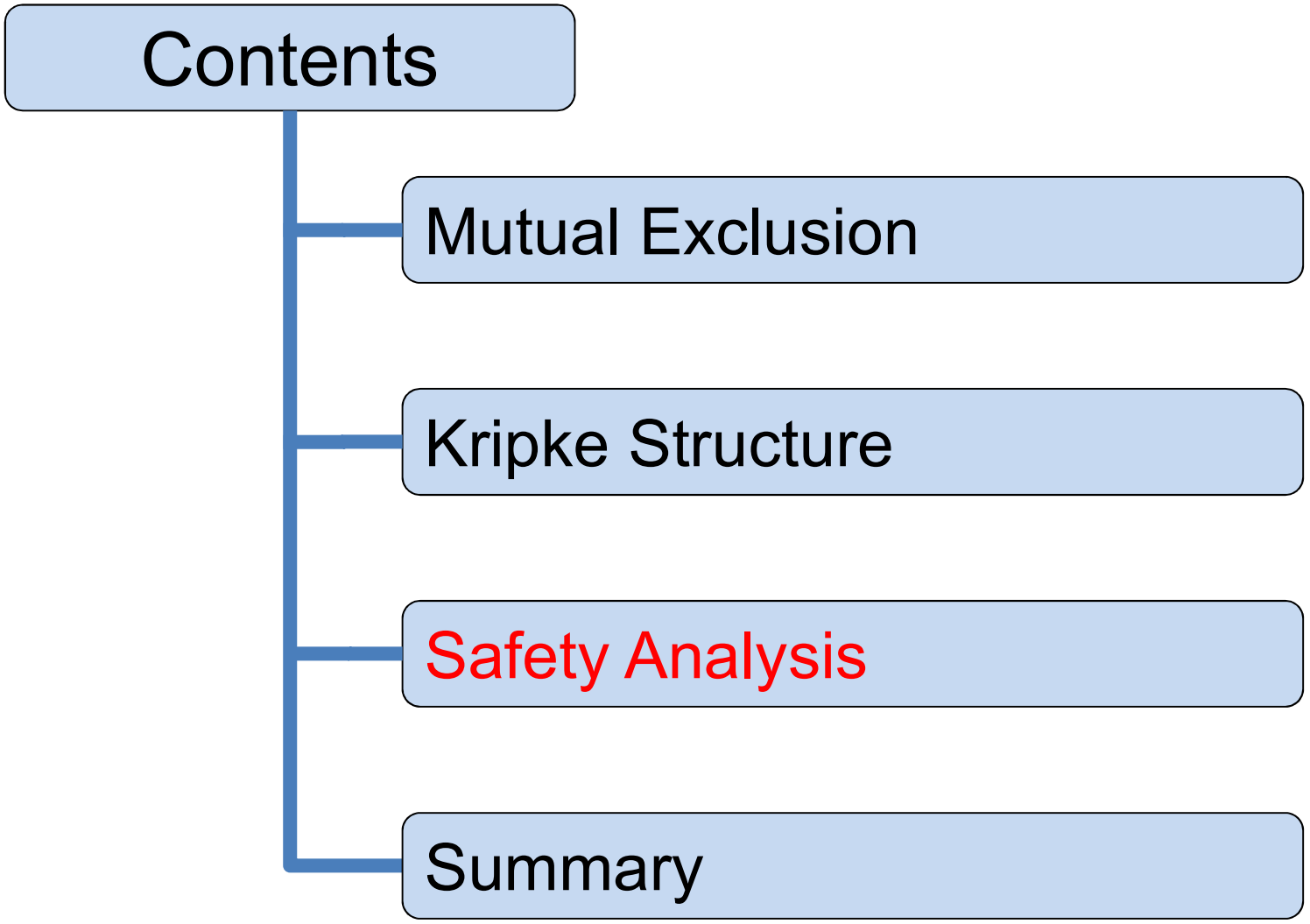
(wait,b,x,y,t)

(CR,NCR,x,y,t)

(CR,wait,x,y,t)

The set of unsafe states: (CR,CR,x,y,t)

Safety Analysis – An Example



Safety Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

```
BOOL SafetyAnalysis(K,A)
 {   w:=I;
     repeat until w={};
          s:=w.getElement(); if (s not in A) return false;
               visited[s]:=true;
               for each (s' in R(s)),
                    if (visited[s']=false) w.putElement(s');
               w.removeElement(s);
       return true;
   }
```

Safety Analysis

Let K=<S,R,I> be a Kripke structure, and Y $\subseteq$S.

Proposition:

SafetyAnalysis(K,Y) =true

Iff

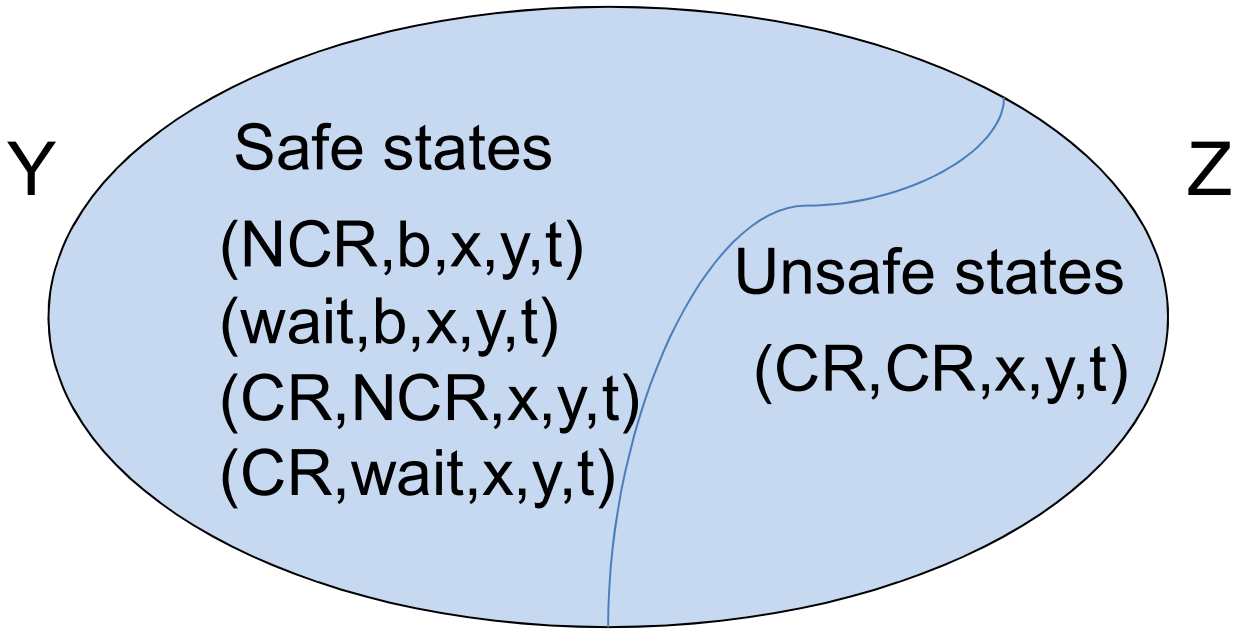
Y is a safety property

Iff

Every reachable state of K is in Y

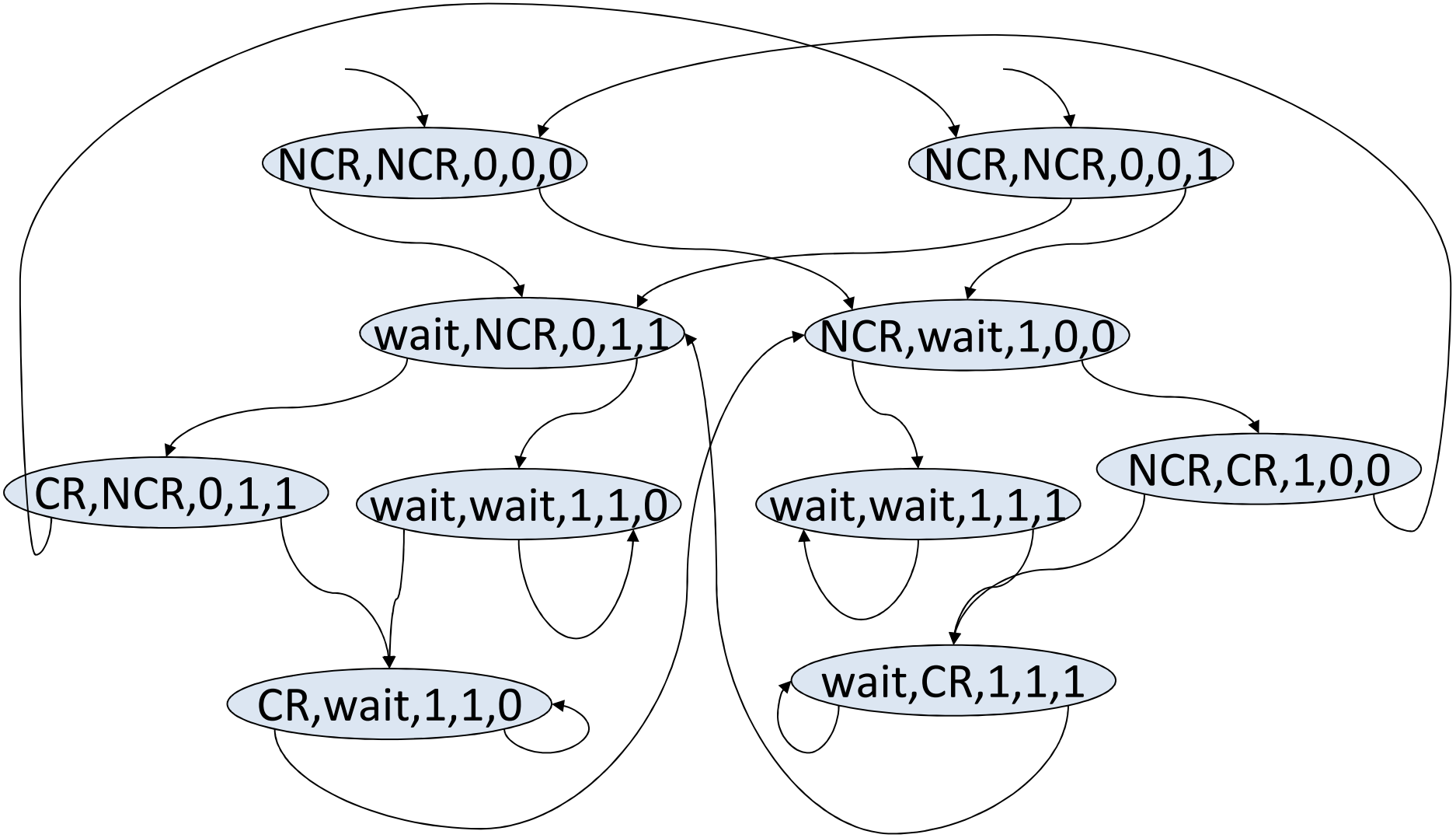
(K is safe with respect to Y)

Safety Analysis

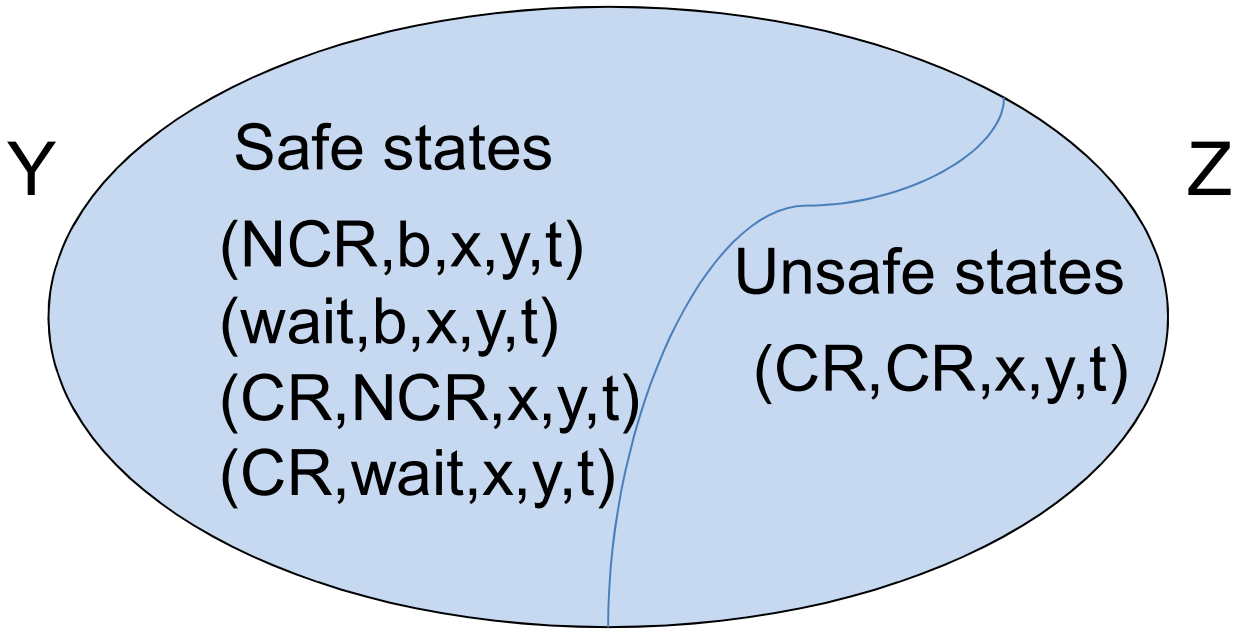


SafetyAnalysis(K,Y)=?

