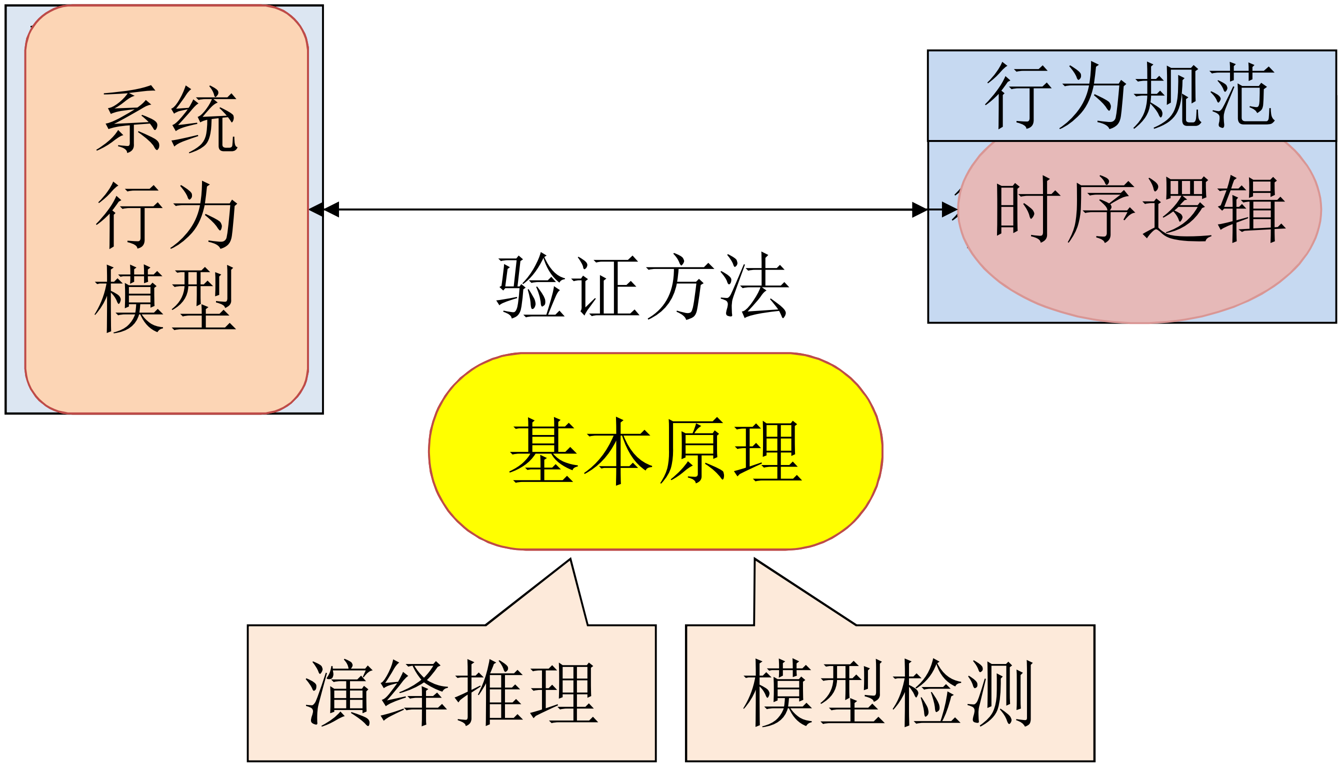
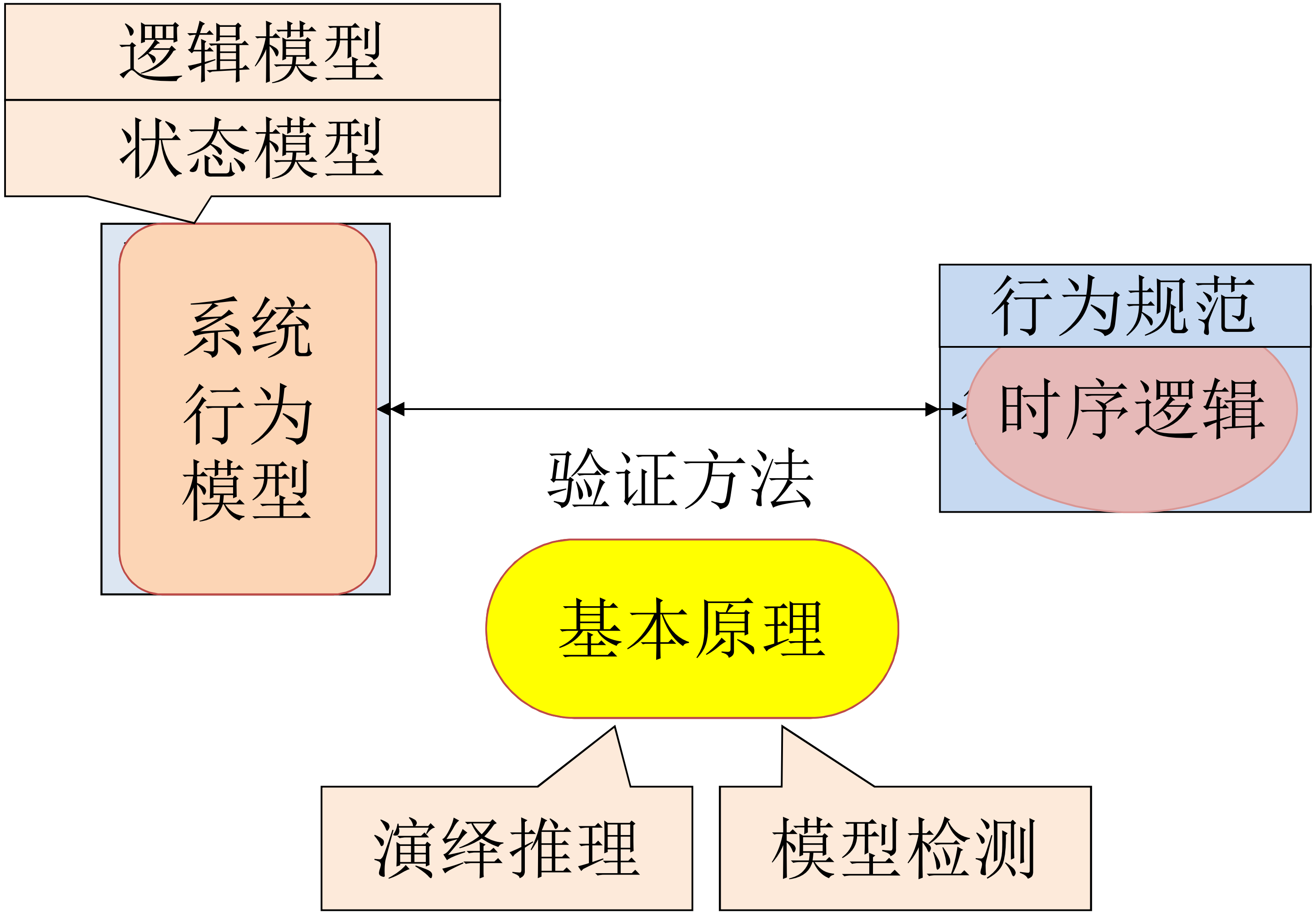
基于迁移标号的迁移系统

中国科学院软件研究所
计算机科学国家重点实验室
张文辉
http://lcs.ios.ac.cn/~zwh/

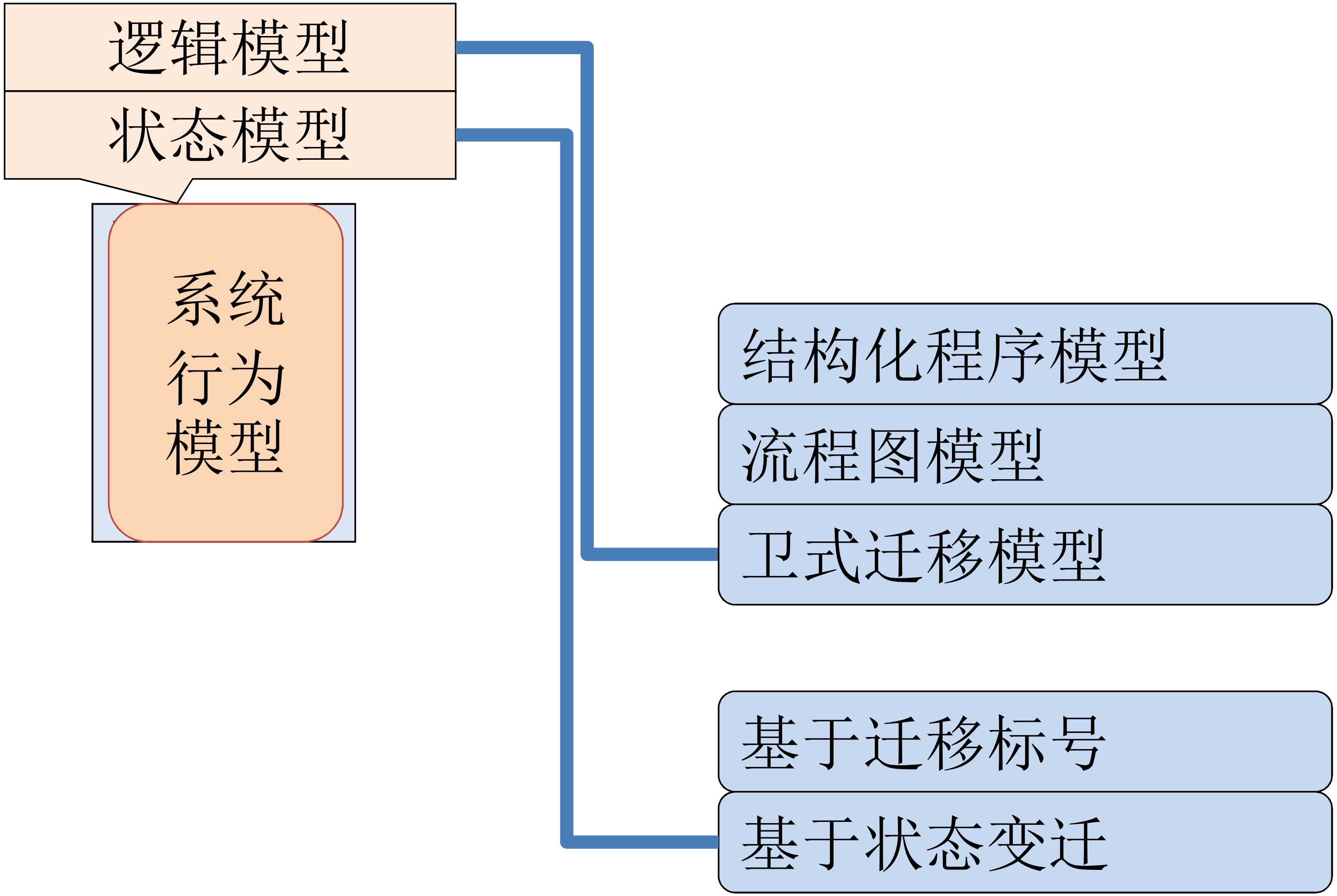
课程内容



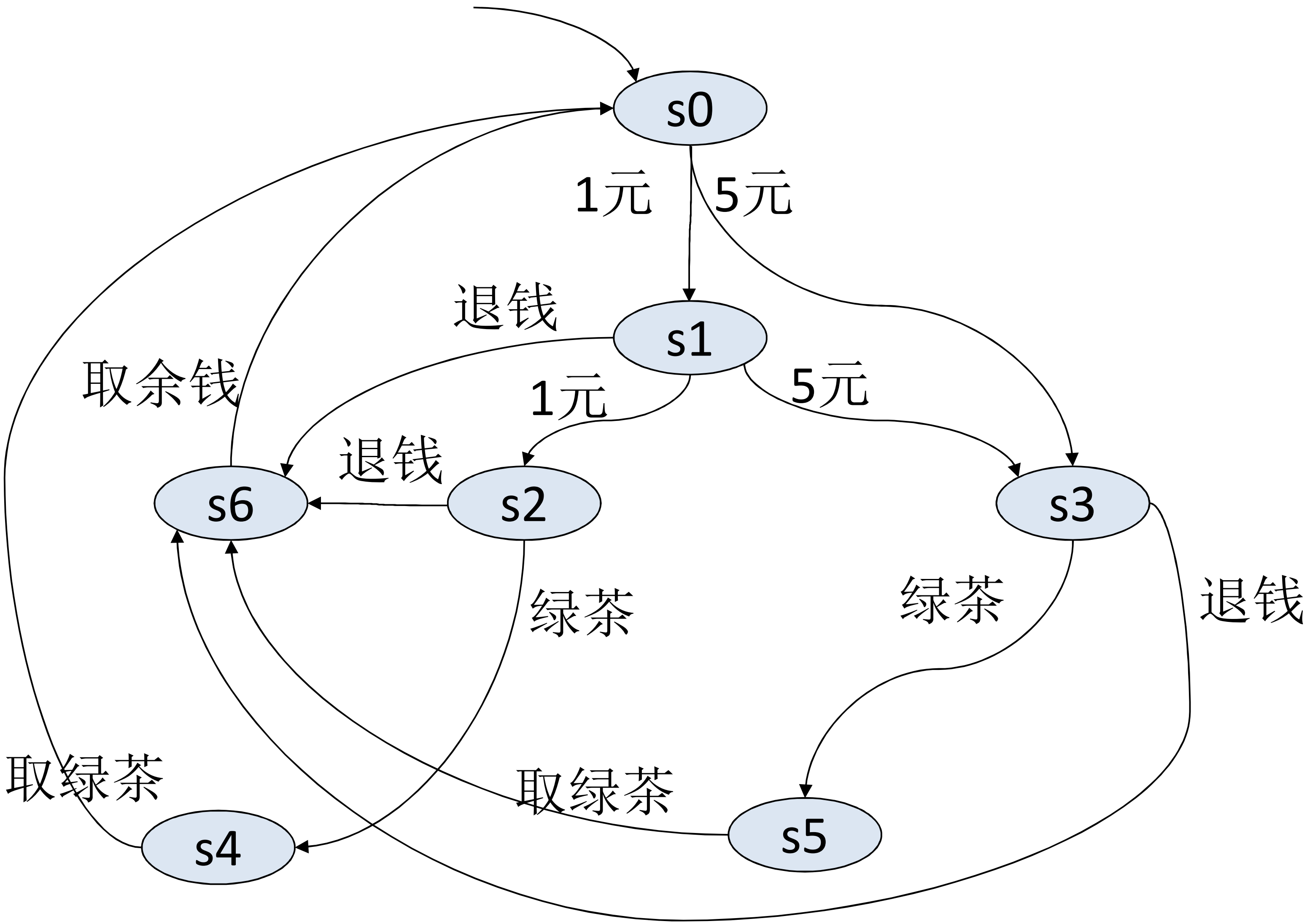
课程内容



课程内容(1)



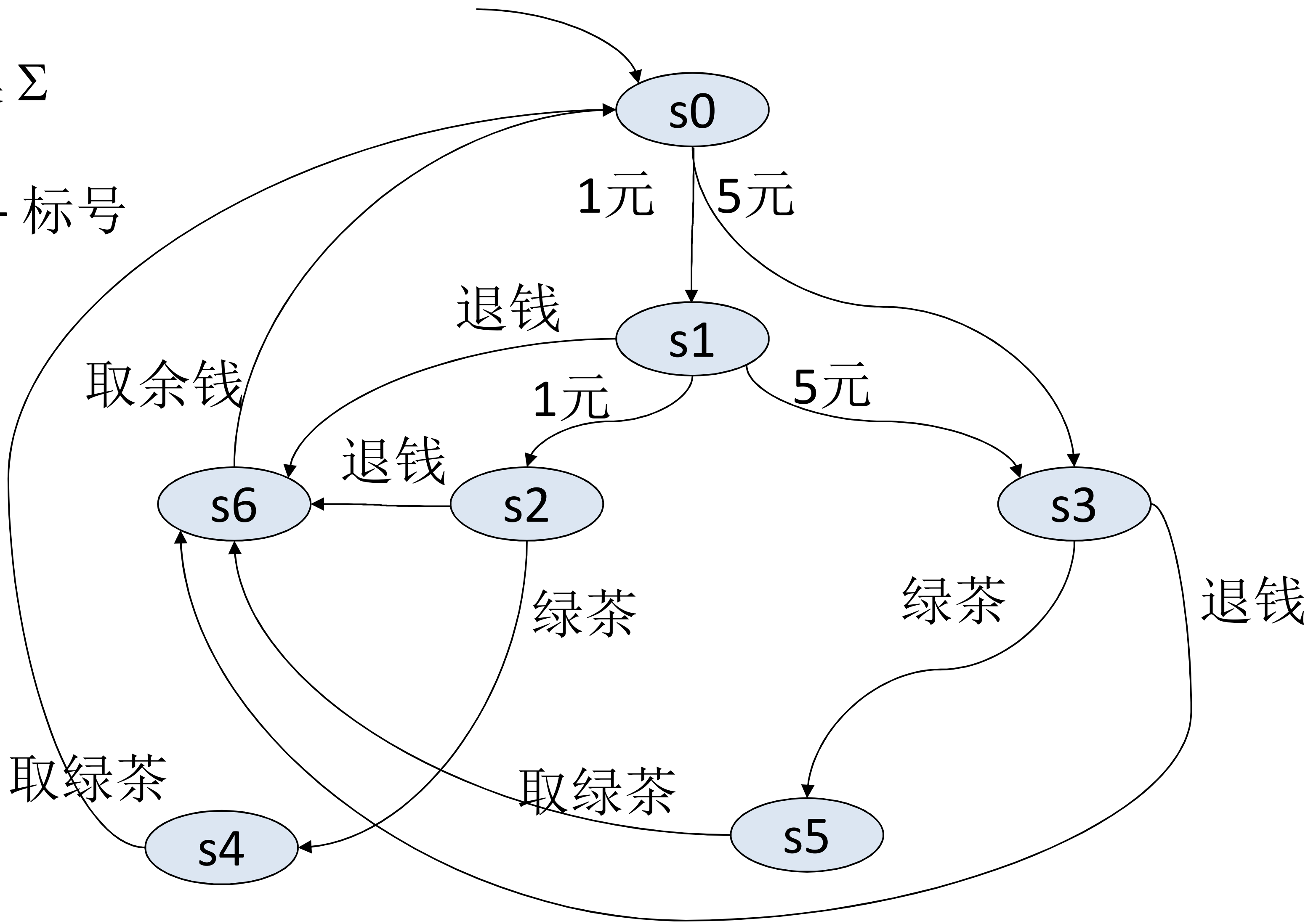
例子：自动售茶机的设计



例子：自动售茶机的设计

标号集 Σ

(S,R,I) + 标号



内容：标号迁移系统与ω-自动机

- 标号迁移系统(LTS)
- Büchi自动机(BA)
- 泛Büchi自动机(GBA)
- ω-自动机(BA,GBA,MA,SA,RA,PA)

(I) Labeled Transition Systems

Definition

A labeled transition system is a quadruple $<\Sigma, S, \Delta, I>$

- Σ : A finite set of symbols
- S : A finite set of states
- $\Delta \subseteq S \times \Sigma \times S$: A transition relation
- $I \subseteq S$: A set of initial states

Remark：
Let $R = \{ (s,s') \mid (s,a,s') \in \Delta \}$.
Then (S,R,I) is a Kripke structure, and $\Delta : R \rightarrow (2^{\Sigma} \setminus \emptyset)$

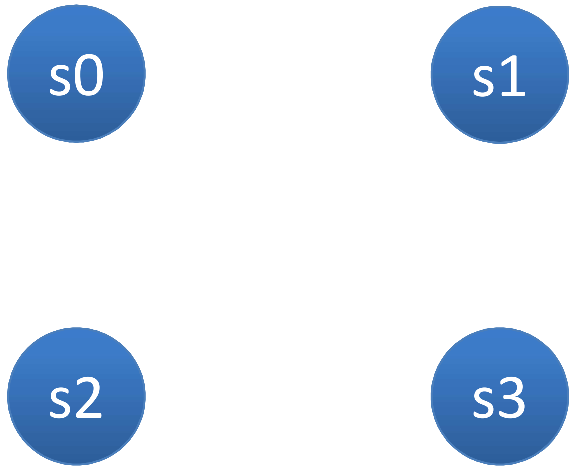
(I) Labeled Transition Systems

- Basic Concepts
  - Labels, States, Labeled Transition Relation, Initial States
  - Words, Runs
  - Language
- Deterministic vs Non-deterministic LTS
- Comparison with Labeled KS

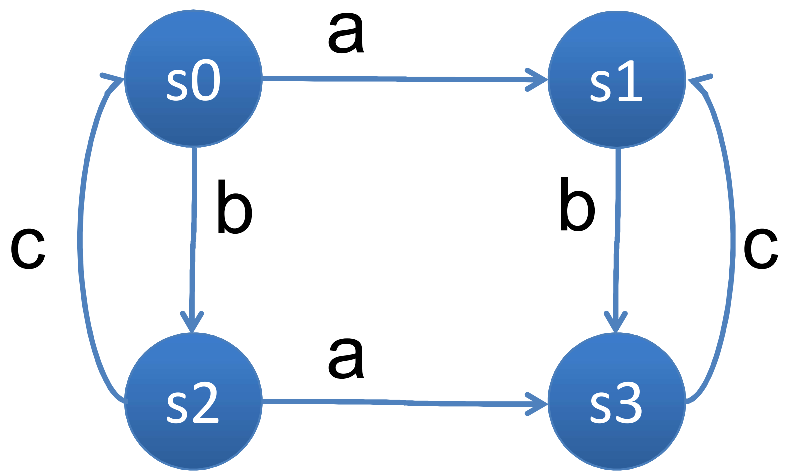
Example: Σ

{a,b,c}

Example: S



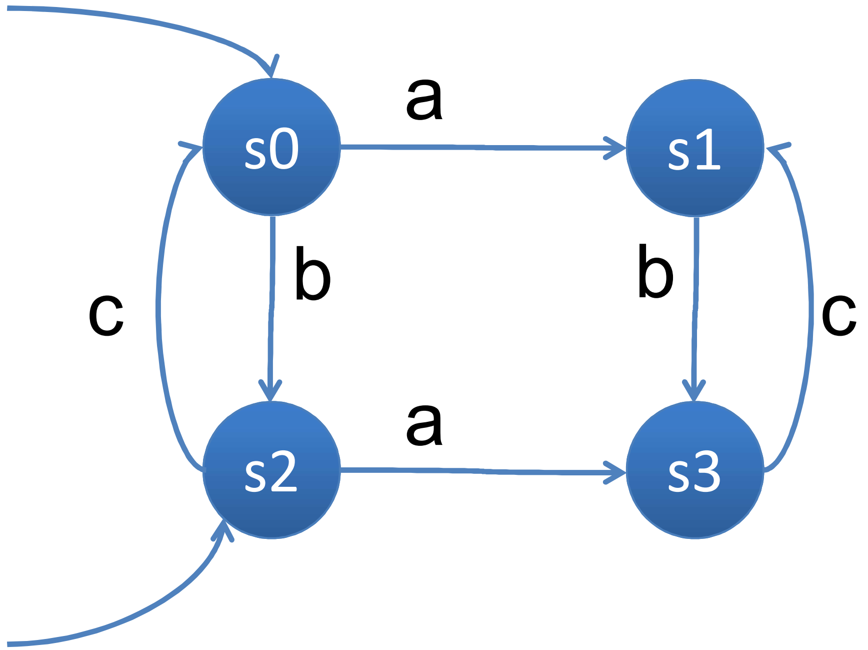
Example: Δ



Notation

$s \rightarrow^{a} s'$: $(s,a,s') \in \Delta$

Example: I



Words, Runs on Words, Runs

Given a LTS $A=<\Sigma,S,\Delta,I>$

A word is an infinite sequence of Σ

Let $w=w[1]w[2]w[3]\ldots\ldots\in\Sigma^\omega$ be a word.

Definition

A **run** of A on w is an infinite sequence $s_0\, s_1\, s_2\, \ldots.$ of S such that $s_0 \in I$, and $(s_i\, ,w[i+1],s_{i+1}) \in\Delta$ for all $i \geq 0$.

Definition

A **run** of A is an infinite sequence $s_0\, s_1\, s_2\, \ldots.$ of S such that there is a w and $s_0\, s_1\, s_2\, \ldots.$ is a run on w.

Words over Runs

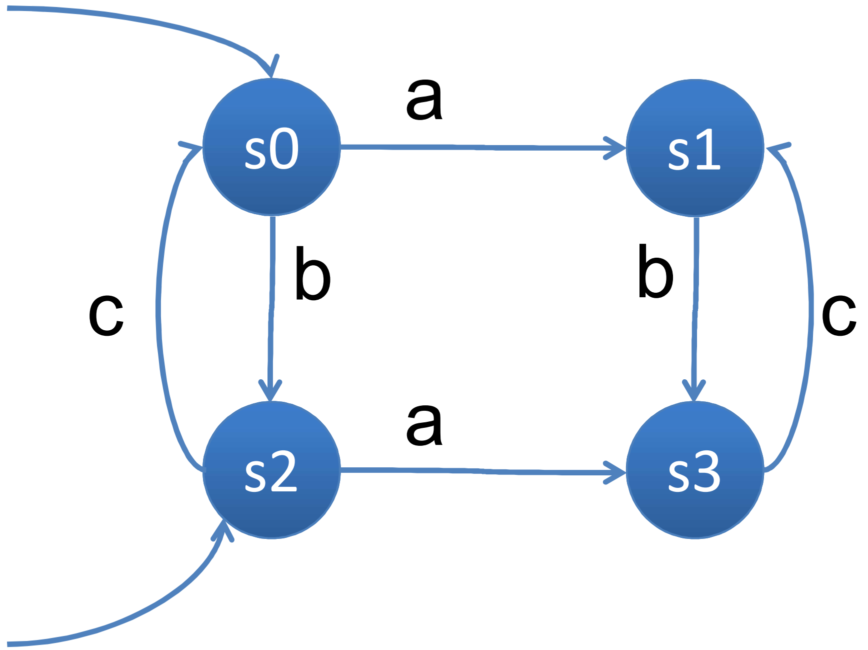
Definition

A **word** over a run r of A is

an infinite sequence of Σ: a_1a_2 ....

such that r is a run on a_1a_2 ....

Example: Words, Runs



words: a^ω, $(bc)^\omega$, $a(bc)^\omega$

runs: $(s0s2)^\omega$, $s0(s1s3)^\omega$

Language

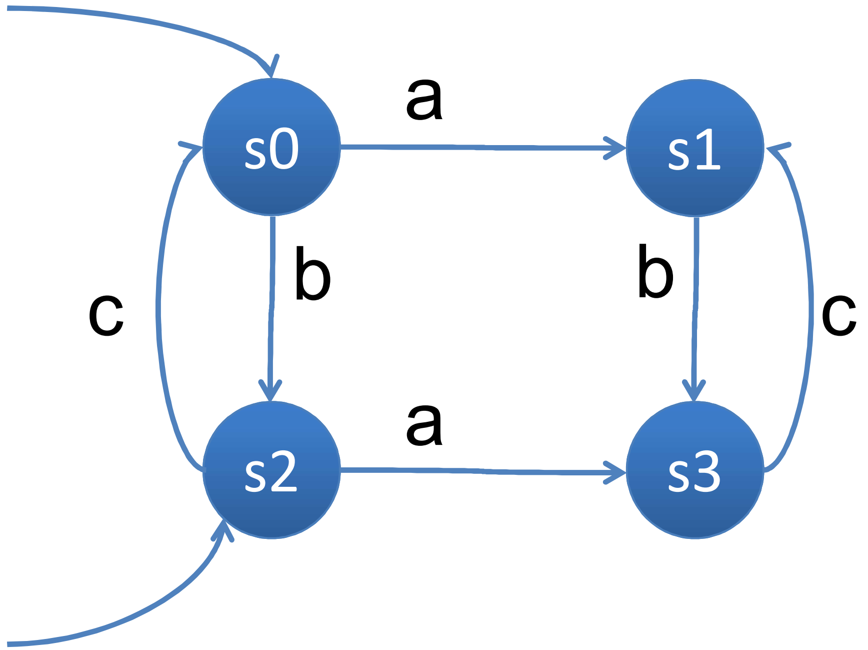
Definition

The language of A is

the set of words over runs of A.

The language of A is denoted L(A).

Example: Language



words over runs:

$(bc)^\omega$, $(cb)^\omega$,

$(bc)^*a\,(bc)^\omega$, $(bc)^*ba(cb)^\omega$, $(cb)^*a\,(cb)^\omega$, $(cb)^*ca(bc)^\omega$

Deterministic vs Non-deterministic LTS

Given $A = \langle \Sigma, S, \Delta, I \rangle$

Definition

A is deterministic, if

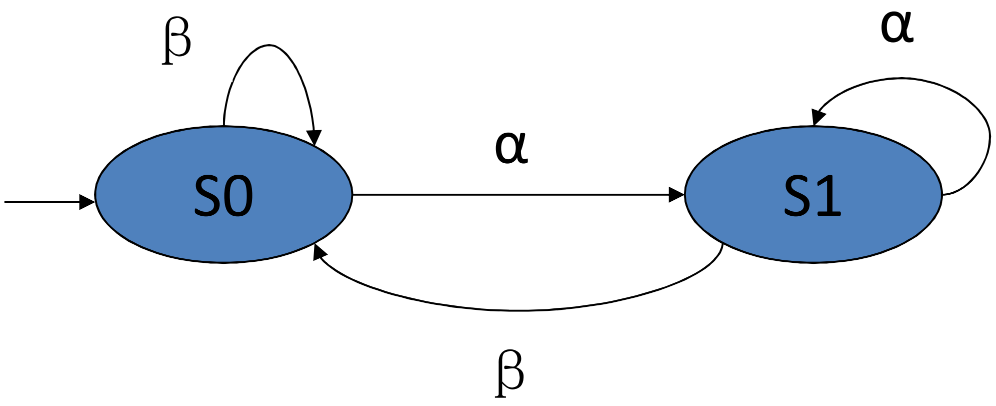
$|I| = 1$ and $|\Delta(s,a)| \leq 1$ for all $s \in S$ and $a \in \Sigma$.

Theorem

For a deterministic LTS,

for any word w, there is at most one run on w.