Safety Analysis



Safety Analysis



SafetyAnalysis(K,Y)=true ⇔
Every reachable state is a Y-state (safe state) ⇔
K is safe w.r.t. Y

Deductive Proof of the Safety Property

(NCR,b,x,y,t)

(wait,b,x,y,t)

(CR,NCR,x,y,t)

(CR,wait,x,y,t)

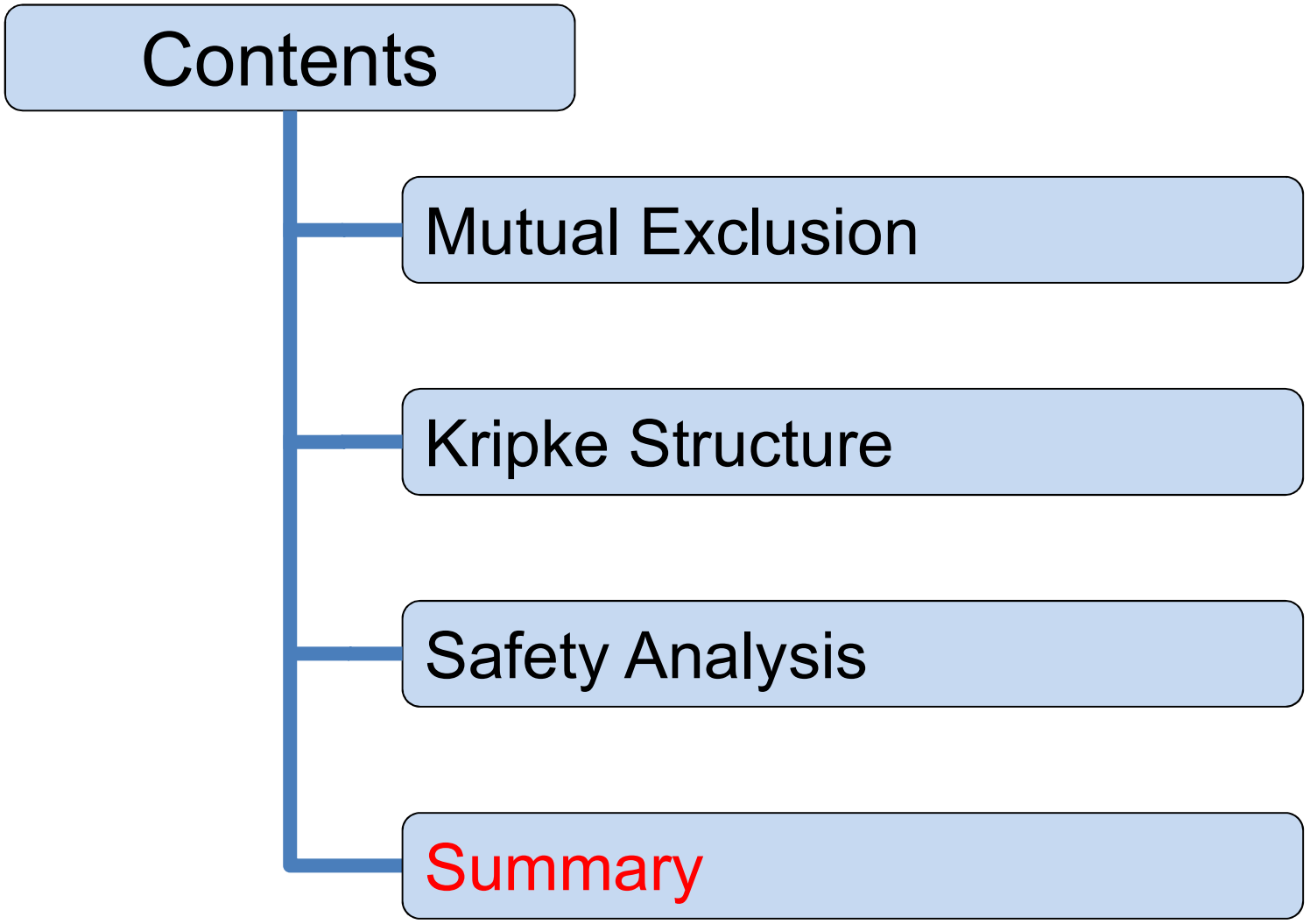
Is this set (Y) a tranition invariant?

No, e.g., (CR,wait,x,y,1)→ (CR,CR,x,y,1)

Inductive Set (X)

(NCR,NCR,0,0,t) (NCR,wait,1,0,t) (wait,NCR,0,1,t) (NCR,CR,1,0,t) (CR,NCR,0,1,t) (wait,wait,1,1,t) (CR,wait,1,1,0) (wait,CR,1,1,1)	$I \subseteq X$ $s \in X \Rightarrow ((s \rightarrow s') \Rightarrow s' \in X)$ $X \subseteq Y$ ---------- Y is a safety property

Safety Analysis – An Example



Correctness of the Design

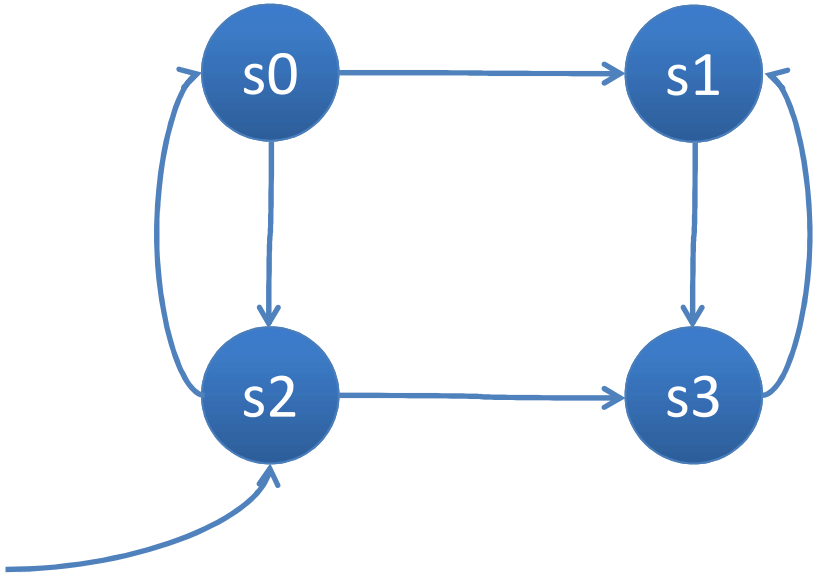
- How do we know that the design is correct?
  - We have to be sure that the bad state is not reachable in all possible executions of the algorithm
  - We may use state exploration (model checking) techniques, or deductive proof methods
  - We have shown that the bad states are not reachable

(III) Avoidability Property

Let Y be a set of states.

Y is avoidable, if
there a computation that does not reach any Y-state.

Example: {s1,s3}√, {s0}√, {s0,s1} x



Avoidability Problem

Given a set $A \subseteq S$.

Is A an avoidability property?

Avoidability Problem

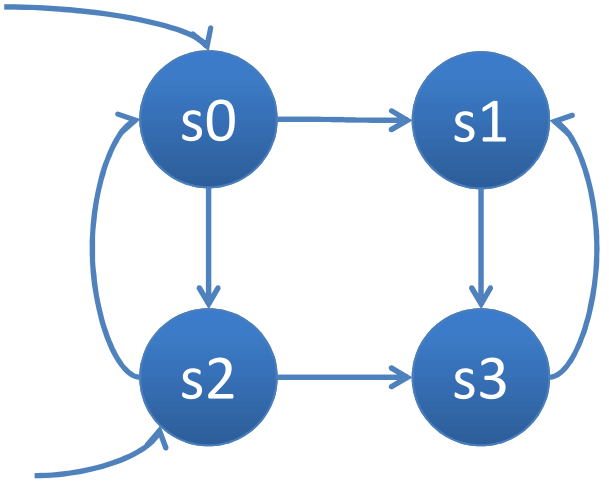
Lemma

For a finite state system <S,R,I>:

A is avoidable

iff

there is a ($\neg$A)-computation



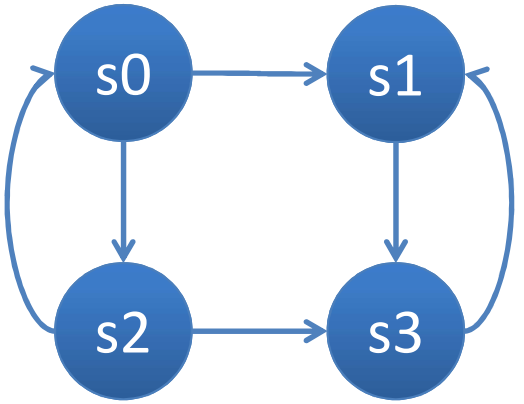
Example:

A = { s1,s3}

($\neg$A)-computation: $(s0s2)^{\omega}$

Directed Graphs, G=(V,E)

- Strongly connected graphs
- Subgraphs
- Strongly connected subgraphs
- Derived subgraphs
- Strongly connected components
- Non-trivial Strongly connected components



Avoidability Problem

Let $R|Y = R \cap (Y \times Y)$

A is avoidable

iff

there is a $(\neg A)$-computation

iff

there is a $(\neg A)$-path starting from I that reaches a non-trivial strongly connected component of $< \neg A, R|\neg A >$.

Avoidability Problem

For a finite state system K=<S,R,I>:

Let $K|X = \langle X, R|X, I \cap X \rangle$

Let $K' = \langle S', R', I' \rangle = K|\neg A$

A is avoidable, iff

there is a reachable non-trivial SCC of (S',R') in K' .

Tarjan Algorithm (scctarjan)

algorithm scctarjan； **input:** $G = (V, E)$； **output:** A set of SCCs;

index := 0; *S* := empty;

for each *v* **in** *V* **do if** (*v*.index is undefined) strongconnect(*v*) **endif endfor**

function strongconnect； **input:** *v*； **output:** A set of SCCs;

v.index := *v*.lowlink := *index; index := index +1*; *S*.push(*v*);

for each (*v*, *w*) **in** *E* **do**

if (*w*.index is undefined) **then**

strongconnect(*w*); *v*.lowlink := min(*v*.lowlink, *w*.lowlink)

else if (*w* is in *S*) **then**

v.lowlink := min(*v*.lowlink, *w*.index)

endif

endfor

if (*v*.lowlink = *v*.index) **then** // SCCok(v);

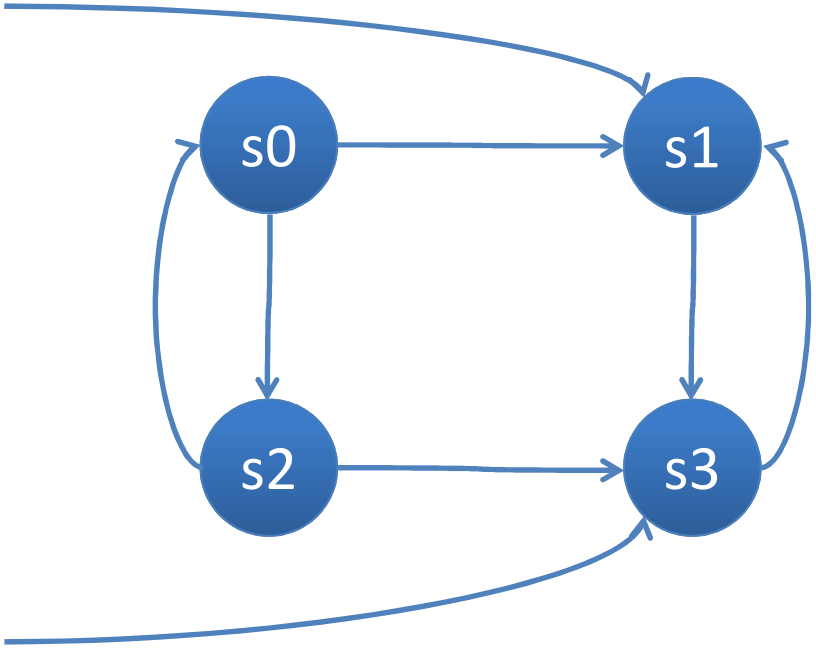
currentSCC := empty;

repeat *w* := *S*.pop(); add *w* to currentSCC; **until** (*w* = *v*);

output currentSCC

endif

Example:



{s0,s2}, {s1,s3}

Correctness and Complexity

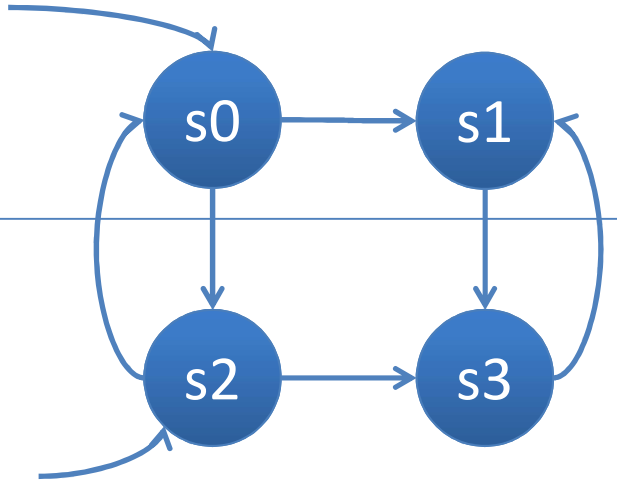
Given G=(V,E).