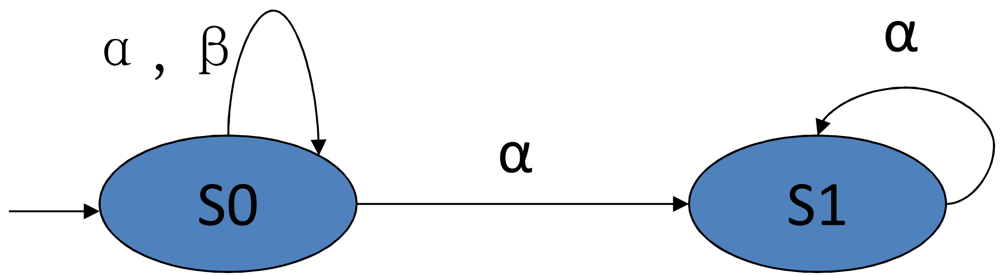
Deterministic LTS



β α β β α α

s0s0s1s0s0s1s1

Non-deterministic LTS



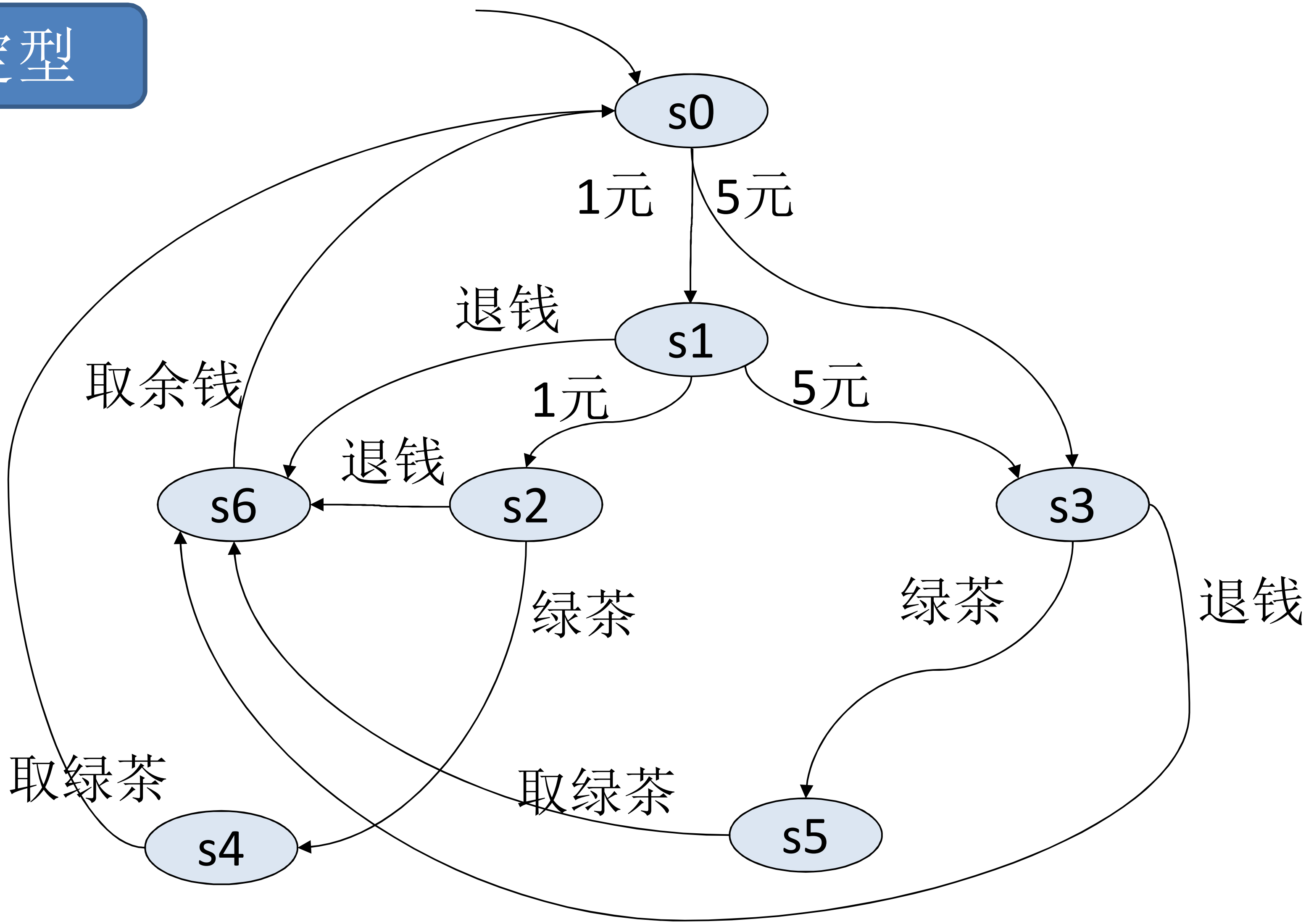
β α β β α α

s0s0s0s0s0s0

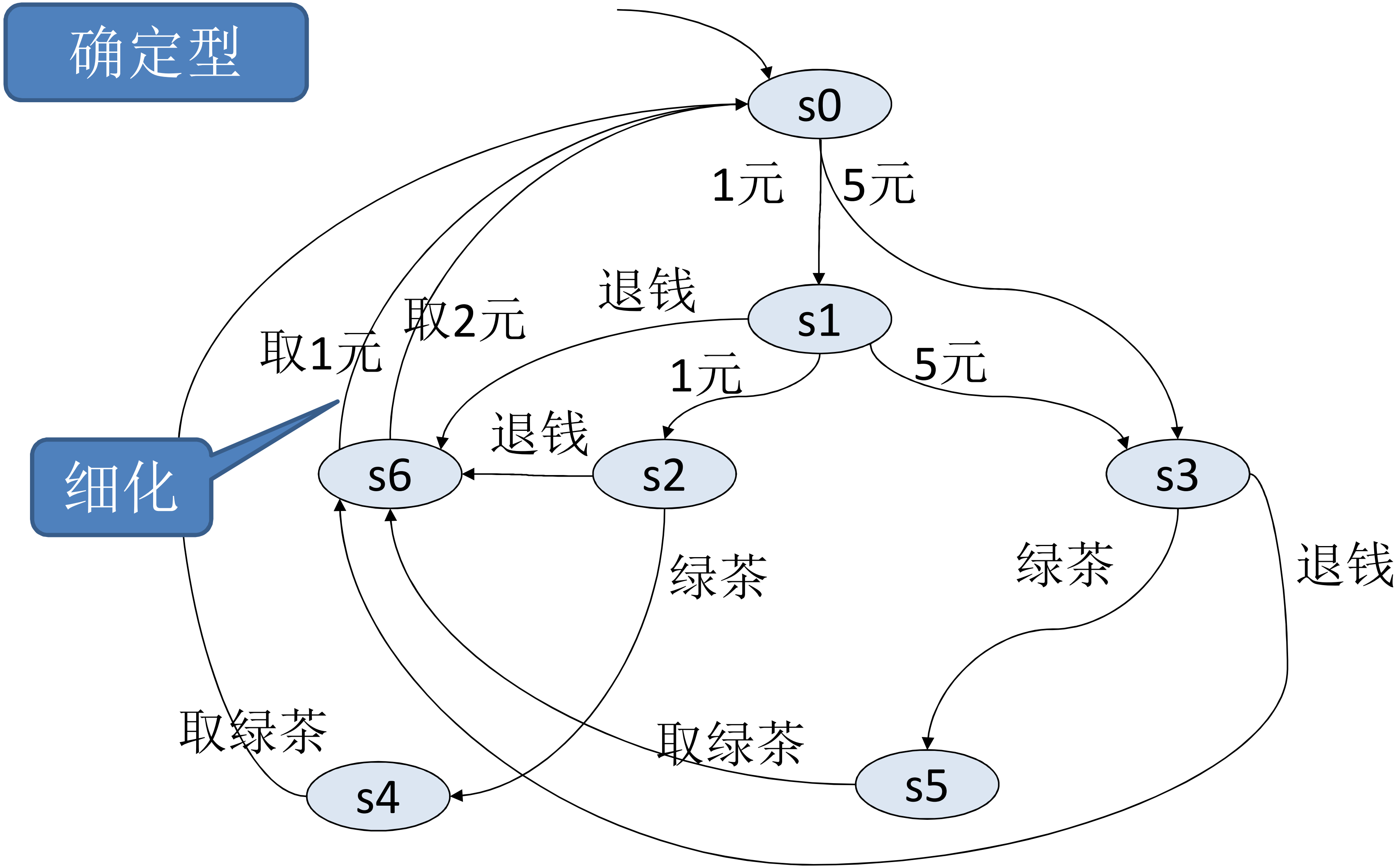
s0s0s0s0s0s1

例子：自动售茶机的设计

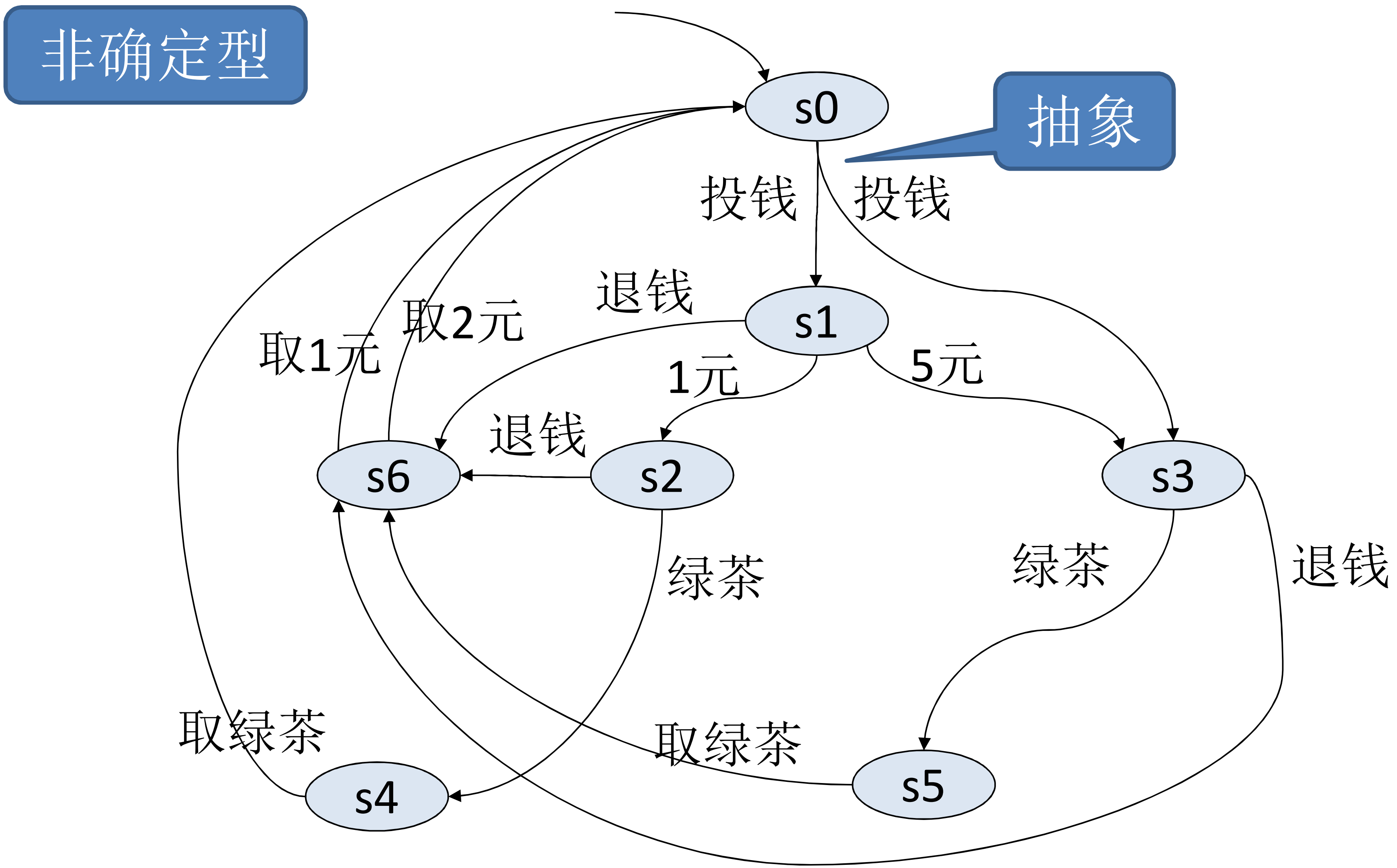
确定型



例子：自动售茶机的设计



例子：自动售茶机的设计



非确定型
抽象
s0
投钱
投钱
s1
退钱
取2元
取1元
1元
5元
退钱
s6
s2
s3
绿茶
绿茶
退钱
取绿茶
取绿茶
s4
s5

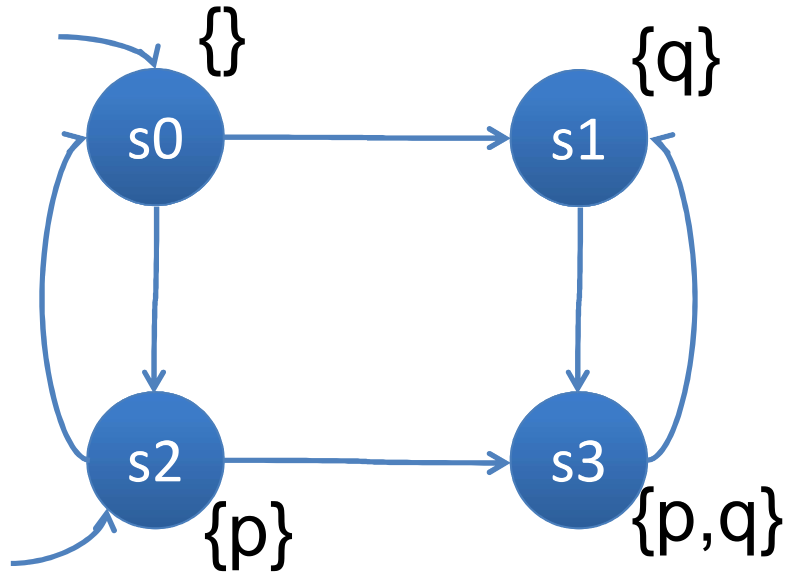
Comparison with Labeled KS

AP

K = <S,R,I,L>

L: S → 2^{AP}

K = <S,R,I> + <AP,L>

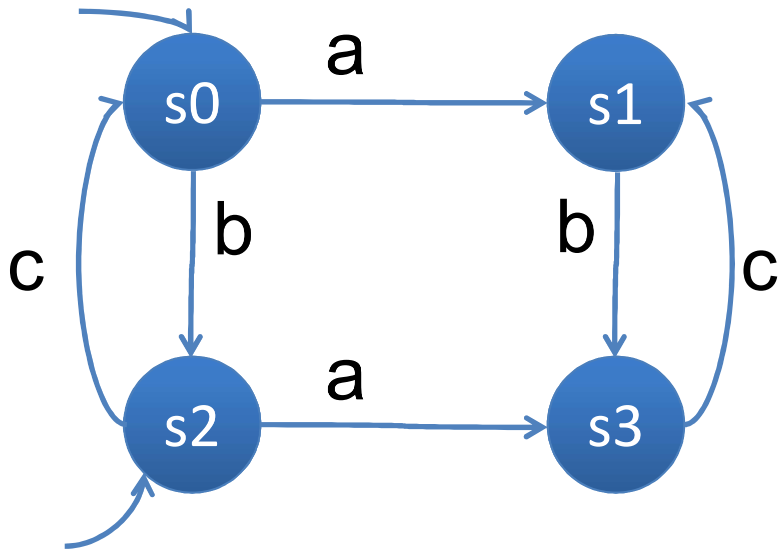


LTS = <Σ,S,Δ,I>

R = { (s,s') | (s,a,s') ∈ Δ }

Δ : R → (2^{Σ} \ ∅)

LTS = <S,R,I> + <Σ,Δ>



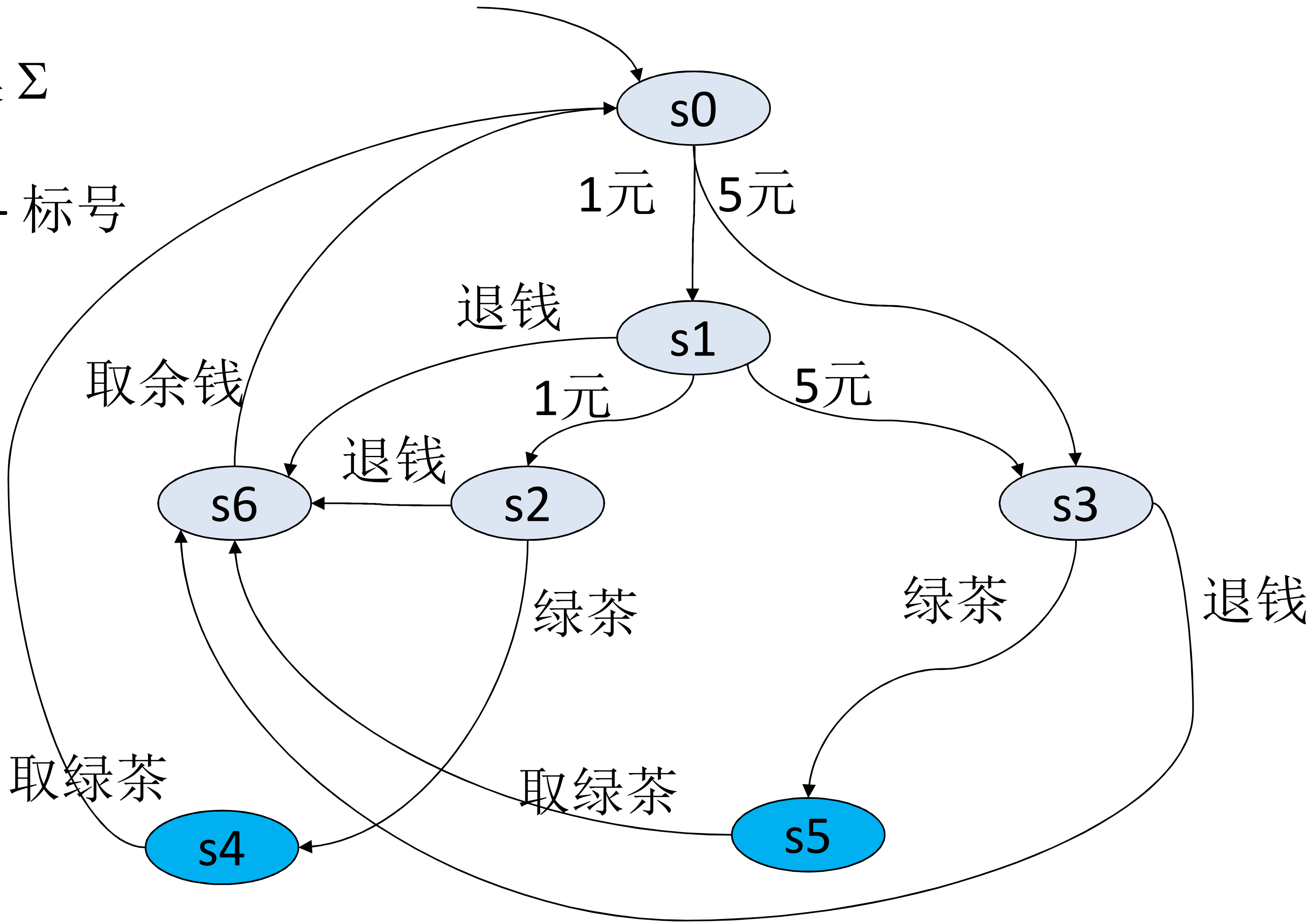
(II) Büchi Automata

LTS + Acceptance Condition

例子：自动售茶机的设计

标号集 Σ

(S,R,I) + 标号



Büchi Automata

Definition

A Büchi automaton (BA) is a quintuple $<\Sigma,S,\Delta,I,F>$

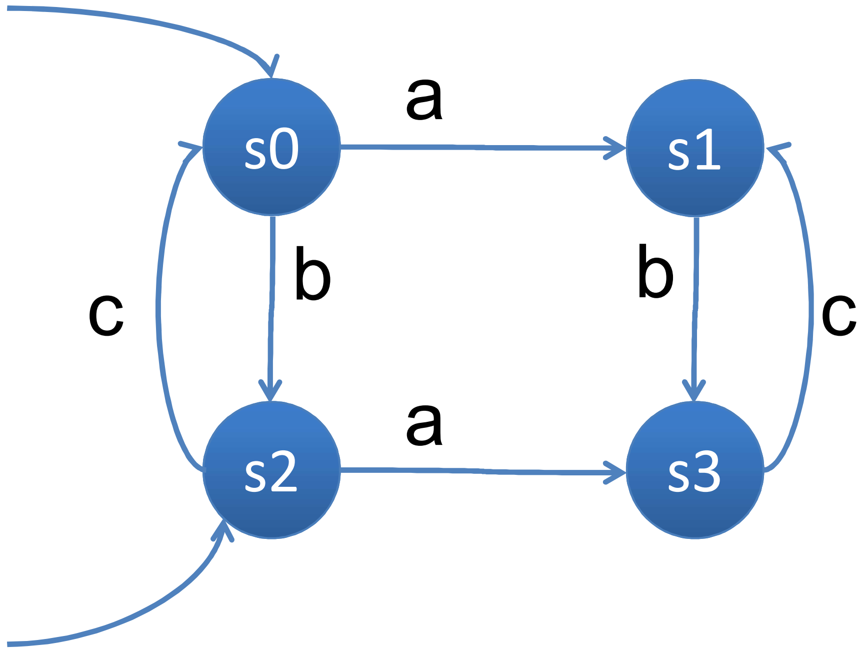
– $<\Sigma,S,\Delta,I>$ is a labeled transition system