The output of scctarjan(G) is a set of SCCs that is a partition of G.

The complexity of scctarjan(G) is O(|V|+|E|).

Avoidability Analysis

Let K=<S,R,I> and A $\subseteq$S.




```
BOOL AvoidabilityAnalysis(K,A)
{     K':=(S',R',I'):= K|¬A;
      G:=(S',R');
      scclist:=scctarjan(G);
      w:={};
      for each (e in scclist) if (nontrivial(e)) w:=w∪e;
      return ReachbilityAnalysis(K',w);
}
```

Avoidability Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

Proposition:

AvoidabilityAnalysis(K,A) =true

iff

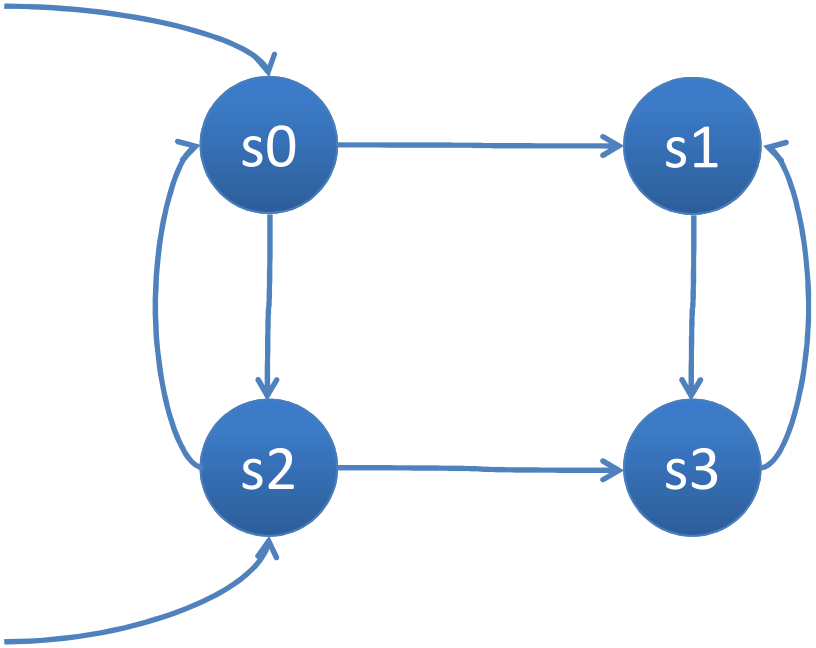
A is an avoidability property

Inevitability

Given a set Y.

Y is an inevitability property, if
every computation reaches a Y-state

Example: {s1,s3}, {s2,s3}



Avoidability & Inevitability

Inevitability is a negation of avoidability.

Proposition
Y is an inevitability property of K, iff
Y is not an avoidability property.

Inevitability Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

```
BOOL InevitabilityAnalysis(K,A)
{    K':=(S',R',I'):=K|¬A;
     G:=(S',R');
     scclist:=scctarjan(G);
     w:={};
     for each (e in scclist) if (nontrivial(e)) w:=w ∪ e;
     return (not ReachbilityAnalysis(K',w));
 }
```

Inevitability Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

Proposition:

InevitabilityAnalysis(K,A) =true

Iff

A is an inevitability property

Inevitability & Avoidability

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

InevitabilityAnalysis(K,A) =true $\Leftrightarrow$
AvoidabilityAnalysis(K,A) =false

Deductive Inevitability Analysis

Proof of Inevitability

If there is a sequence of sets of states:

$X_0, X_1,\ldots,X_n$ such that

$I \subseteq X_0$,

$R(X_i \backslash Y) \subseteq X_{i+1}$, for i=0,1,…,n-1

$X_n \subseteq Y$,

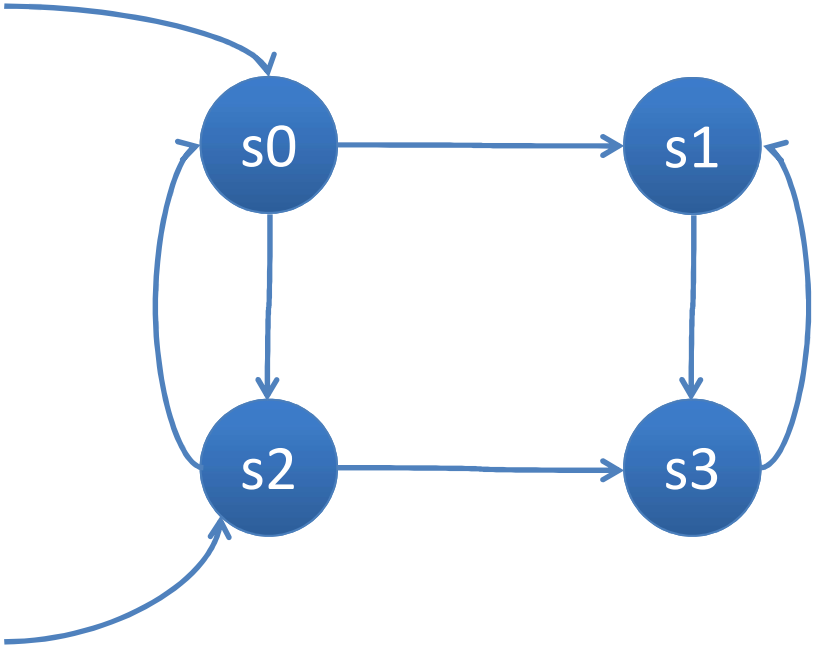
Then Y an inevitability property.

Completeness

For finite state systems,

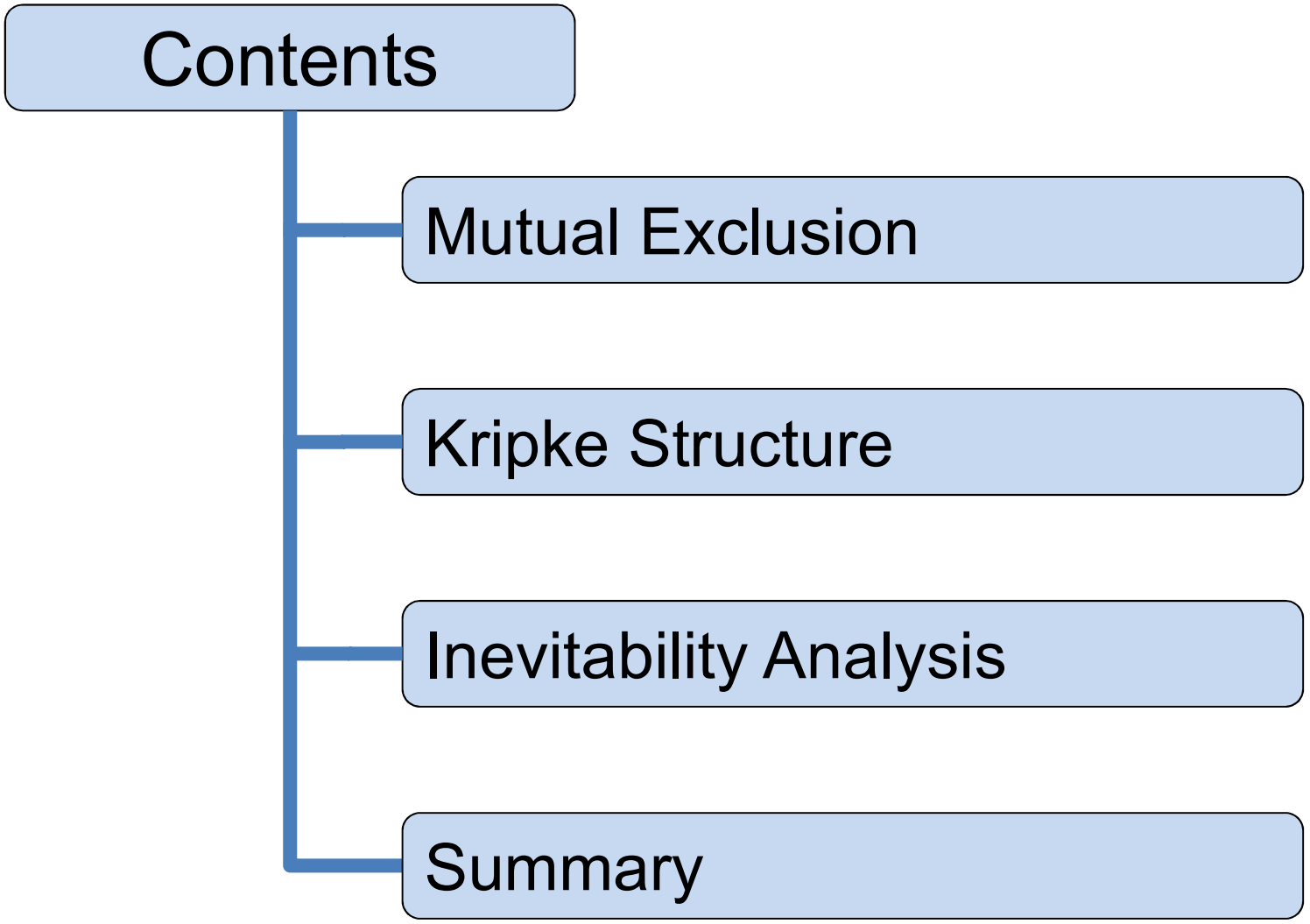
if the conclusion holds, then such a sequence exists.

Example: Y={s2,s3}

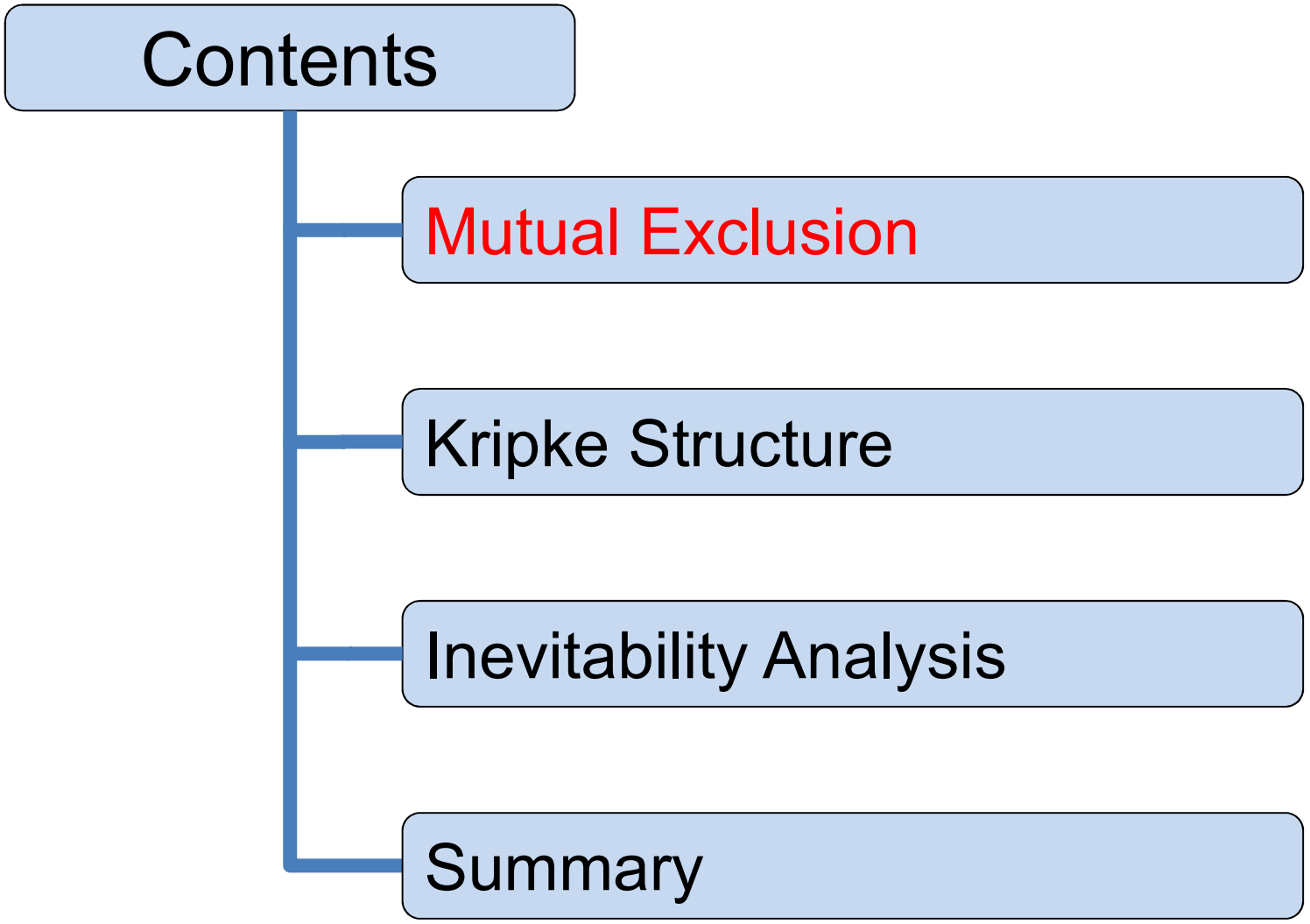


X_0={s0,s2}, X_1={s1,s2}, X_2={s3}

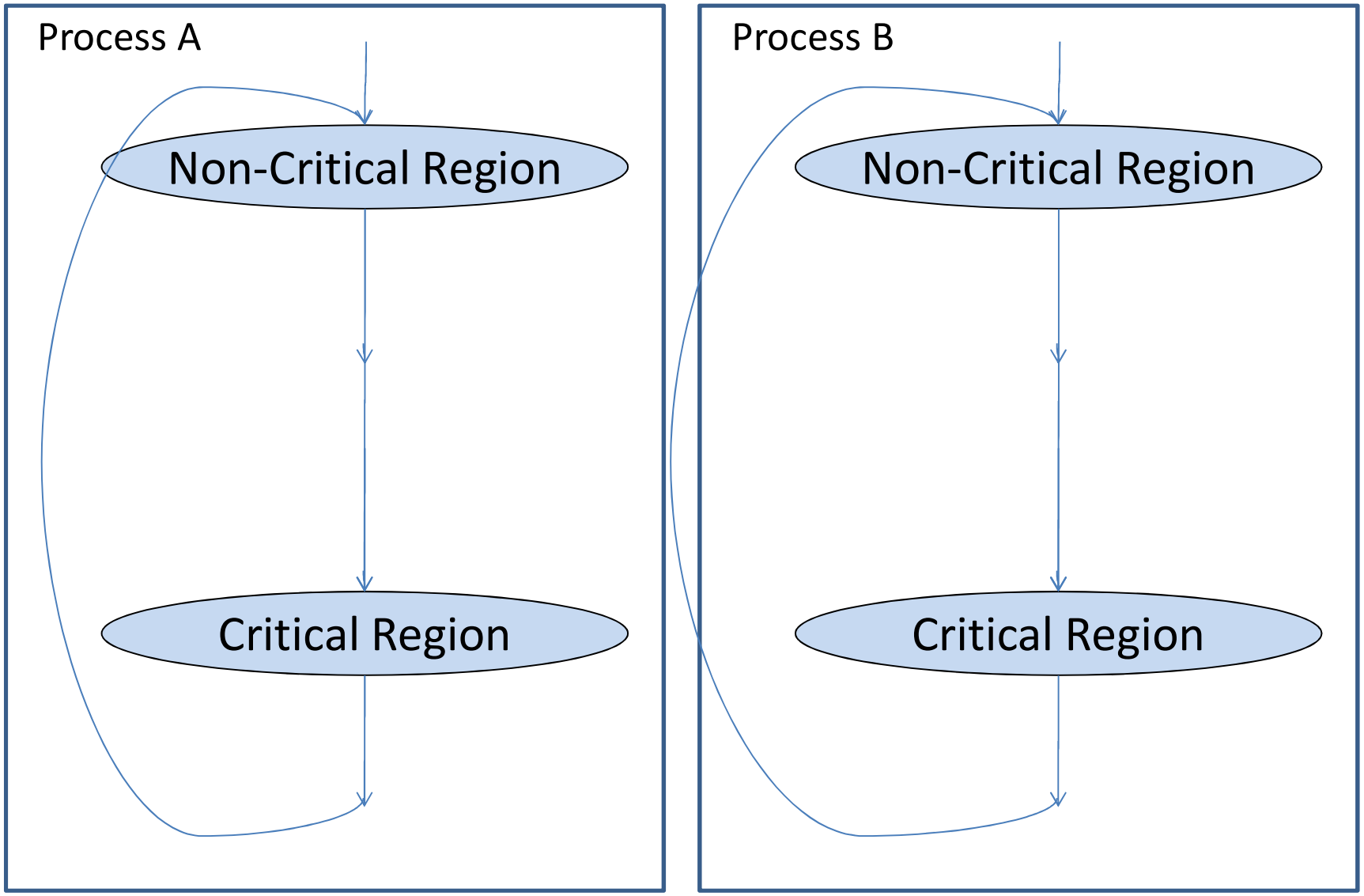
Inevitability Analysis – An Example



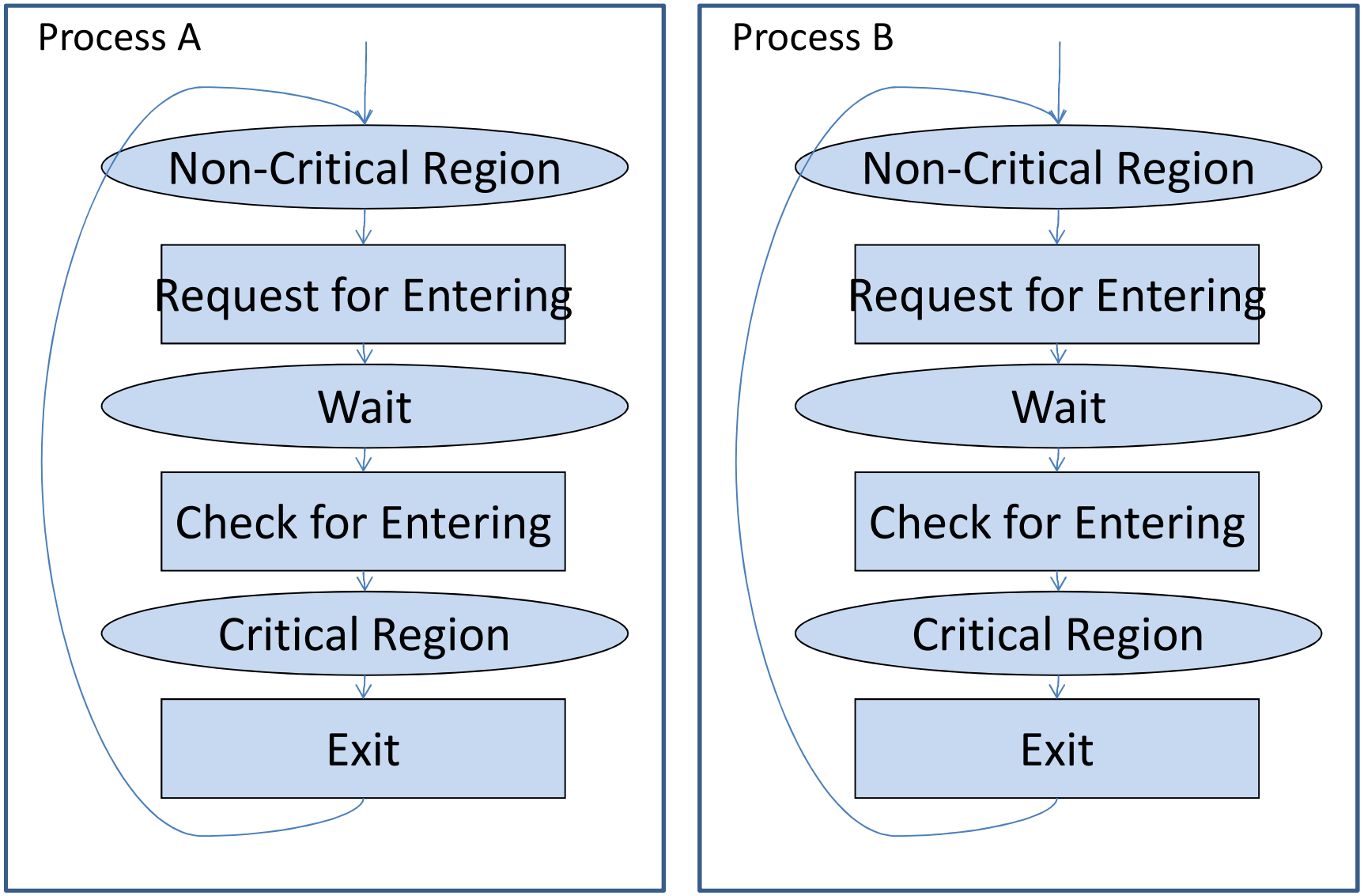
Inevitability Analysis – An Example



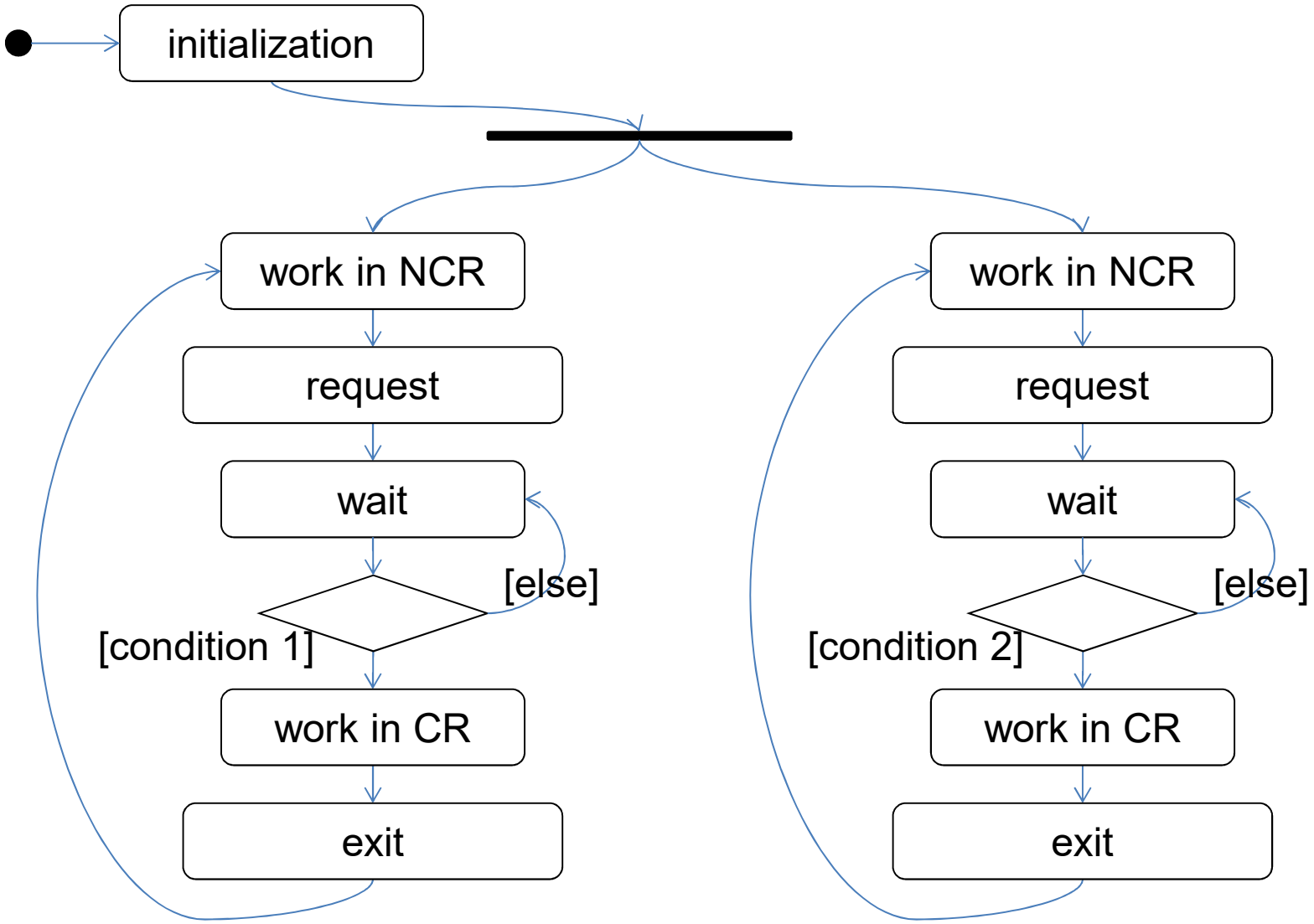
Example: Mutual Exclusion



Example: Mutual Exclusion



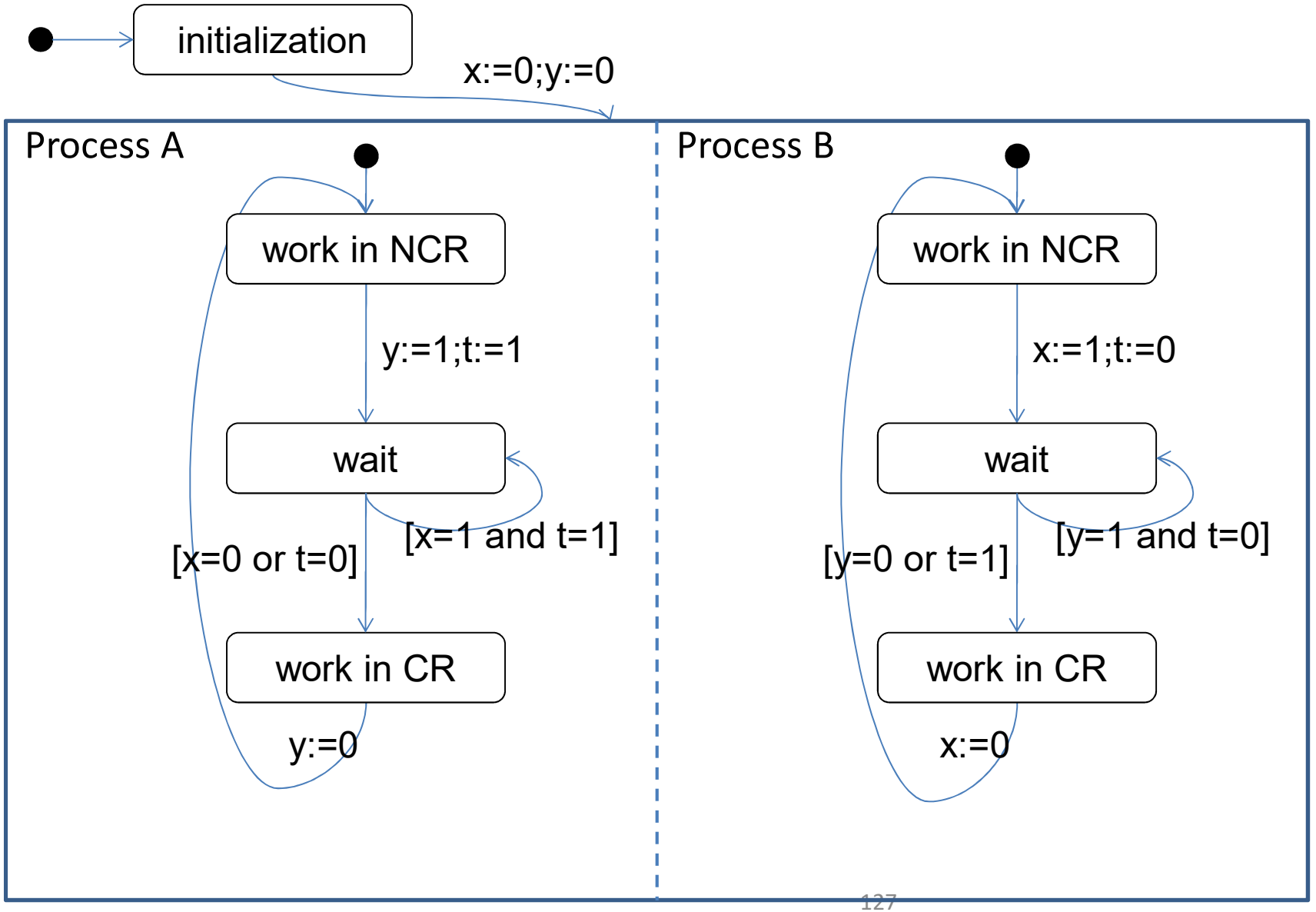
Design of Mutual Exclusion (Activity)



Design of Mutual Exclusion

- Purpose:
  - ensure that not both processes are working in the critical region (CR)
- Mechanism:
  - use shared variables
  - y=1: the first process is applying for entering CR or it is in CR
  - x=1: the second process is applying for entering CR or it is in CR
  - t=(i-1): the i-th process has priority for entering CR

Design of Mutual Exclusion (State)



Correctness of the Design

- Ensure that not both processes are working in the critical region (CR)
- Ensure that at least one process reaches CR

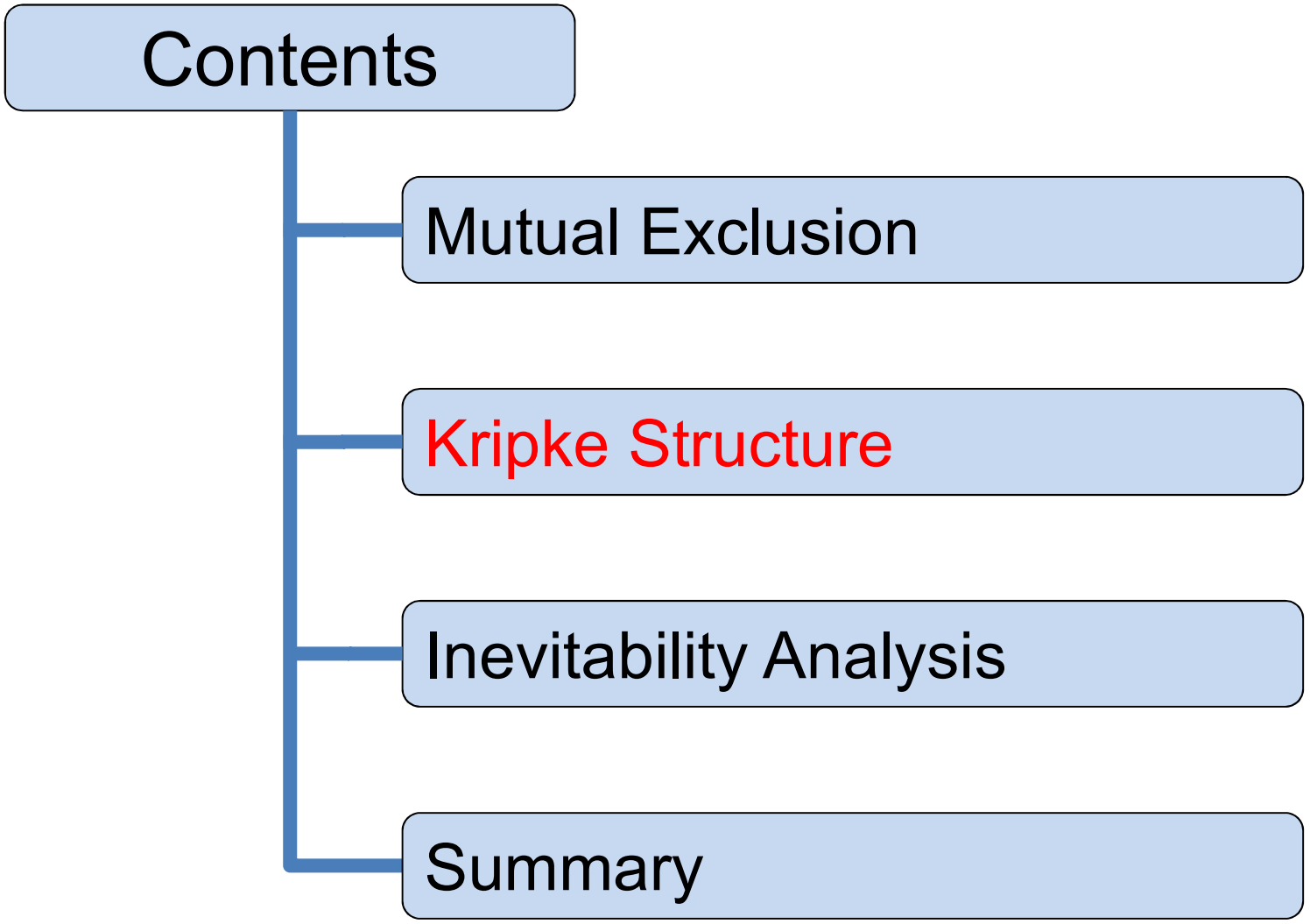
Combined States of the Two Processes

Process A	Process B	x,y,t	Remark
NCR	NCR		
NCR	wait		
NCR	CR		Good states
wait	NCR		
wait	wait		
wait	CR		Good states
CR	NCR		Good states
CR	wait		Good states
CR	CR		Good states

Correctness of the Design

- How do we know that the design is correct?