– $F \subseteq S$: A set of acceptance states

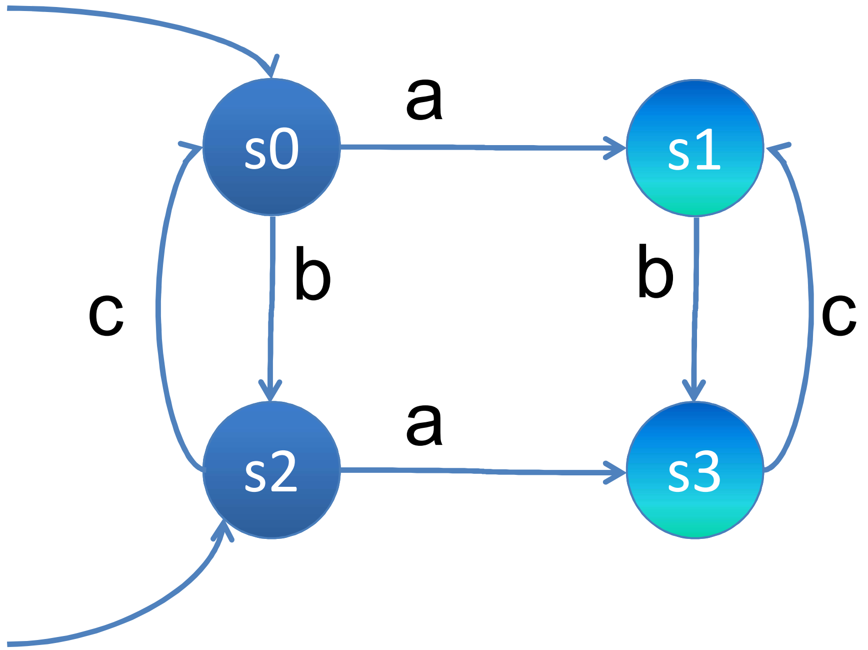
Büchi Automata

- Basic Concepts
  - Labels, States, Labeled Transition Relation, Initial States
  - Words, Runs
  - Accepting Runs, Accepting Words, Language
  - Comparison with LTS
- Emptiness
- Basic Operations
- Language Inclusion

Example: Σ, S, Δ, I



Example: F={s1,s3}



Words, Runs on Words, Runs

Given a LTS A=$<\Sigma,S,\Delta,I>$

A word is an infinite sequence of Σ

Let w=w[1]w[2]w[3]……$\in\Sigma^\omega$ be a word.

Definition

A **run** of A on w is an infinite sequence $s_0\, s_1\, s_2$ …. of S such that $s_0 \in I$, and $(s_i, w[i+1], s_{i+1}) \in \Delta$ for all $i \geq 0$.

Definition

A **run** of A is an infinite sequence $s_0\, s_1\, s_2$ …. of S such that there is a w and $s_0\, s_1\, s_2$ …. is a run on w.

Words over Runs

Definition

A **word** over a run r of A is

an infinite sequence of Σ: a_1a_2 ....

such that r is a run on a_1a_2 ....

Accepting Runs, Accepting Words

Let $\inf(\pi)$ be the set of states
that appear infinitely many times on π.

Definition
An **accepting run** of A is a run π of A
such that $\inf(\pi) \cap F \neq \emptyset$.

Definition
An **accepting word** of A is
a word over some accepting run of A.

Language

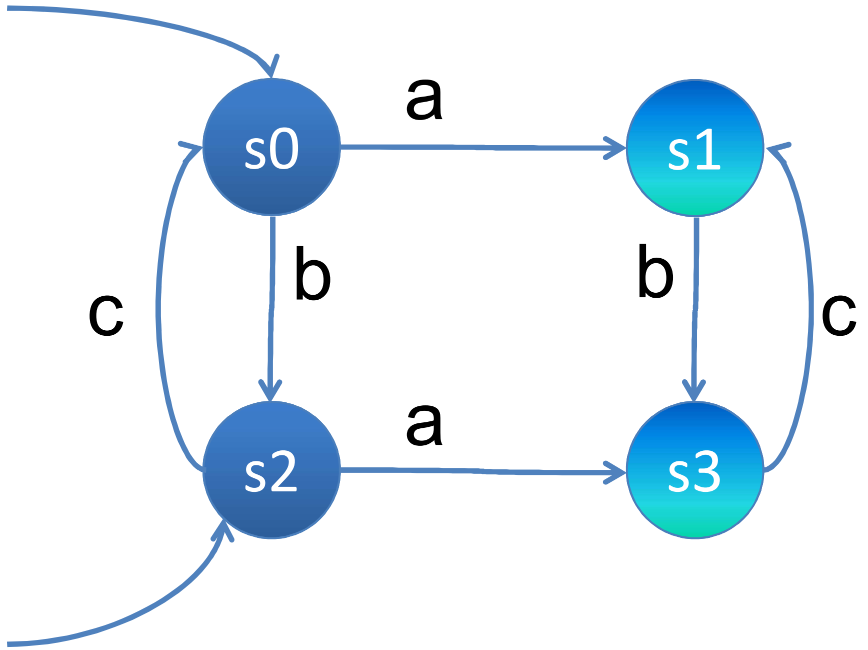
Definition

The language of A is

the set of accepting words of A.

The language of A is denoted L(A).

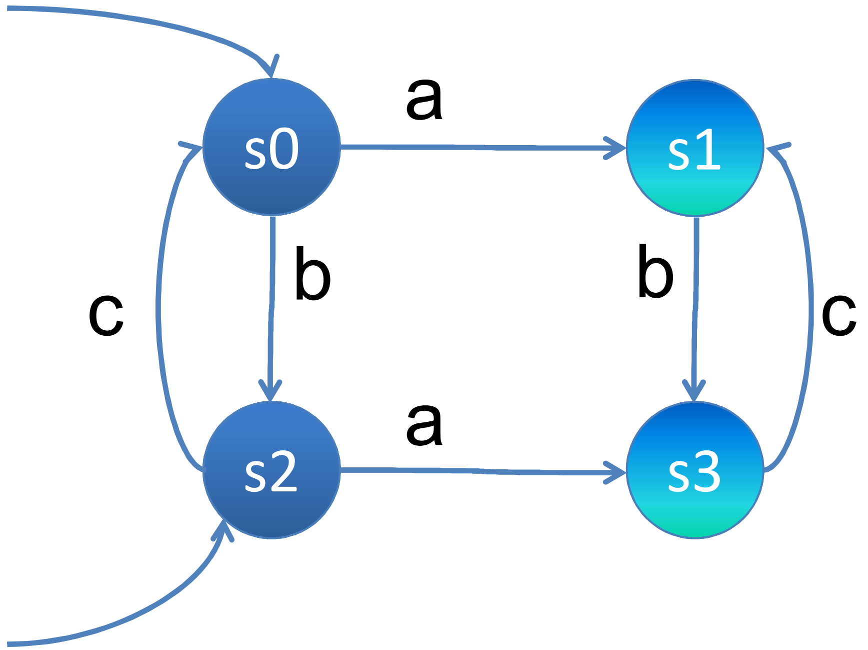
Example: Words, Runs



words: a^ω, $(bc)^\omega$, $a(bc)^\omega$

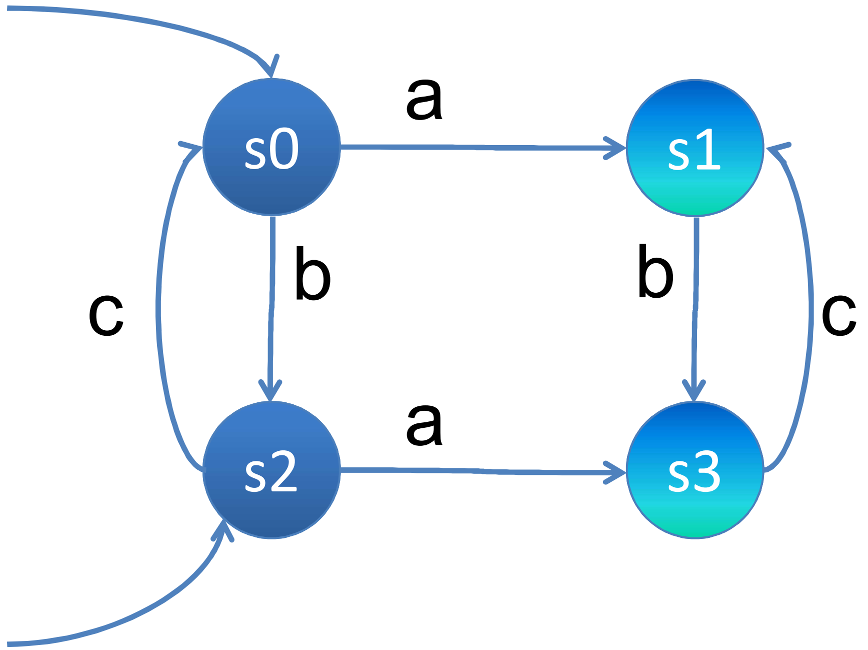
runs: $(s0s2)^\omega$, $s0(s1s3)^\omega$

Example: Accepting Runs



words:	a^ω,	$(bc)^\omega$,	$a(bc)^\omega$
runs:		$(s0s2)^\omega$,	$s0(s1s3)^\omega$
accepting runs:			$s0(s1s3)^\omega$

Example: Accepting Words and Language



accepting words:

$(bc)^*a\,(bc)^\omega$, $(bc)^*ba(cb)^\omega$, $(cb)^*a\,(cb)^\omega$, $(cb)^*ca(bc)^\omega$

Comparison with LTS

Given an LTS: M=<Σ,S,Δ,I>

- Σ : A finite set of symbols
- S : A finite set of states
- $\Delta \subseteq$ S x Σ x S : A transition relation
- I $\subseteq$ S : A set of initial states

Let A =<Σ,S,Δ,I,S> be a BA.

Then L(M) = L(A).

Emptiness Problem

Let A be a BA.

$L(A)=\emptyset$?

Emptiness Check

Given $A=<\Sigma,S,\Delta,I,F>$

Define $R=\{ (s,s') \mid (s,a,s') \in \Delta, s \in \Sigma\}$

L(A) is empty iff $<S,R,I,\{F\}>$ is empty.

A better algorithm is known as double DFS.

ddfs

Basic Operations

Preliminaries: Ramsey Theorem

Union

Intersection

Complementation

Ramsey Theorem

Two Colors

A group of 6 people:

3 of them know each other or do not know each other

A complete graph with 6 vertices, edges with 2 colors:

there is a triangle of which the 3 edges has the same color

$R(3,3)=6$

$R(4,4)=18$ (a complete subgraph with 4 vertices)

$R(5,5)\leq 48$

Three Colors

A complete graph with 17 vertices, edges with 3 colors:

there is a triangle of which the 3 edges has the same color

$R(3,3,3)=17$

Infinite Number of Vertices

A complete graph with infinite number of vertices,
edges with finite number of colors:
there is a complete subgraph with infinite number of vertices such that the edges of the graph are colored with the same color

Ramsey Theorem

A *k–coloring C of* $[X]^n$ *is a function from* $[X]^n$ *into a set* of size *k.*

H is homogeneous for C if C is constant on $[H]^n$*, i.e. all n–element subsets of H are assigned the same color by* C.

Ramsey Theorem *RT(n,k)*

Every *k–coloring of* $[\mathbf{N}]^n$ *has* an infinite homogeneous set.

Proof (by induction on *n*)

For $n = 1$: *[X]* is infinite, k is finite ➔ OK

Assuming n=r+1 and the theorem is true for $n \leq r$:

Given a *C*-coloring of the $(r + 1)$-element subsets of *X*.

Let a_0 be an element of *X* and let $Y = X \setminus \{a_0\}$. We have a *C*-coloring of the *r*-element subsets of *Y*, by deleting a_0 from each $(r + 1)$-element subset of *X*.

By the induction hypothesis, there exists an infinite subset Y_1 of *Y* such that every *r*-element subset of Y_1 is colored the same color in the induced coloring.

Proof

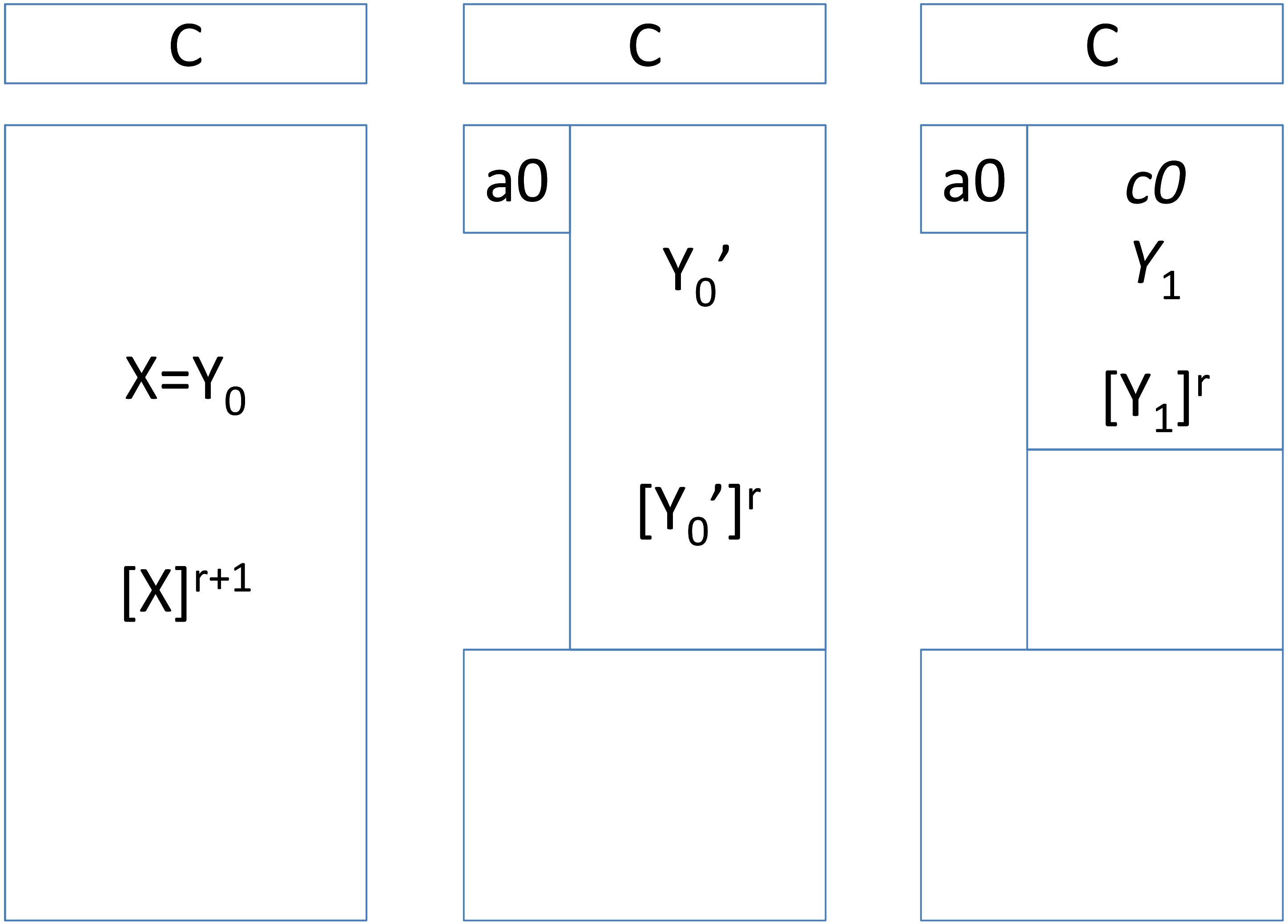
There is an element a_0 and an infinite subset Y_1 such that all the $(r + 1)$-element subsets of X consisting of a_0 and r elements of Y_1 have the same color.