  - We have to be sure that the good states are inevitable, i.e., reachable in all possible executions of the algorithm
  - We may use state exploration (model checking) techniques or deductive proof methods

Inevitability Analysis – An Example



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A Kripke structure is a triple K=<S,R,I>

- S : A finite set of states
- $R \subseteq S \times S$: A total transition relation
- $I \subseteq S$: A set of initial states

Domains of Process and Variable States

Process A	Process B	x	y	t
NCR	NCR	1	1	1
wait	wait	0	0	0
CR	CR			

(a,b,x,y,t)

The Set of States: S

$\{(a,b,x,y,t) \mid a,b \in \{NCR,wait,CR\} \text{ and } x,y,t \in \{0,1\}\}$

Transition Relation: R

(NCR,b,x,y,t)	→ (wait,b,x,1,1)
(wait,b,0,y,t)	→ (CR,b,0,y,t)
(wait,b,x,y,0)	→ (CR,b,x,y,0)
(wait,b,1,y,1)	→ (wait,b,1,y,1)
(CR,b,x,y,t)	→ (NCR,b,x,0,t)
(a,NCR,x,y,t)	→ (a,wait,1,y,0)
(a,wait,x,0,t)	→ (a,CR,x,0,t)
(a,wait,x,y,1)	→ (a,CR,x,y,1)
(a,wait,x,1,0)	→ (a,wait,x,1,0)
(a,CR,x,y,t)	→ (a,NCR,0,y,t)

The Set of Initial States: I

{ (NCR,NCR,0,0,0), (NCR,NCR,0,0,1) }

Good States

(CR,b,x,y,t)

(a,CR,x,y,t)

The set of other states:

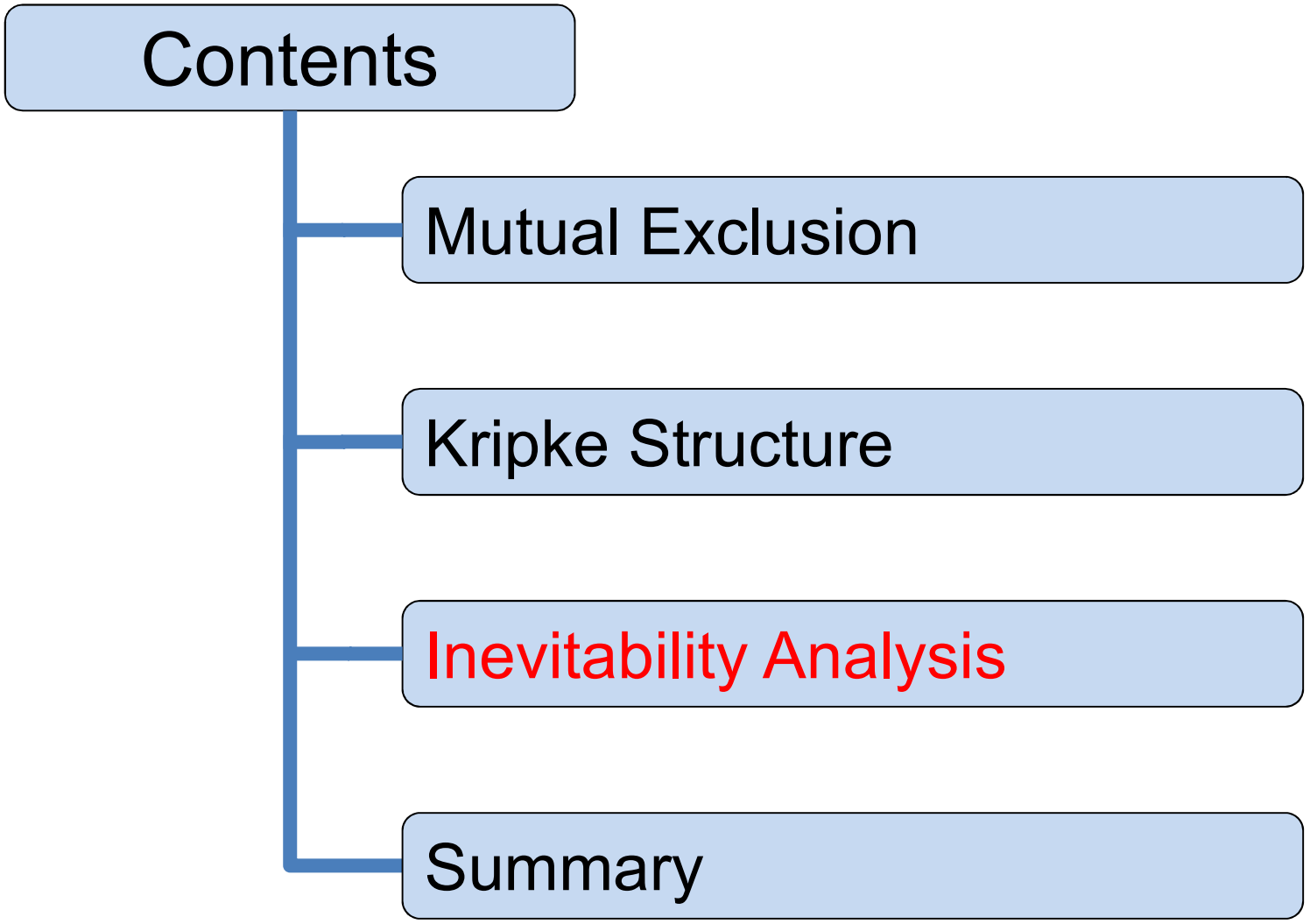
(NCR,NCR,x,y,t)

(wait,wait,x,y,t)

(wait,NCR,x,y,t)

(NCR,wait,x,y,t)