By the same argument, there is an element a_1 in Y_1 and an infinite subset Y_2 of Y_1 with the same properties.

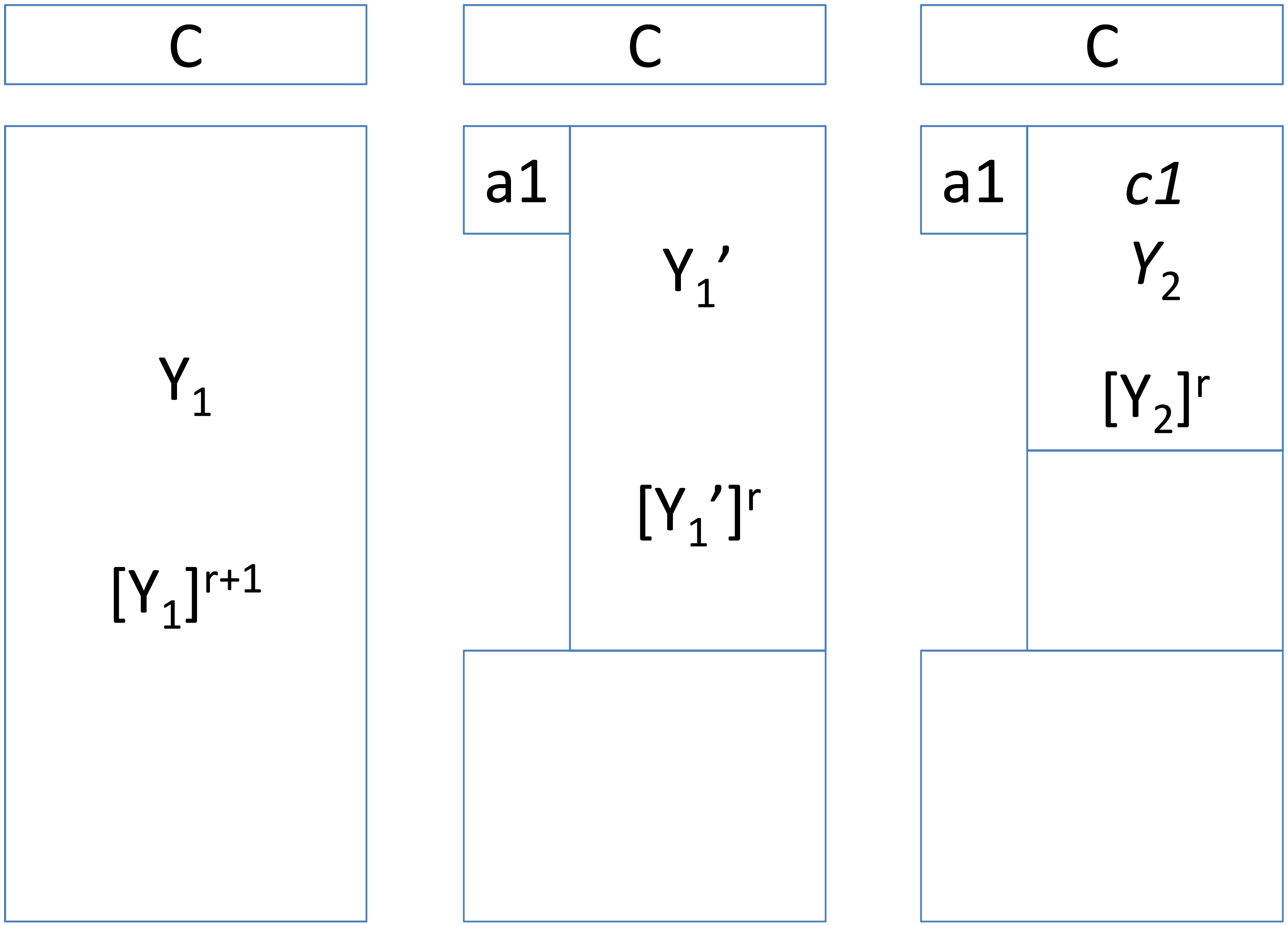
Inductively, we obtain a sequence $\{a_0, a_1, a_2, \ldots\}$ such that the color of each $(r + 1)$-element subset $(a_{i(1)}, a_{i(2)}, \ldots, a_{i(r+1)})$ with $i(1) < i(2) < \ldots < i(r + 1)$ depends only on the value of $i(1)$.

Further, there are infinitely many values of $i(n)$ such that this color will be the same. Take these $a_{i(n)}$'s to get the desired monochromatic set.

Proof



Proof



Proof

Then we have

a0,a1,a2,a3,…
c0,c1,c2,c3,…

Let J_m ($1 \leq m \leq k$) be the set of a_j
such that c_m is consistent with the selection of a_j .

Then one of such is an infinite set. Let it be Z.

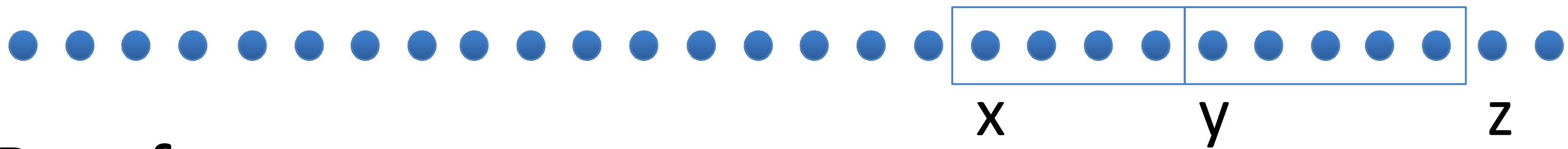
Then C is constant on $[Z]^n$

Corollary

Suppose that Σ^* is divided into finitely many equivalent classes.

Let $w=w[1]w[2]w[3]w[4]\ldots$ be an infinite word over Σ.

Then there is a pair of equivalent classes U,V such that $w \in U.V^{\omega}$.



Proof.

Let $f(x,y)$ = equivalent class of $w[x\ldots y-1]$ for $y>x\geq 1$.

Then the corollary follows from Ramsey theorem for pairs.

Basic Operations

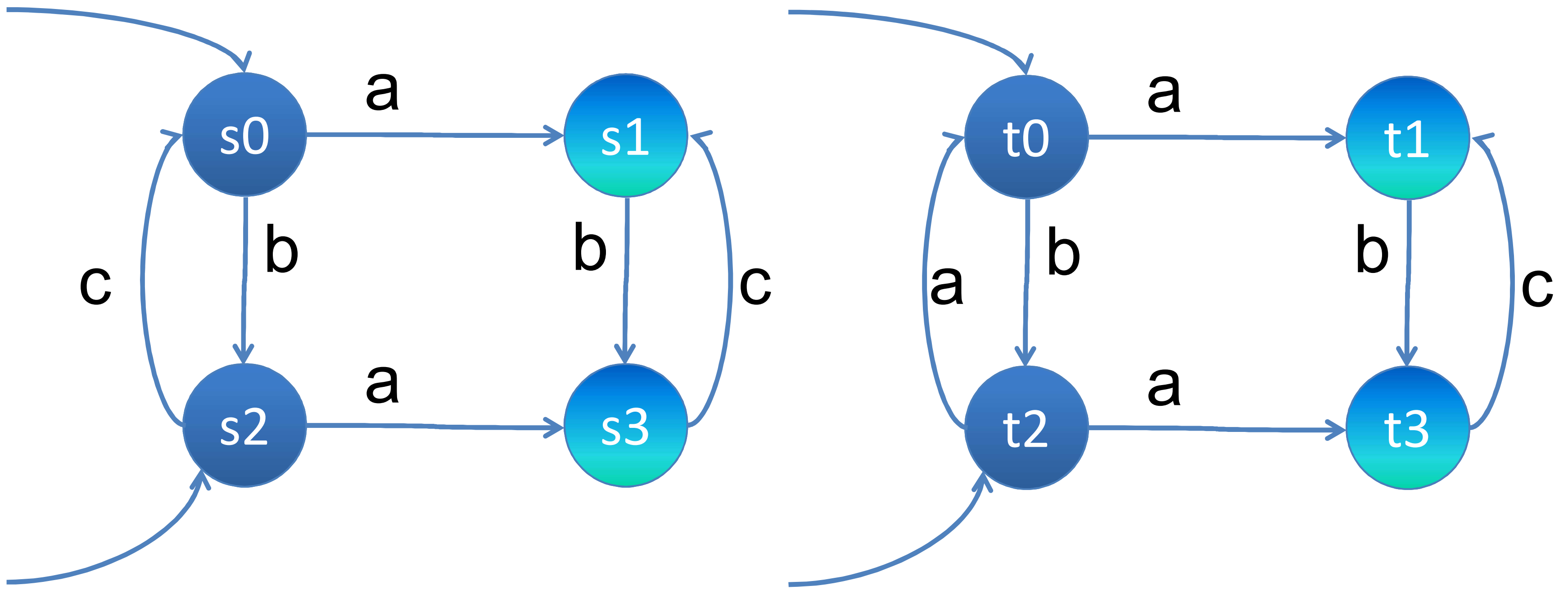
Preliminaries: Ramsey Theorem

Union

Intersection

Complementation

Union: Example



Union

Given two BAs

$A_1 = <\Sigma, S_1, \Delta_1, I_1, F_1>$, $A_2 = <\Sigma, S_2, \Delta_2, I_2, F_2>$.

Suppose that S_1 and S_2 are disjoint.

Define $A_1 \cup A_2 = <\Sigma, S, \Delta, I, F>$ where

$S = S_1 \cup S_2$

$\Delta = \Delta_1 \cup \Delta_2$

$I = I_1 \cup I_2$

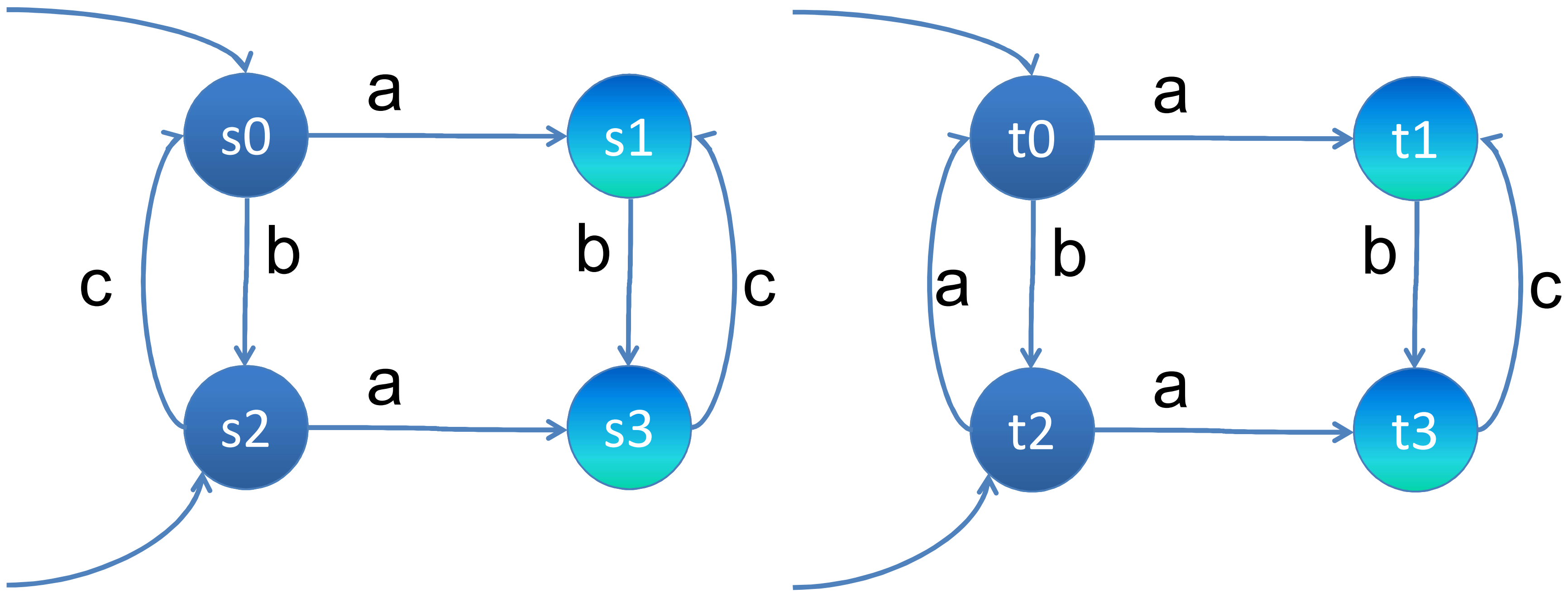
$F = F_1 \cup F_2$

Union

Theorem

$L(A_1 \cup A_2) = L(A_1) \cup L(A_2)$

Intersection: Example



Intersection

Given BAs

$A_1=<\Sigma,S_1,\Delta_1,I_1,F_1>$, $A_2=<\Sigma,S_2,\Delta_2,I_2,F_2>$.

Attempt 1: Define $A_1 \cap A_2 = <\Sigma,S,\Delta,I,F>$ where

$S = S_1 \times S_2$

$\Delta = \{ ((s_1,s_2),a,(s_1',s_2')) \mid (s_1,a,s_1') \in \Delta_1, (s_2,a,s_2') \in \Delta_2 \}$

$I = I_1 \times I_2$

$F = ?$

Intersection

Given BAs

$A_1 = <\Sigma, S_1, \Delta_1, I_1, F_1>$, $A_2 = <\Sigma, S_2, \Delta_2, I_2, F_2>$.

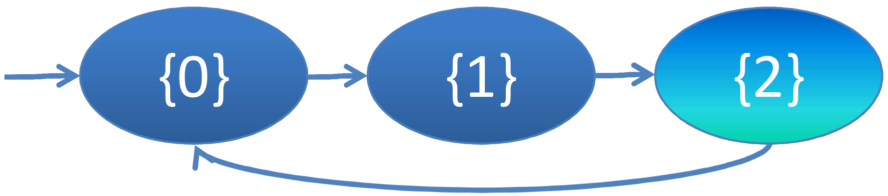
Define $A_1 \cap A_2 = <\Sigma, S, \Delta, I, F>$ where

$S = S_1 \times S_2 \times \{0,1,2\}$

$\Delta = \{ ((s_1,s_2,i),a,(s_1',s_2',j)) \mid (s_1,a,s_1') \in \Delta_1, (s_2,a,s_2') \in \Delta_2, ?? \}$

$I = I_1 \times I_2 \times \{0\}$

$F = S_1 \times S_2 \times \{2\}$



Intersection

Δ =

$\{ ((s_1,s_2,0),a,((s_1',s_2',0)) \mid (s_1,a,s_1')\in\Delta_1, (s_2,a,s_2')\in\Delta_2 \} \cup$
$\{ ((s_1,s_2,0),a,((s_1',s_2',1)) \mid (s_1,a,s_1')\in\Delta_1, (s_2,a,s_2')\in\Delta_2, s_1\in F_1\} \cup$

$\{ ((s_1,s_2,1),a,((s_1',s_2',1)) \mid (s_1,a,s_1')\in\Delta_1, (s_2,a,s_2')\in\Delta_2 \} \cup$
$\{ ((s_1,s_2,1),a,((s_1',s_2',2)) \mid (s_1,a,s_1')\in\Delta_1, (s_2,a,s_2')\in\Delta_2, s_2\in F_2\} \cup$

$\{ ((s_1,s_2,2),a,((s_1',s_2',0)) \mid (s_1,a,s_1')\in\Delta_1, (s_2,a,s_2')\in\Delta_2 \}$

Intersection

Theorem

$L(A_1 \cap A_2) = L(A_1) \cap L(A_2)$

Complementation

The set of BAs is closed under complementation.

Given $A=<\Sigma,S,\Delta,I,F>$.

There exists a BA B such that

$L(B) = \Sigma^{\omega} \setminus L(A)$

Proof

Definition

A congruence '~' over a set of strings is an equivalence relation such that

(x1 ~ y1 and x2 ~ y2) ➔ x1.x2 ~ y1.y2

Proof

Given $A=<\Sigma,S,\Delta,I,F>$.

Define $\approx$ over Σ^*.

$u \approx v$, iff for all q,q': $q \xrightarrow{u(F)} q'$ and $q \xrightarrow{v(F)} q'$.

Proof

$\approx$ is a congruence, i.e.,

u1≈v1 and u2≈v2 implies u1u2≈v1v2

The number of such equivalence classes is finite.

Proof

Suppose that U,V are equivalent classes.

Lemma

$U.V^{\omega} \subseteq L(A)$ or $U.V^{\omega} \subseteq \overline{L(A)}$.

Lemma

Let w be an infinite word over Σ.

Then there is a pair of equivalent classes U,V such that $w \in U.V^{\omega}$.

Proof

Theorem

$\overline{L(A)}$ can be represented by a Büchi automaton.

Proof.

Each of $U.V^{\omega} \subseteq \overline{L(A)}$ can be represented by an Büchi automata. The union of such automata is also representable by a Büchi automaton.

We have: $\overline{L(A)} = \cup \{ U.V^{\omega} \mid U.V^{\omega} \cap L(A) = \emptyset \}$.

Proof

Reference

D. A. Peled. Software Reliability Methods.

2001. pp.151-152.

Language Inclusion

Let A and B be a BAs.

$L(A) \subseteq L(B)$?

$L(A) \cap (\Sigma^{\omega} \setminus L(B)) = \emptyset$?

$L(A) \cap L(\neg B) = \emptyset$?

$A \cap \neg B = \emptyset$?

Example: Properties and Emptiness

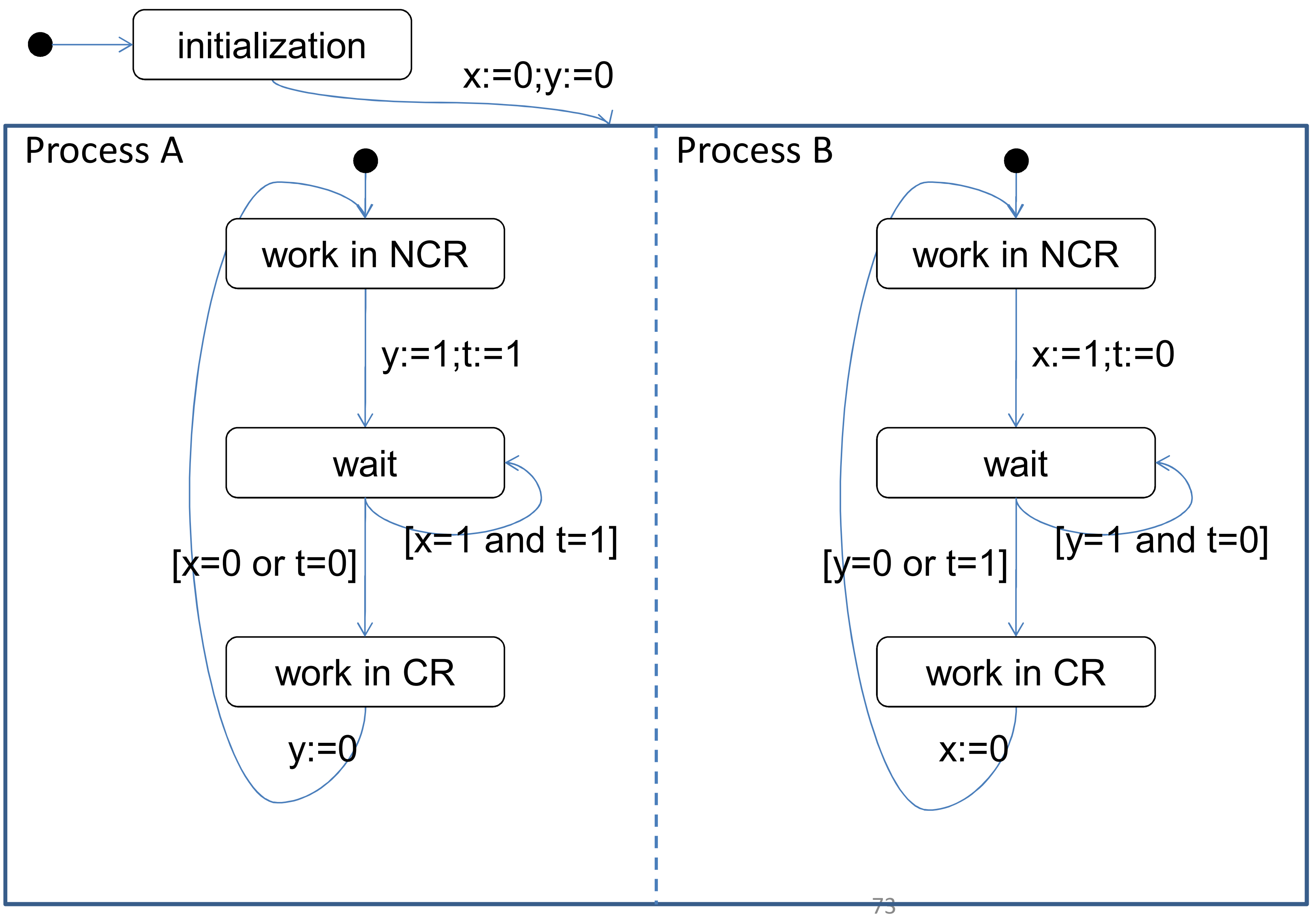
Contents

Example: Properties and Emptiness

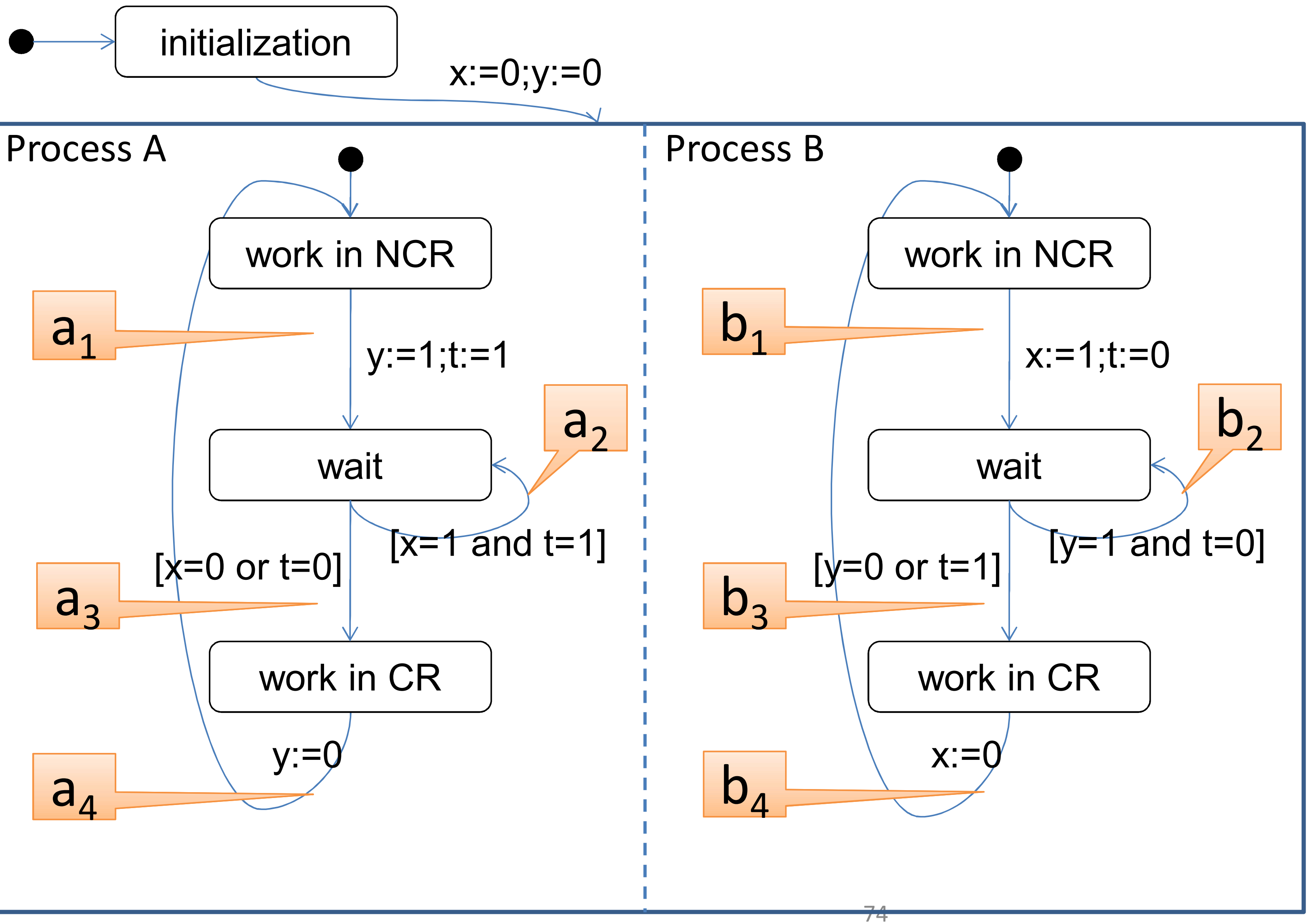
Contents

- Mutual Exclusion
- Büchi Automata
- Emptiness Checking
- Summary

Design of Mutual Exclusion



Design of Mutual Exclusion



Example: Properties and Emptiness

Contents

Büchi Automaton

Definition

A Büchi automaton is a quintuple <Σ,S,Δ,I,F>

– Σ : A finite set of symbols

– S : A finite set of states

– $\Delta \subseteq S \times \Sigma \times S$: A transition relation

– $I \subseteq S$: A set of initial states

– $F \subseteq S$: A set of acceptance states

The Set of Actions: Σ

$\{ a_i, b_i | \ i=1,2,3,4 \}$

The Set of States: S

$\{(a,b,x,y,t) \mid a,b \in \{NCR, wait, CR\} \text{ and } x,y,t \in \{0,1\}\}$

Transition Relation: R

(NCR,b,x,y,t)	→a1	(wait,b,x,1,1)
(wait,b,1,y,1)	→a2	(wait,b,1,y,1)
(wait,b,0,y,t)	→a3	(CR,b,0,y,t)
(wait,b,x,y,0)	→a3	(CR,b,x,y,0)
(CR,b,x,y,t)	→a4	(NCR,b,x,0,t)
(a,NCR,x,y,t)	→b1	(a,wait,1,y,0)
(a,wait,x,1,0)	→b2	(a,wait,x,1,0)
(a,wait,x,1,t)	→b3	(a,CR,x,1,t)
(a,wait,x,y,1)	→b3	(a,CR,x,y,1)
(a,CR,x,y,t)	→b4	(a,NCR,0,y,t)

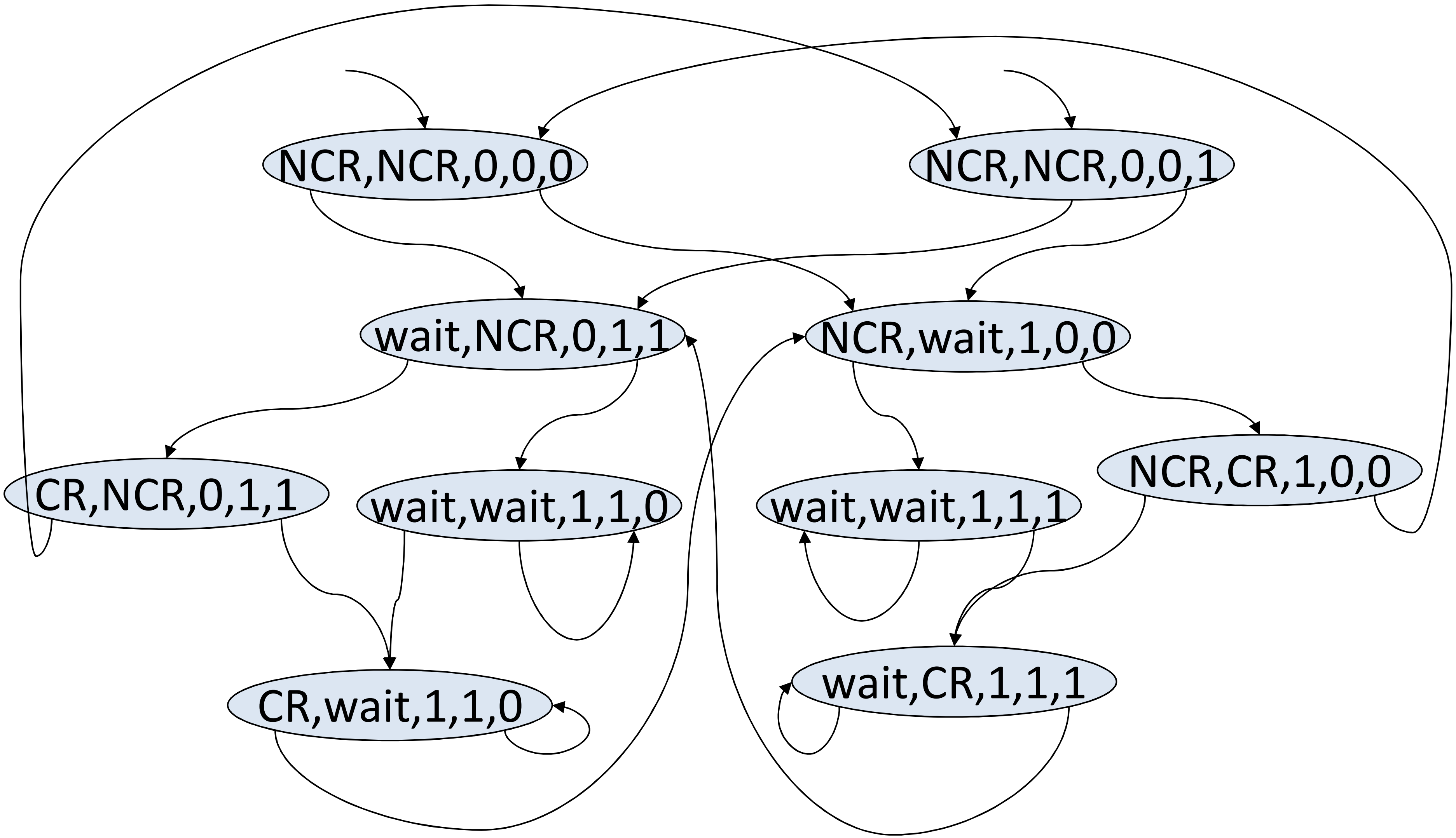
The Set of Initial States: I

{ (NCR,NCR,0,0,0), (NCR,NCR,0,0,1) }

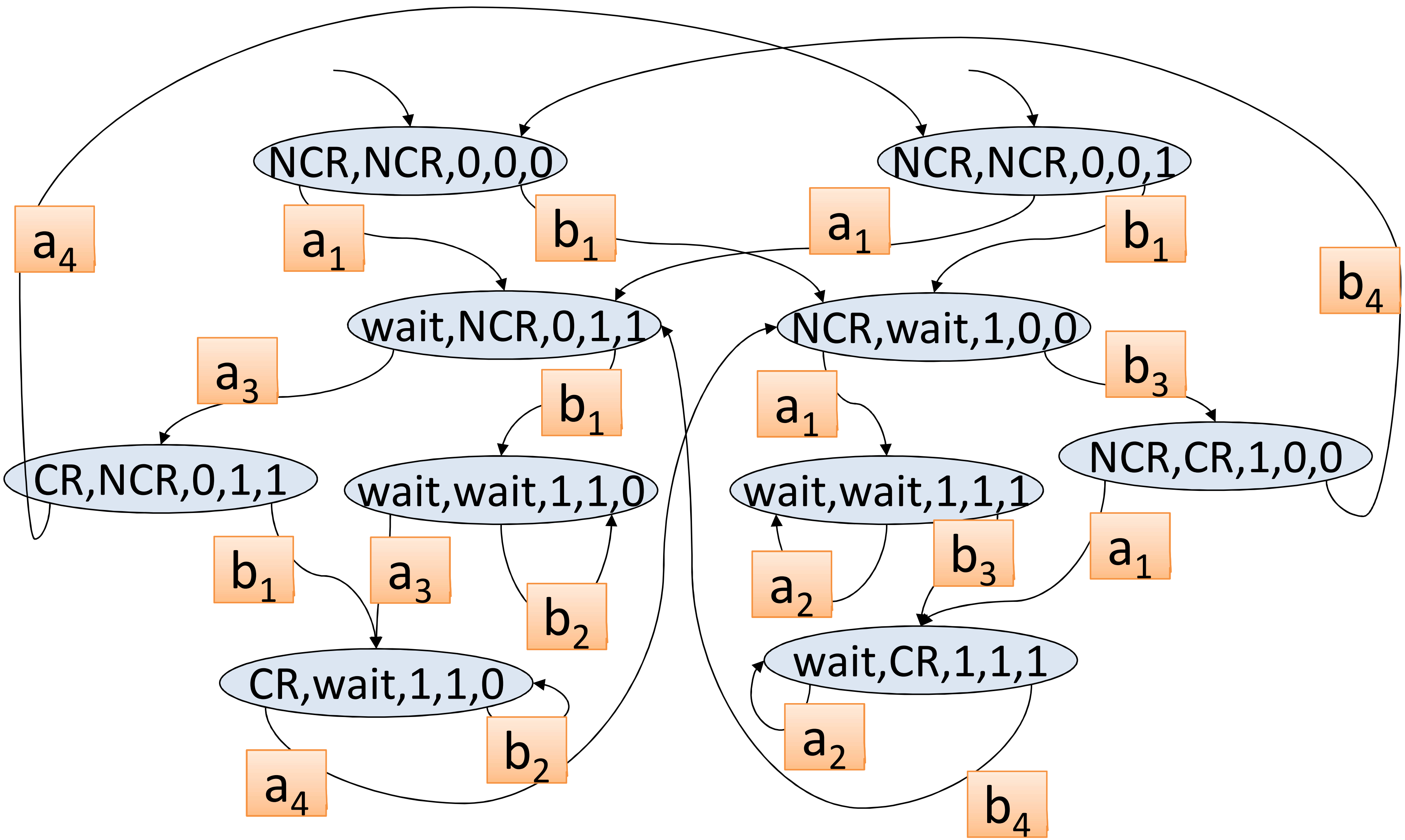
The Set of Accepting States: F

F = S

Büchi Automata



Büchi Automata



Specification of a Safety Property

$(\Sigma \backslash \{a3,b3\})^{\omega}$

$(\Sigma \backslash \{a3,b3\})^{*} b3.(\Sigma \backslash \{a3,b3,b4\})^{\omega}$

$(\Sigma \backslash \{a3,b3\})^{*} a3.(\Sigma \backslash \{b3,a3,a4\})^{\omega}$

$(\Sigma \backslash \{a3,b3\})^{*} b3.(\Sigma \backslash \{a3,b3,b4\})^{*}.b4\ldots\ldots$