Inevitability Analysis – An Example



Inevitability Analysis

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     G:=(S',R');
     scclist:=scctarjan(G);
     w:={};
     for each (e in scclist) if (nontrivial(e)) w:=w ∪ e;
     return (not ReachbilityAnalysis(K',w));
 }
```

Inevitability Analysis

Let K=<S,R,I> be a Kripke structure, and A $\subseteq$S.

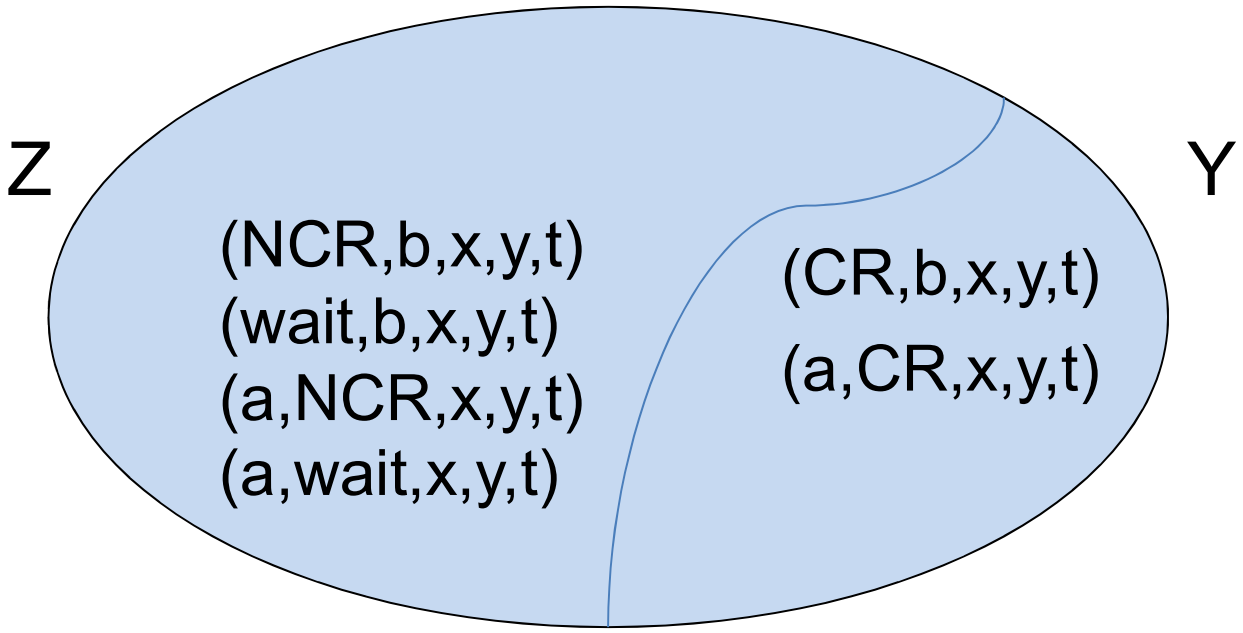
Proposition:

InevitabilityAnalysis(K,A) =true

Iff

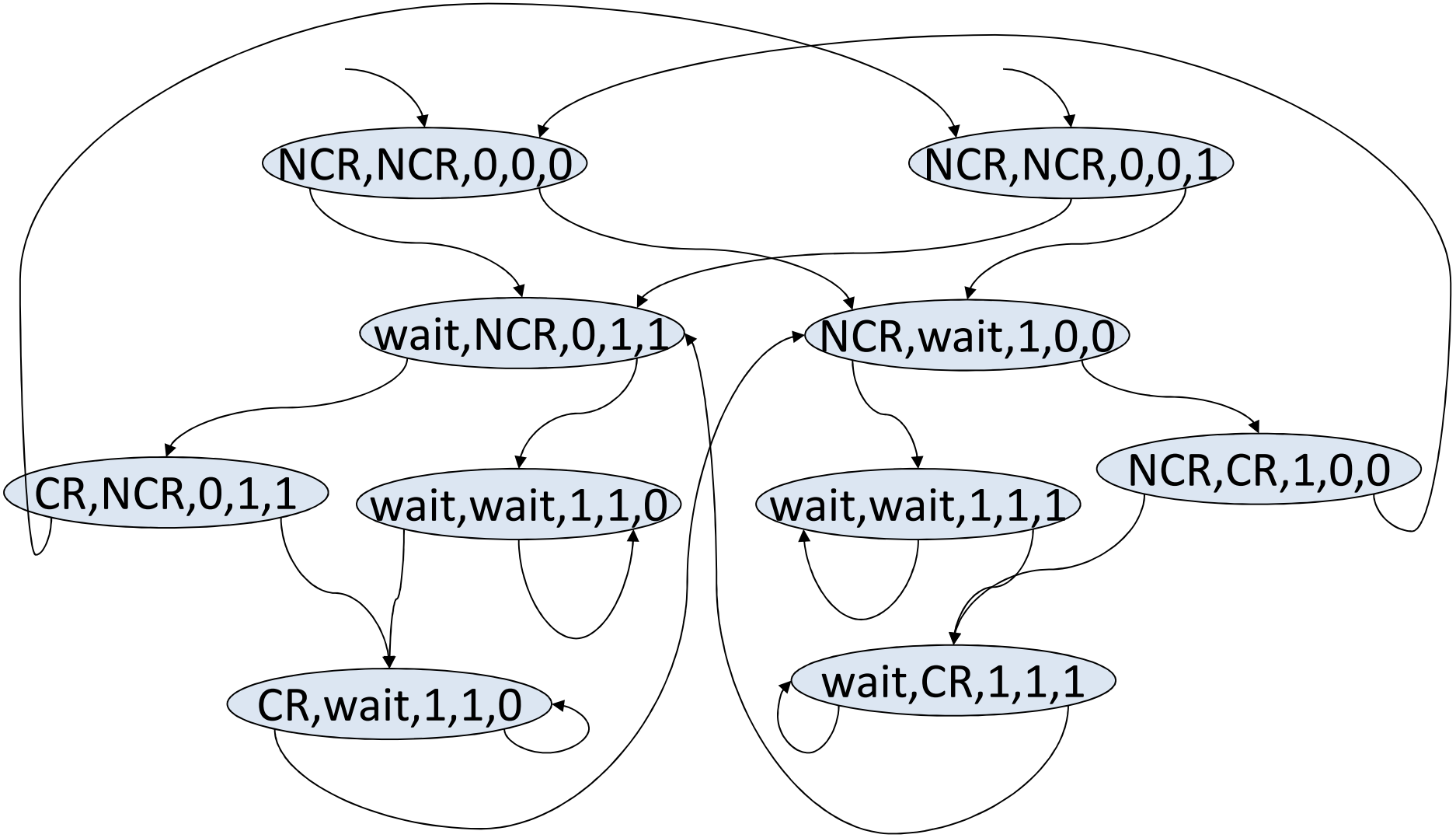
A is an inevitability property

Inevitability Analysis

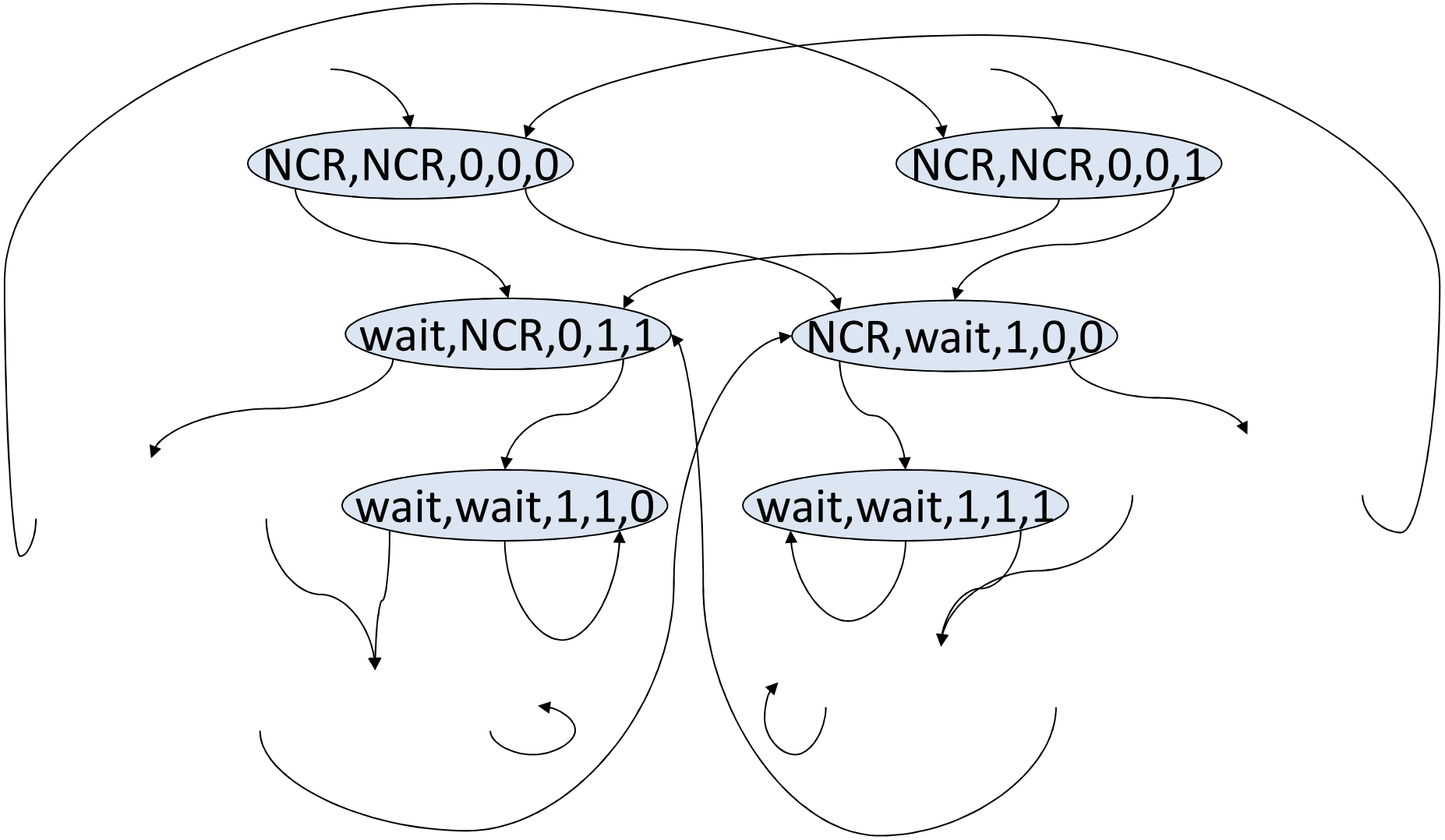


InivitabilityAnalysis(K,Y)=?

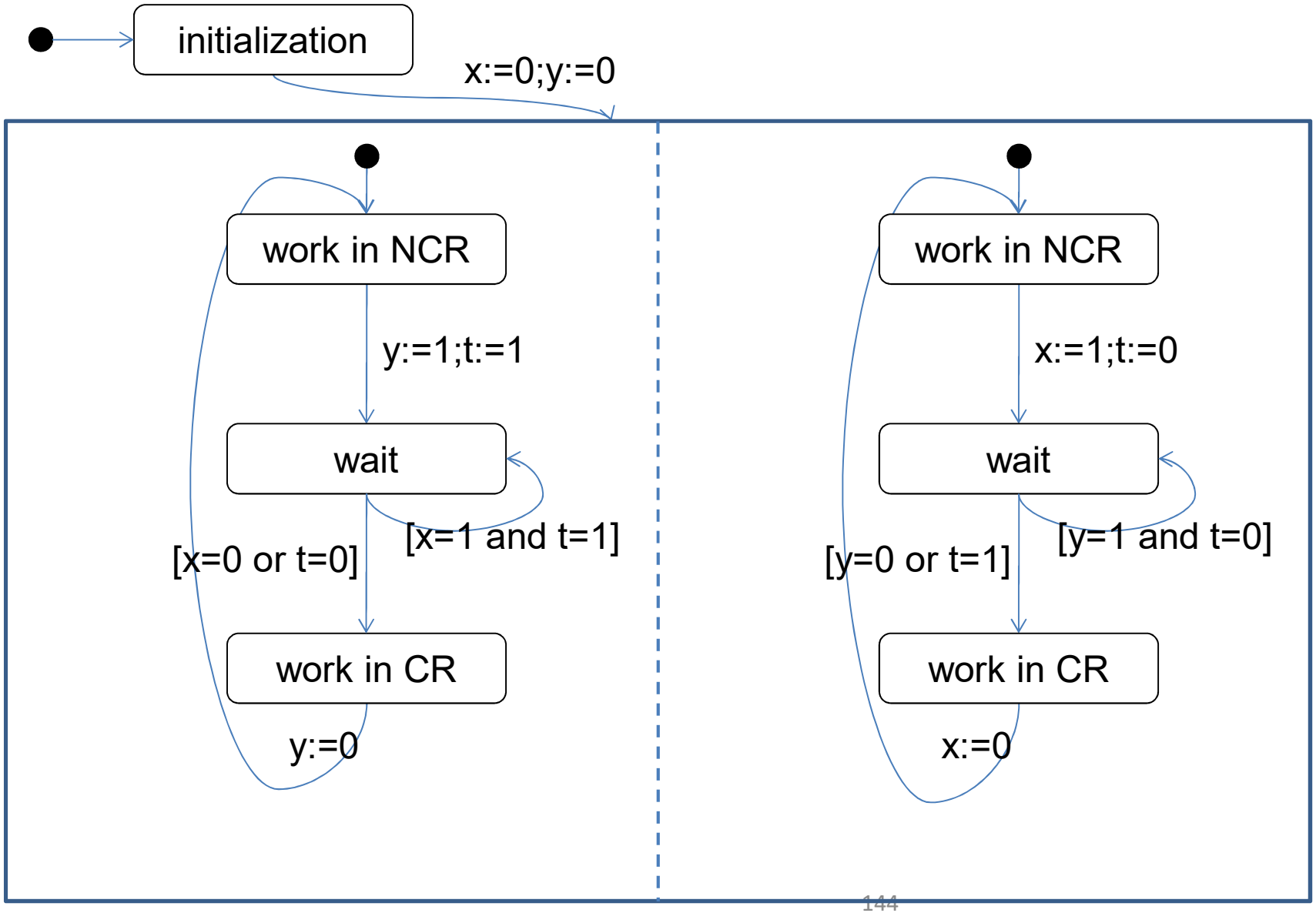
Inevitability Analysis



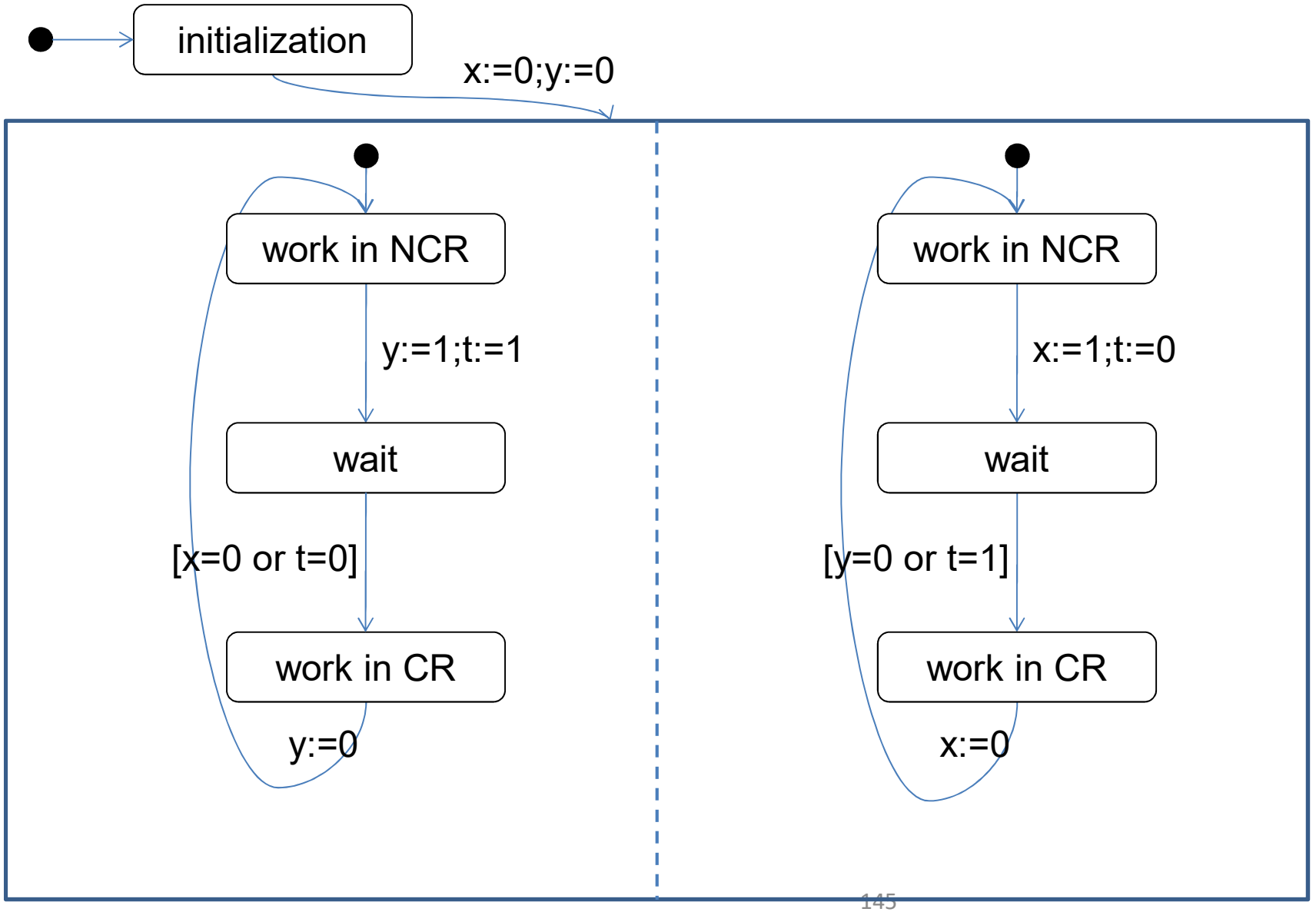
Inevitability Analysis (Returns False ??)



Design of Mutual Exclusion (State)



Revised Model of Mutual Exclusion



Transition Relation: R

(NCR,b,x,y,t) → (wait,b,x,1,1)

(wait,b,0,y,t) → (CR,b,0,y,t)

(wait,b,x,y,0) → (CR,b,x,y,0)

(wait,b,1,y,1) → (wait,b,1,y,1)

(CR,b,x,y,t) → (NCR,b,x,0,t)

(a,NCR,x,y,t) → (a,wait,1,y,0)

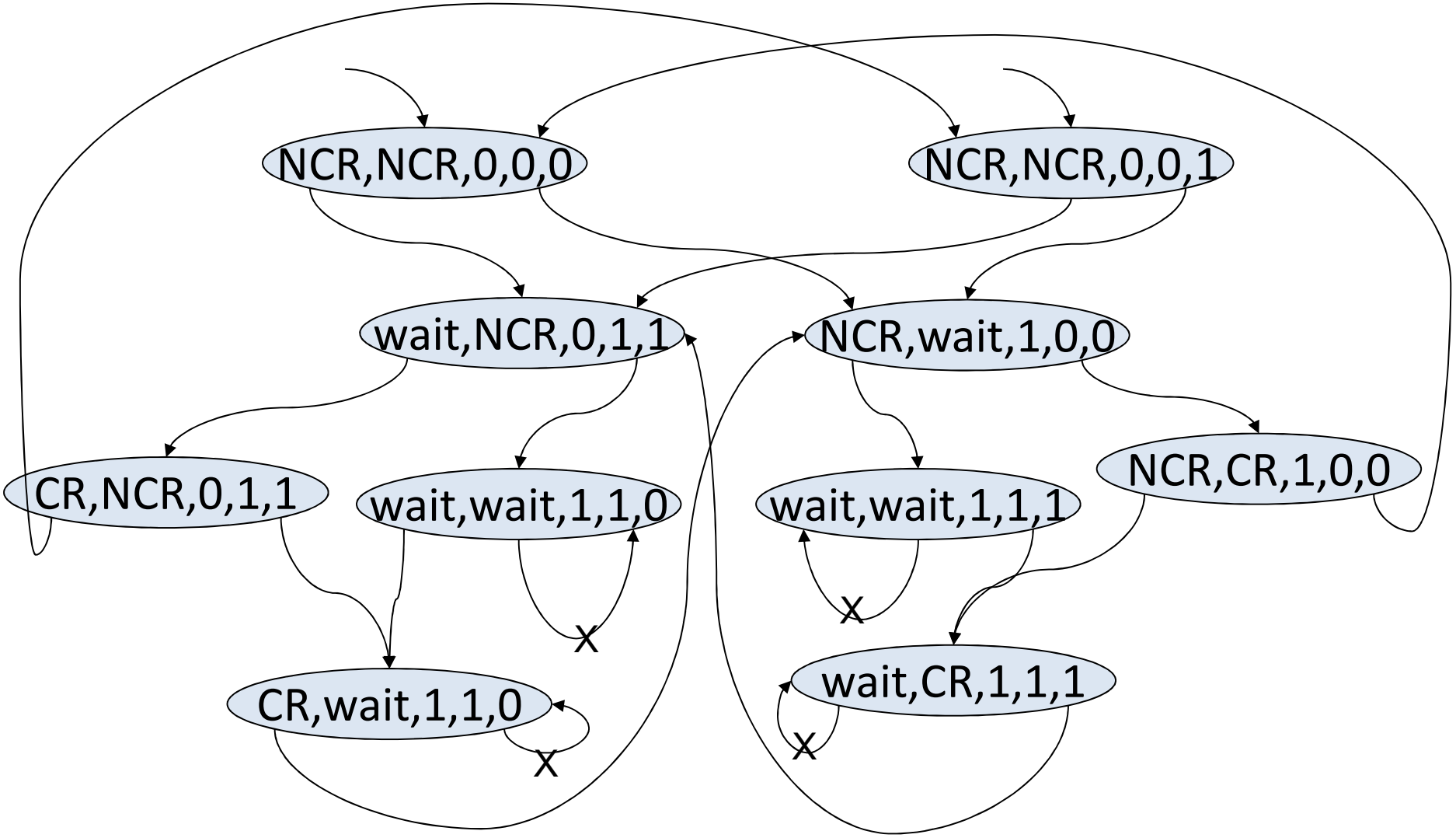
(a,wait,x,1,t) → (a,CR,x,1,t)

(a,wait,x,y,1) → (a,CR,x,y,1)

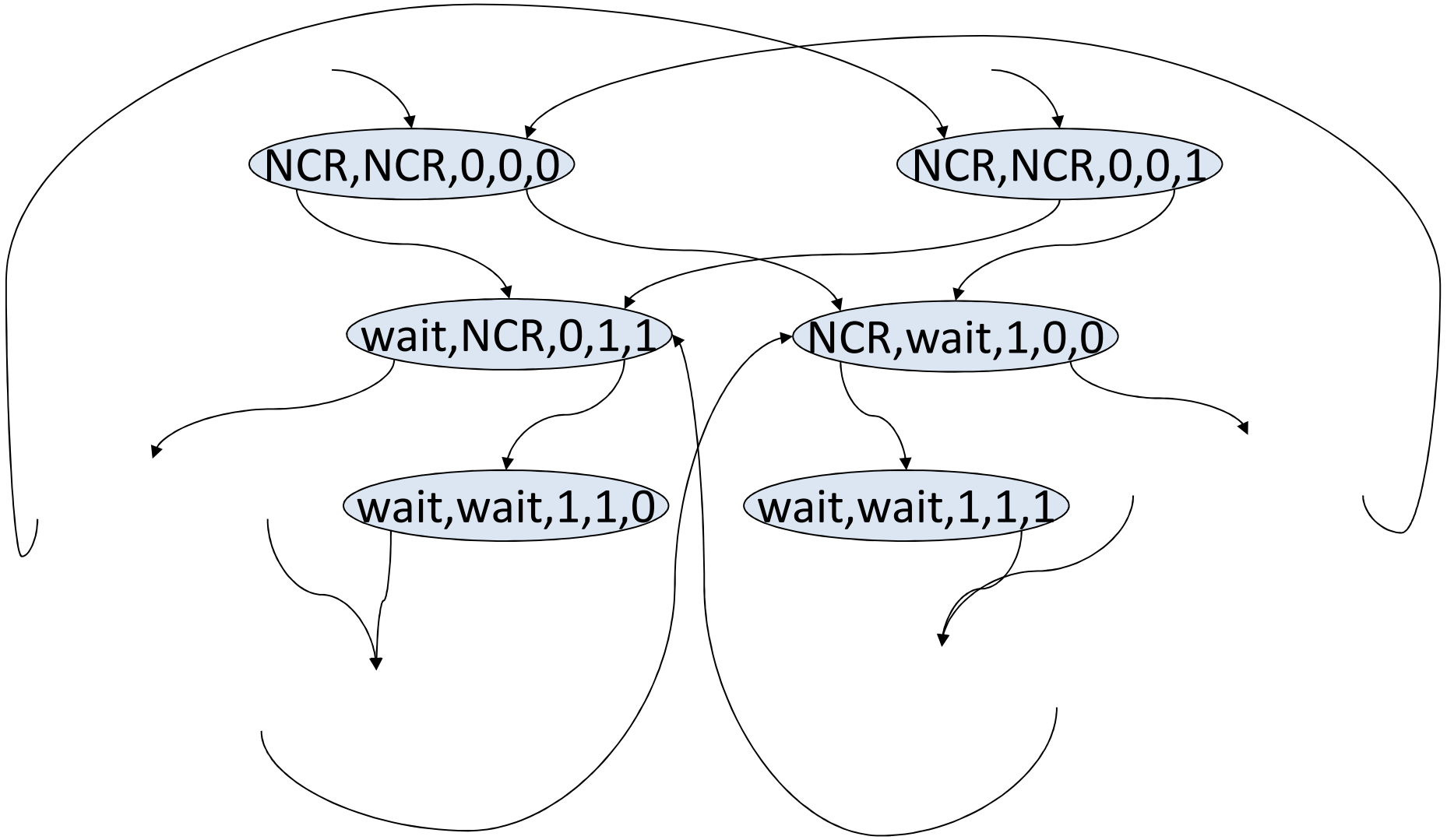
(a,wait,x,1,0) → (a,wait,x,1,0)

(a,CR,x,y,t) → (a,NCR,0,y,t)

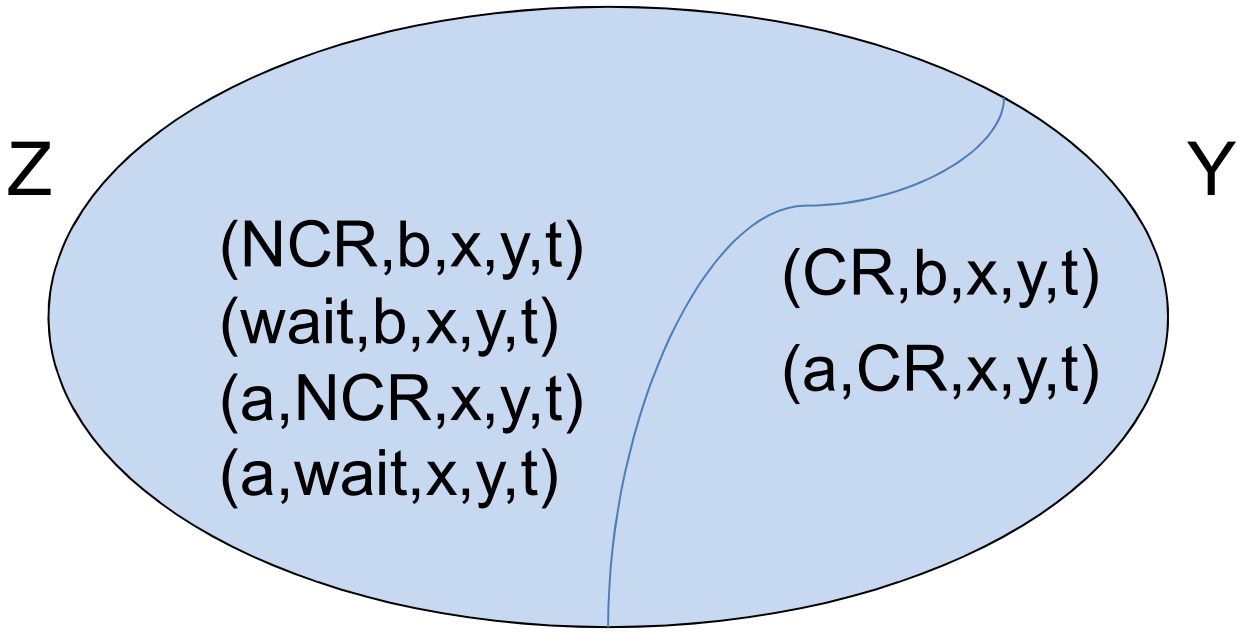
Inevitability Analysis



Inevitability Analysis

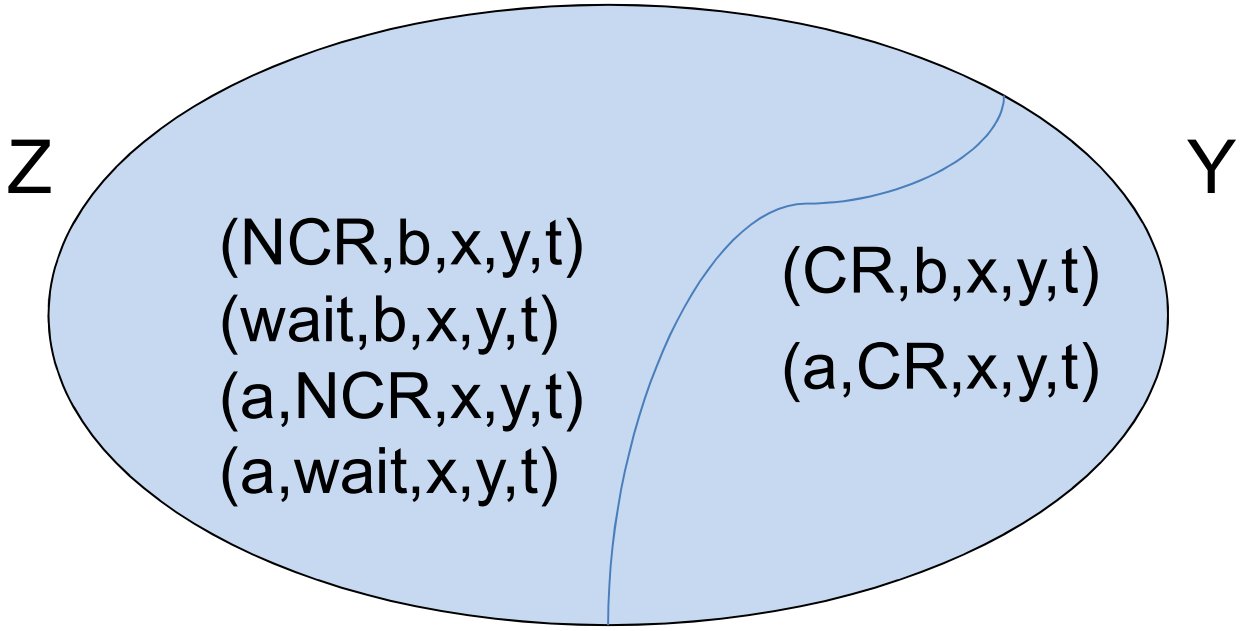


Inevitability Analysis



InivitabilityAnalysis(K’,Y)=?

Inevitability Analysis



InevitabilityAnalysis(K',Y)=true ⇔
Every computation reaches a Y-states ⇔
Y is inevitable in K'

Deductive Proof of Inevitability

X_0 = { (NCR,NCR,0,0,0), (NCR,NCR,0,0,1) }

X_1 = { (wait,NCR,0,1,1), (NCR,wait,1,0,0) }

X_2 = { (wait,wait,1,0,0), (wait,wait,1,1,1) } $\cup$ Y

X_3 = { (CR,wait,1,1,0), (wait,CR,1,1,1) }