$(\Sigma \backslash \{a3,b3\})^{*} a3.(\Sigma \backslash \{b3,a3,a4\})^{*}.a4\ldots\ldots$

Specification of a Safety Property

$X=(\Sigma \backslash \{a3,b3\})$

$Y=(\Sigma \backslash \{a3,b3,b4\})$

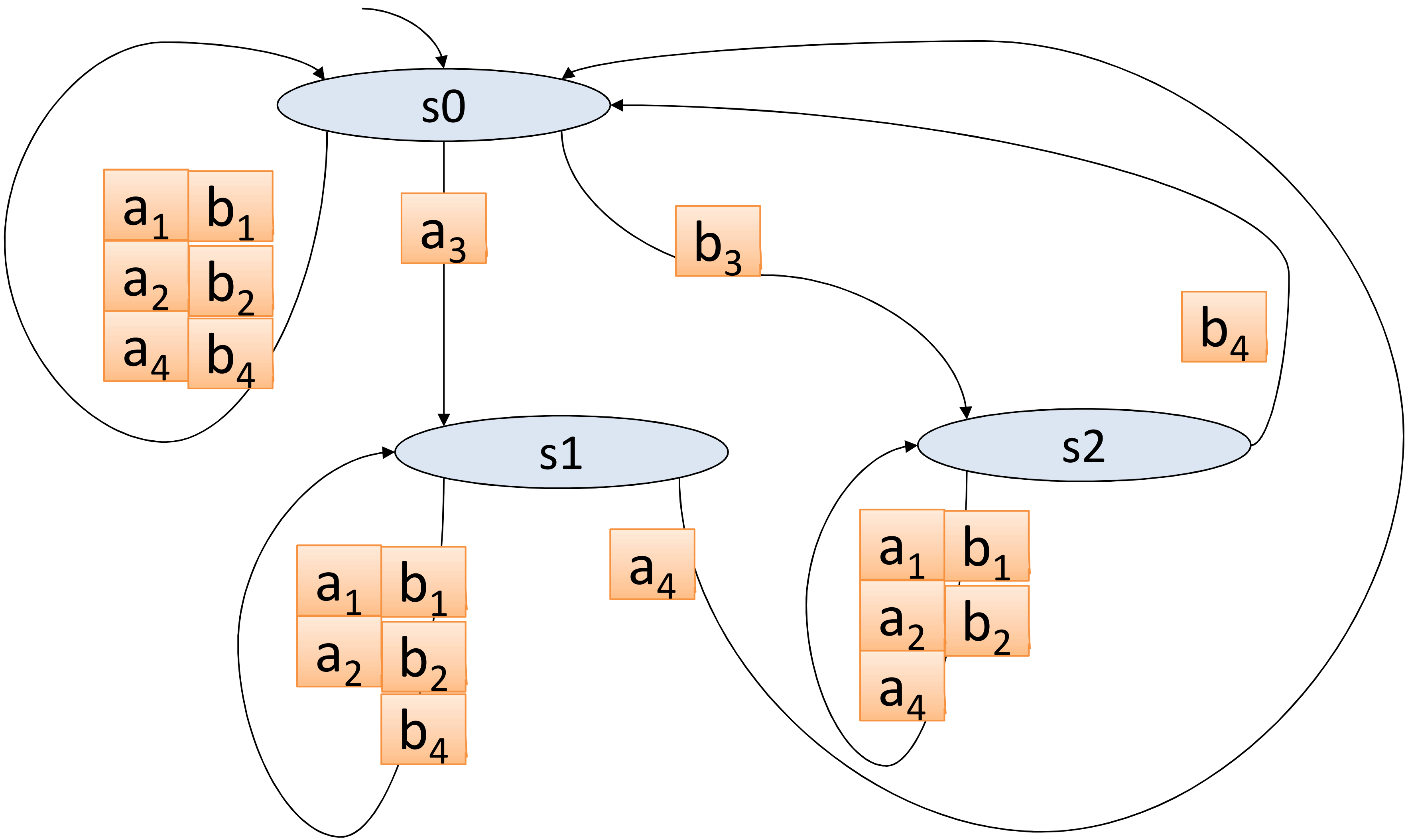
$Z=(\Sigma \backslash \{b3,a3,a4\})$

$U=X^*((b3.Y^*.b4)|(a3.Z^*.a4))$

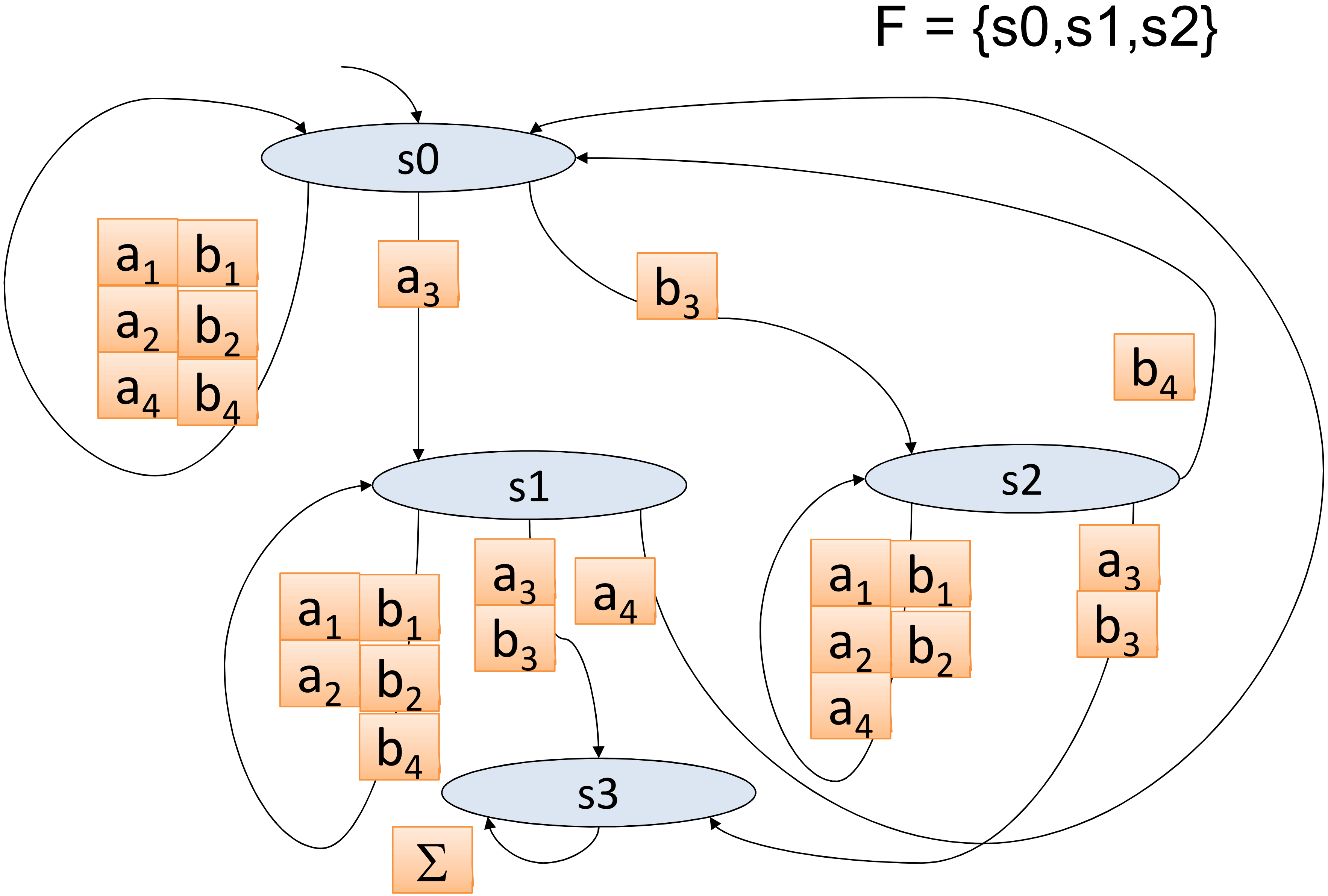
$U^\omega \,|U^*X^\omega|U^*X^*b3.Y^\omega|U^*X^*a4.Z^\omega$

Büchi Automaton of the Safety Spec.

F = {s0,s1,s2}



Büchi Automaton of the Safety Spec.

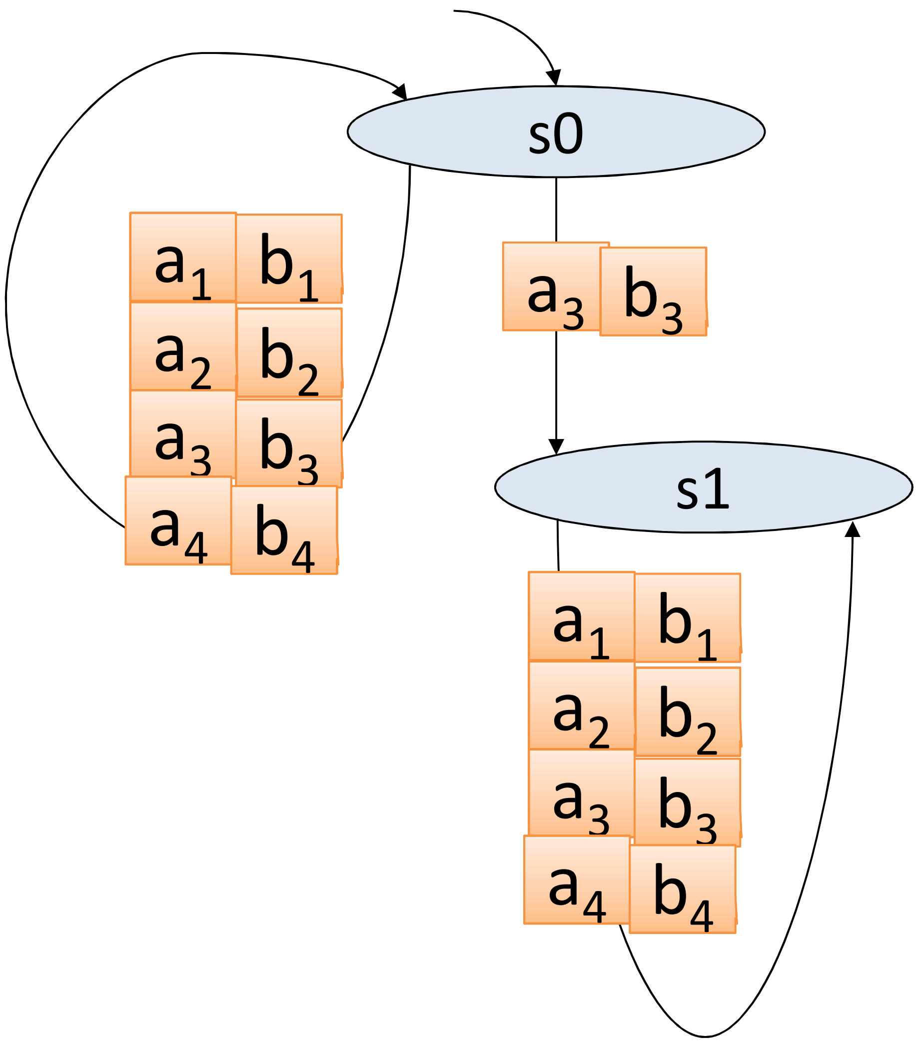


Inevitability Property

$\Sigma^*(a3|b3).\Sigma^\omega$

Specification of an Inevitability Property

$F = \{s1\}$



Example: Properties and Emptiness

Contents

Emptiness Checking

A: model automaton

B: specification automaton

$L(A) \subseteq L(B)$

$L(A) \cap (\Sigma^\omega \setminus L(B)) = \emptyset$

$L(A) \cap L(\neg B) = \emptyset$

$A \cap \neg B = \emptyset$

Example: Properties and Emptiness

Contents

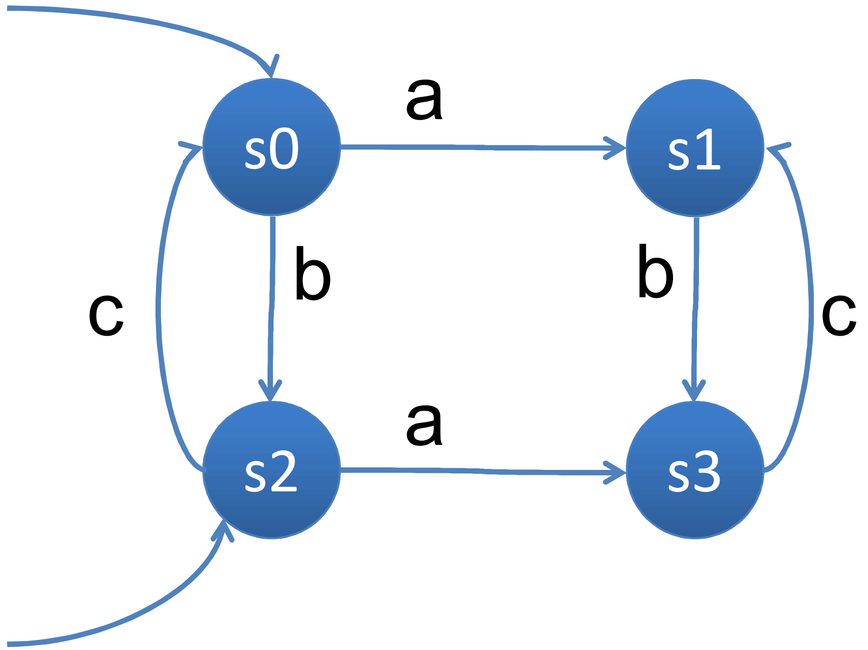
(III) Generalized Büchi Automaton

Definition

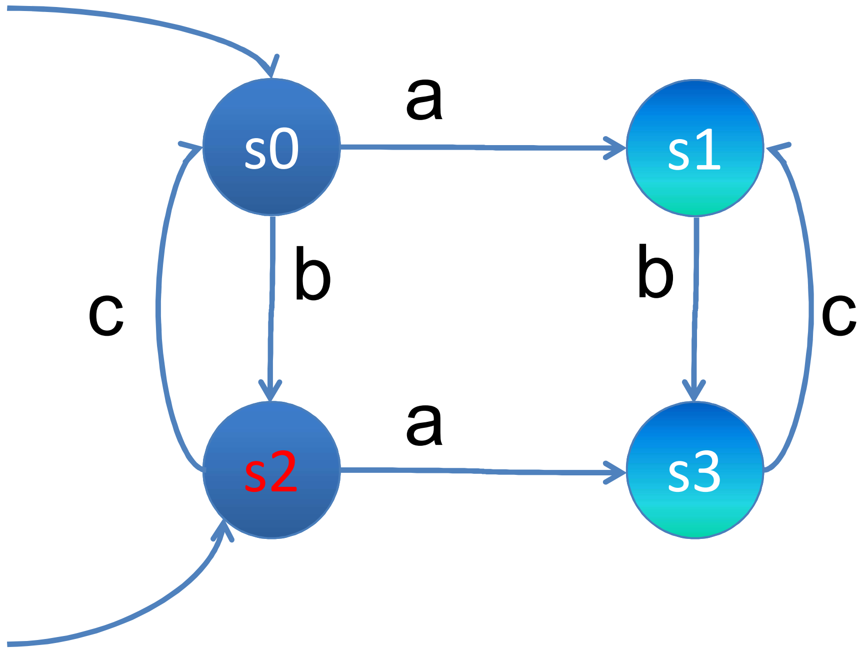
A GBA is a quintuple <Σ,S,Δ,I,F>

- Σ : A finite set of symbols
- S : A finite set of states
- $\Delta \subseteq S \times \Sigma \times S$: A transition relation
- $I \subseteq S$: A set of initial states
- $F \subseteq 2^S$: A set of sets of acceptance states

Example: Σ,S,Δ,I



Example: F={{s1,s3},{s2}}



Basic Concepts

Accepting Runs

Let $\inf(\pi)$ be the set of states that appear infinitely many times on π.