We have

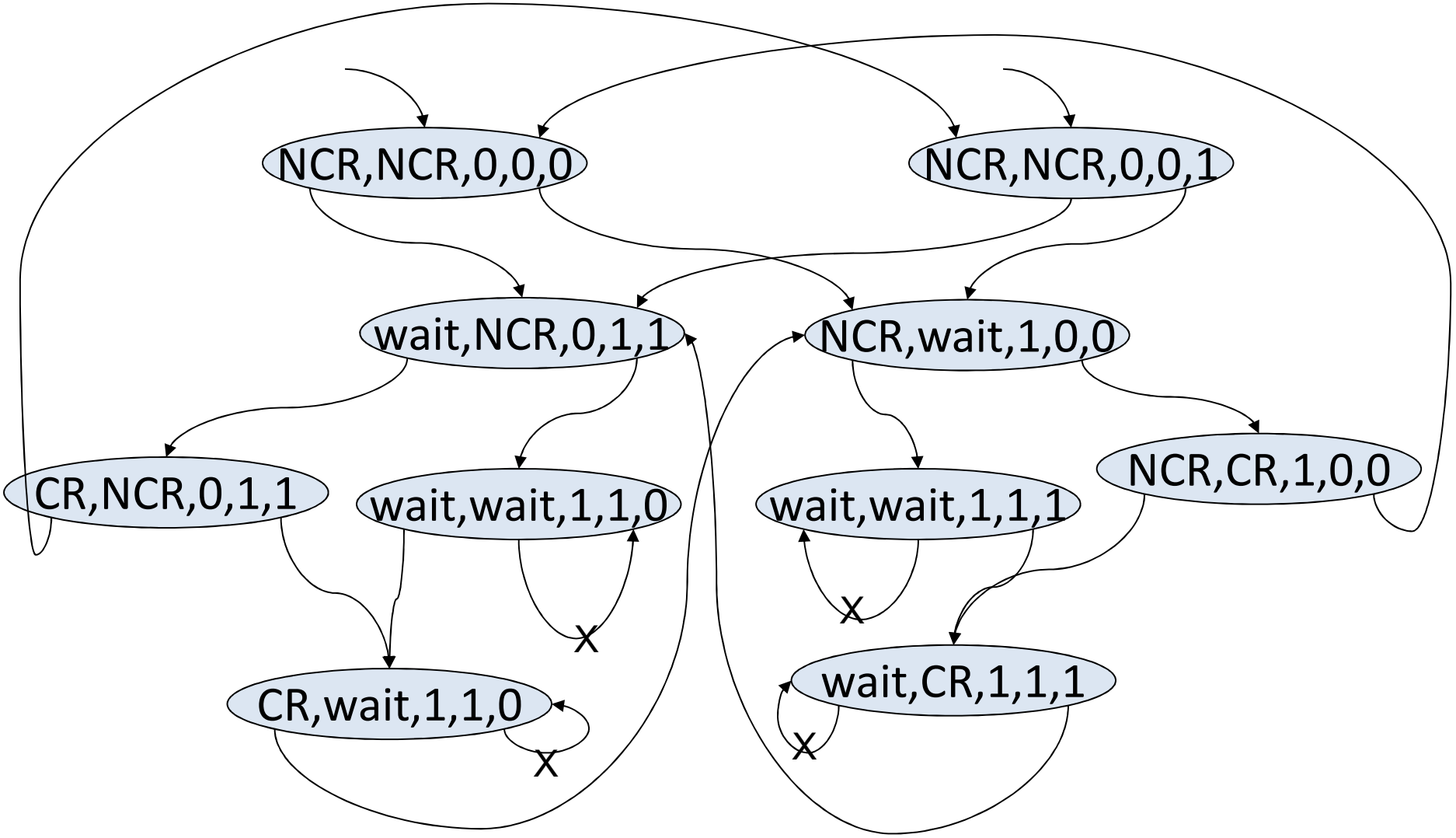
$I \subseteq X_0$,

$R(X_i \backslash Y) \subseteq X_{i+1}$, for i=0,1,2

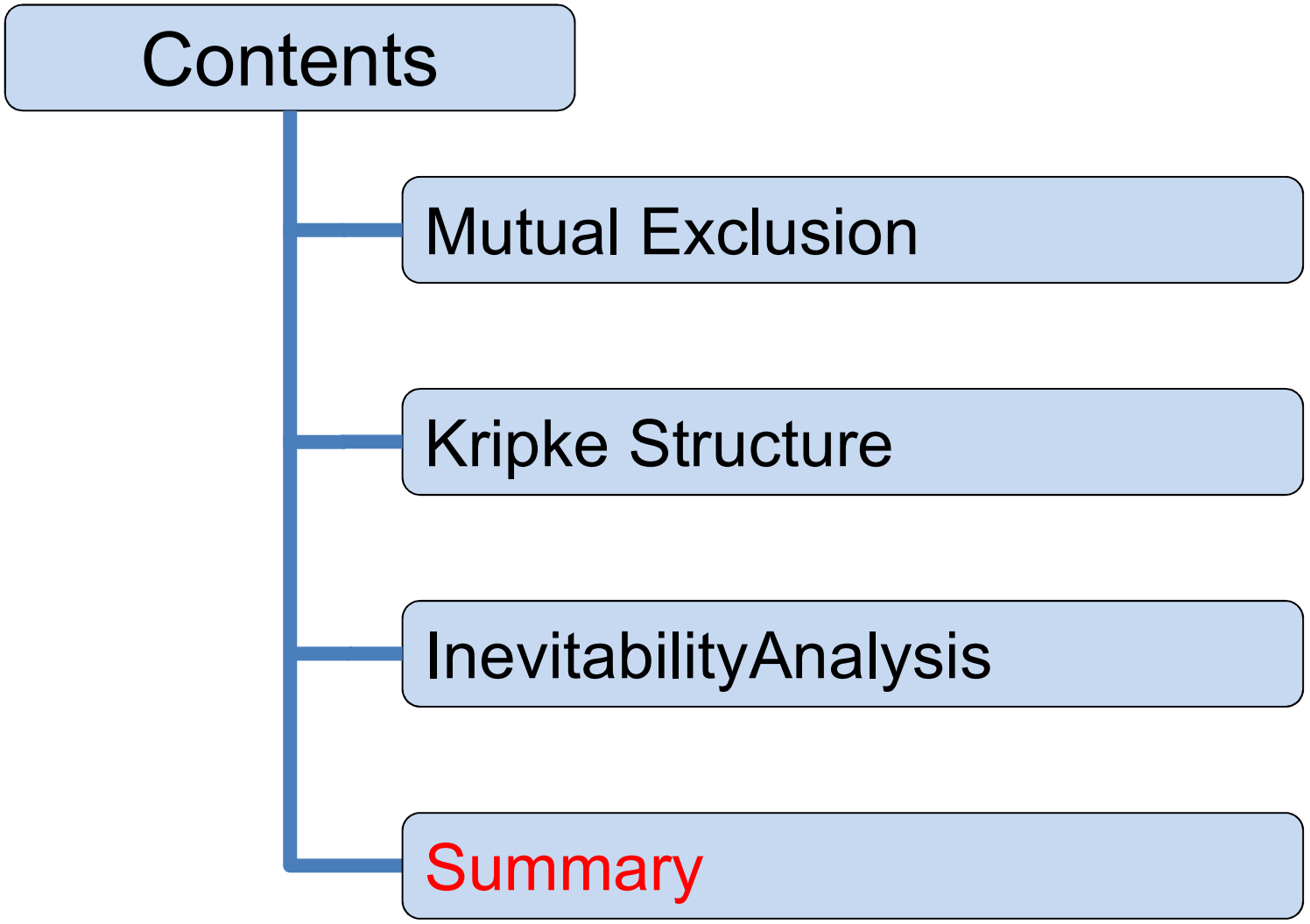
$X_3 \subseteq Y$,

Therefore Y an inevitability property of K'.

Deductive Proof of Inevitability



Inevitability Analysis – An Example



Correctness of the Design

- How do we know that the design is correct?
  - We have to be sure that the good states are inevitable, i.e., reachable in all possible executions of the algorithm
  - We may use state exploration (model checking) techniques or deductive proof methods
  - We have shown that the good states are inevitable.

(IV) Summary

- Kripke结构 --- 基本概念
- 安全性质 --- 模型检测算法、推理方法
- 必达性质 --- 模型检测算法、推理方法

思考：

给定一个Kripke结构K=<S,R,I>和集合B,A⊆S.

(1)

判断以下说法的正确性：

A是可避免性质，当且仅当

K有一条 由I可达 非A非平凡强连通分量的 非A路径。

(2)

定义(B,A)路径为满足以下条件的路径：

至少一个B状态出现在该路径且同时或之后有A状态出现。

设计基于不动点计算的算法以检查K中是否存在

初始状态为起点的(B,A)路径。