Definition

An **accepting run** of A is a run π of A such that for each $f \in F$, $\inf(\pi) \cap f \neq \emptyset$.

Language

Definition

The language of A is

the set of accepting words of A.

The language of A is denoted L(A).

Expressiveness of GBAs

Theorem

Every language expressible by a BA is also expressible by a GBA.

Proof

Given a BA A=<Σ,S,Δ,I,F>.

Let B=<Σ,S,Δ,I,{F}> be a GBA.

Then L(B)=L(A).

Expressiveness of GBAs

Theorem

Every language expressible by a GBA is also expressible by a BA.

Proof

Given a GBA $A=<\Sigma,S,\Delta,I,\{f_1,\ldots,f_n\}>$.

We can construct a BA $B=<\Sigma,S',\Delta',I',F'>$ such that $L(B)=L(A)$.

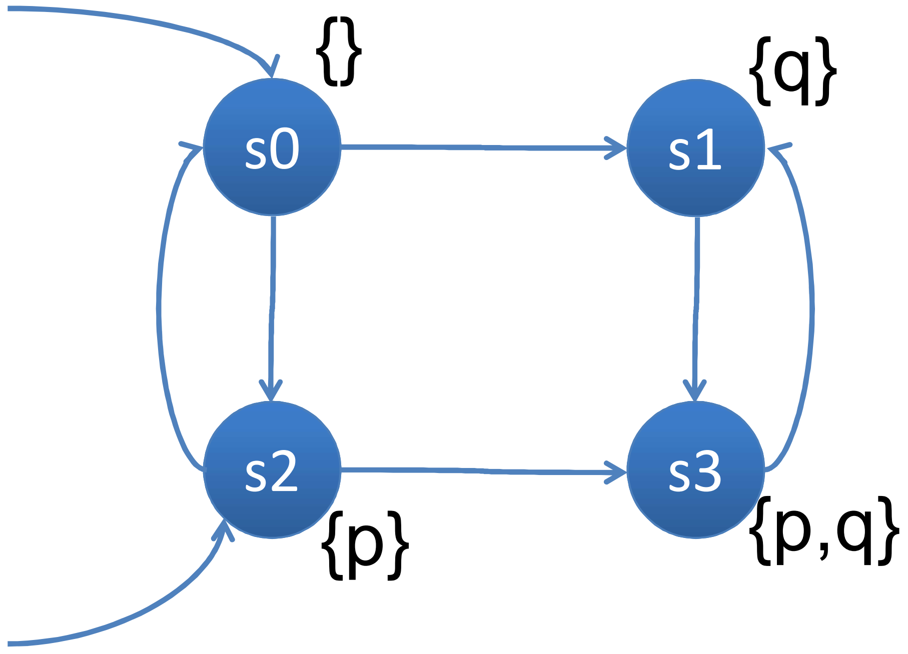
The proof is left as an exercise.

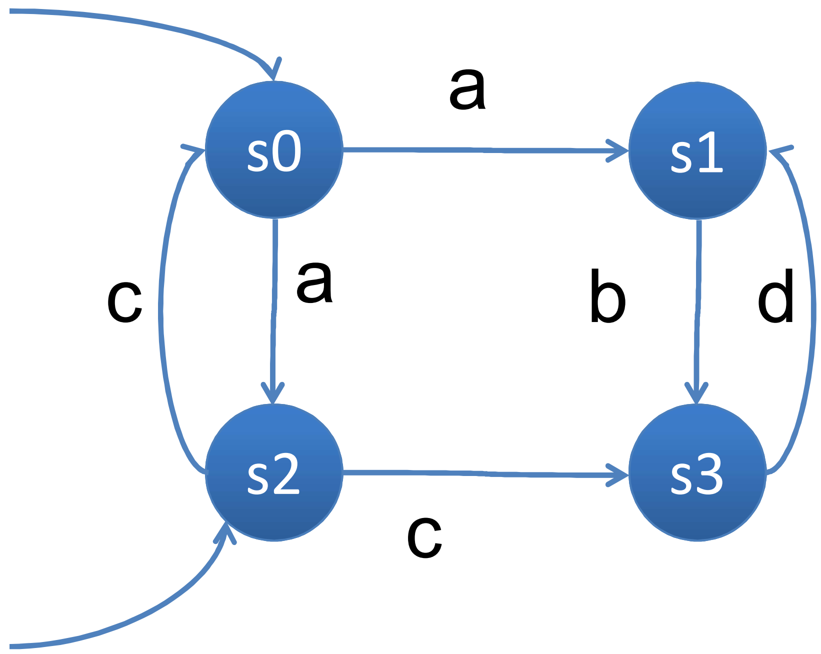
Basic Operations

The set of GBAs is closed under

union, intersection and complementation.

Fair Labeled Kripke Structures and ω-Automata





AP: {p,q}

Σ={a,b,c,d}

Fair Labeled Kripke Structures and ω-Automata

Let AP be given. Let K=<S,R,I,L,Φ> over AP.

Let A=<Σ,S,Δ,I,F> be a GBA.

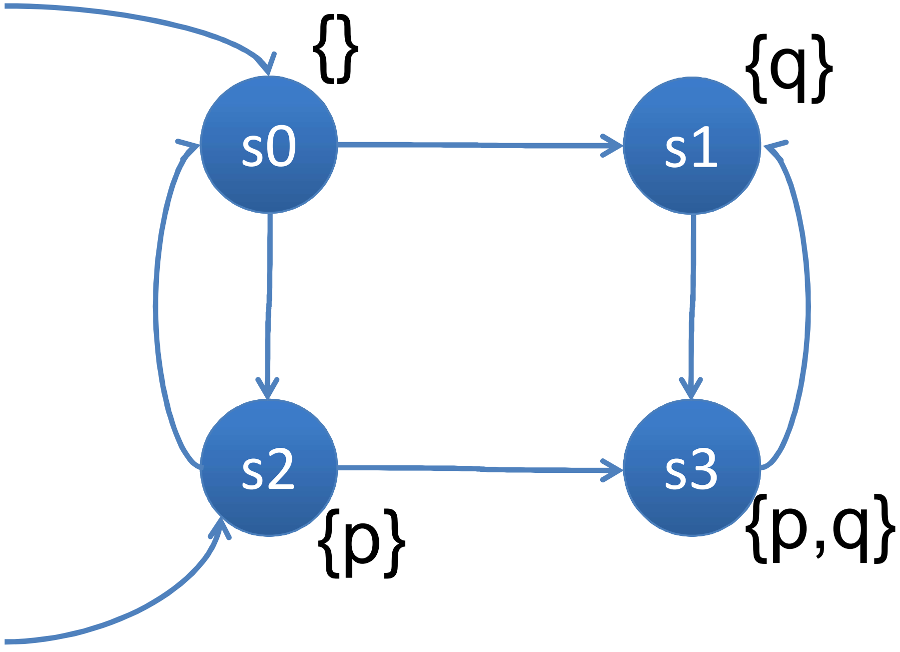
Let ζ be a mapping between 2^{AP} and Σ.

A and K are ζ-equivalent, if L(A)= ζ(L(K)),

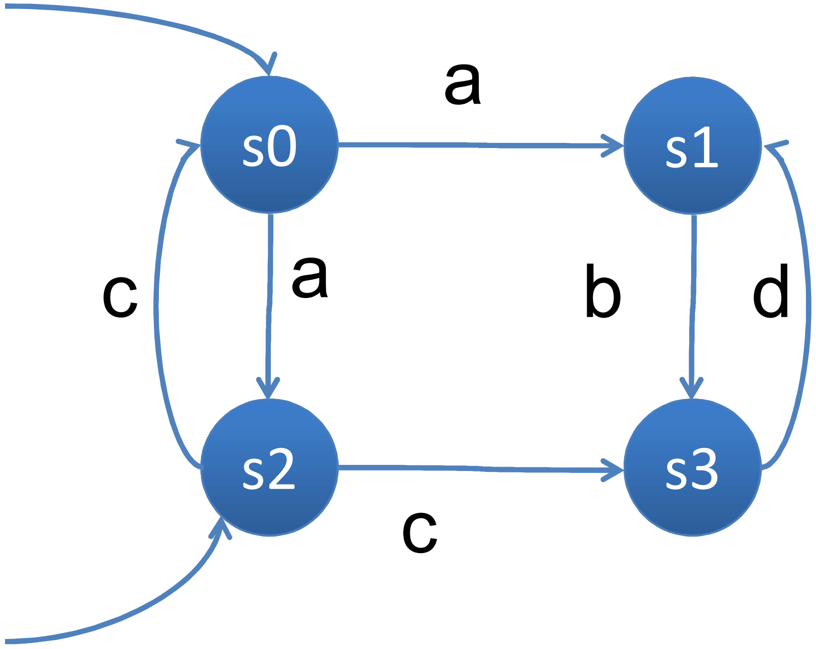
i.e, for every fair computation c of K,

there is an accepting run r of A such that L(r) = ζ(L(c)), and vice versa.

Fair Labeled Kripke Structures and ω-Automata

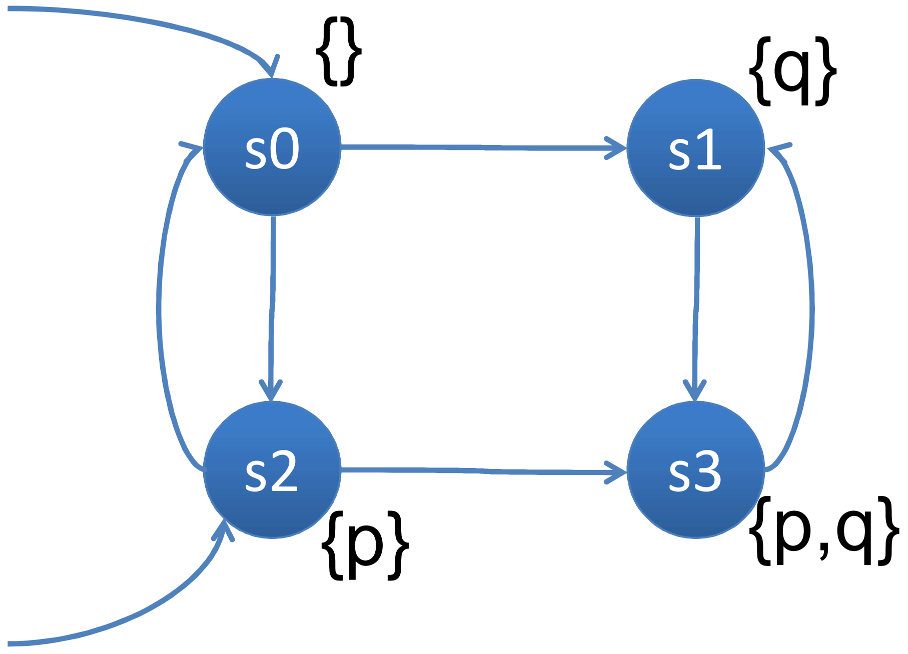


AP: {p,q}

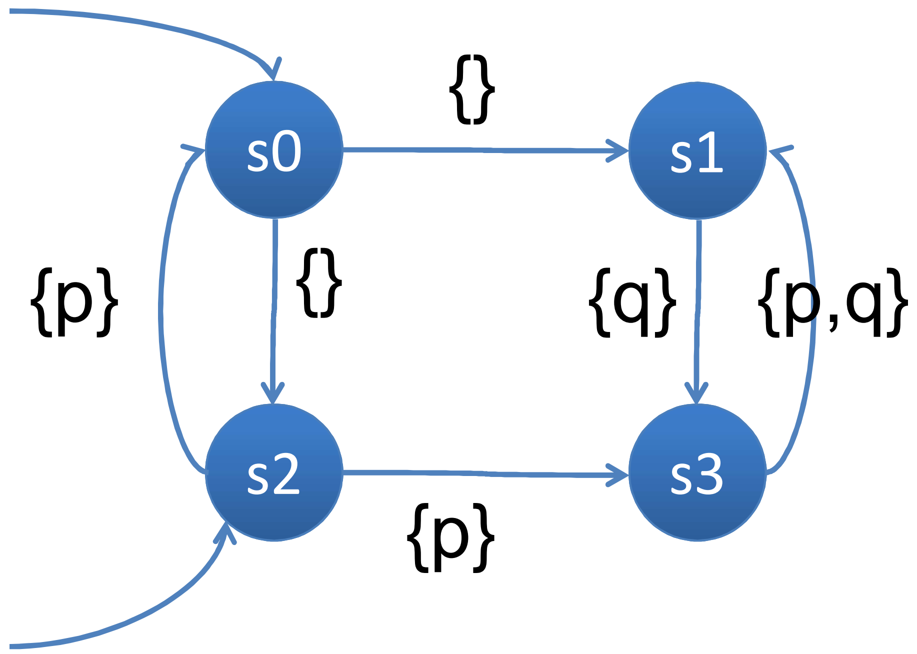


Σ={a,b,c,d}

Fair Labeled Kripke Structures and ω-Automata



AP: {p,q}



Σ={{},{p},{q},{p,q}}

Fair Labeled Kripke Structures → ω-Automata

Let AP be given.

Let K=<S,R,I,L,{φ_1,…, φ_n}>

be a fair labeled Krikpe structure.

Let $\Sigma=2^{AP}$.

Let A=<Σ,S,Δ,I,{f_1,…,f_n}> be a GBA where

Δ={ (s,a,s') | (s,s')$\in$R, a=L(s) } and f_i =[[φ_i]]

Fair Labeled Kripke Structures → ω-Automata

π is a computation of K, iff
L(π) is a word over some runs of A.

π is a fair computation of K, iff
L(π) is an accepting word of A.

L(A) = L(K) (and ζ=ID)

ω-Automata → Fair Labeled Kripke Structures

Let $A=<\Sigma,S,\Delta,I,\{f_1,\ldots,f_n\}>$ be a GBA.

We may defined a corresponding
fair labeled Kripke structure K for A, such that
L(K) = L(A).

ω-Automata → Fair Labeled Kripke Structures

Let $A=<\Sigma,S,\Delta,I,\{f_1,\ldots,f_n\}>$ be a GBA.

$AP= \Sigma \cup \{p_1,\ldots,p_n\}$

$S'= \{ (s,a,s') \mid (s,a,s') \in \Delta \}$

$R' = \{ ((s,a,s'),(s',b,s'')) \mid (s,a,s'),(s',b,s'') \in S' \}$

$I' = \{ (s,a,s') \mid (s,a,s') \in \Delta, s \in I \}$

Initially, $L((s,a,s')) = \{a\}$, add p_i to $L((s,a,s'))$, if $s \in f_i$

$K=<S',R',I',L,\{p_1,\ldots,p_n\}>$

Define $\zeta: 2^{AP} \rightarrow \Sigma$ such that $\zeta(X)=a$ where $\{a\}=X\cap\Sigma$

Then $\zeta(L(K)) = L(A)$

Comparison of LTS and Labeled KS

Let $K=<S,R,I,L>$ be a labeled kripke structure over AP .

Then there is an LTS $A=<\Sigma,S,\Delta,I>$ with $\Sigma=2^{AP}$

such that $L(A)=L(K)$.

Let $A=<\Sigma,S,\Delta,I>$ be an LTS.

Then there is a labeled kripke structure

$K=<S',R,I',L>$ over $AP = \Sigma$

such that $\zeta(L(K)) = L(A)$ where $\zeta(\{a\})=a$ for all $a\in\Sigma$

(IV) ω-Automata (BA,GBA,MA,SA,RA,PA)

ω-Automaton

Defines a language

Recognizes whether a word is in the language

Emptiness

Language inclusion → emptiness

Correctness → language inclusion

$M \models \varphi$ --- $[[M]] \subseteq [[\varphi]]$

Büchi Automaton (BA)

Definition

A BA is a quintuple $<\Sigma,S,\Delta,I,F>$

- Σ : A finite set of symbols
- S : A finite set of states
- $\Delta \subseteq S \times \Sigma \times S$: A transition relation
- $I \subseteq S$: A set of initial states
- $F \subseteq S$: A set of acceptance states

Büchi Automaton (BA)

Definition

A BA is a quintuple $<\Sigma,S,\Delta,I,F>$

– Σ : A finite set of symbols – S : A finite set of states – $\Delta \subseteq S \times \Sigma \times S$: A transition relation – $I \subseteq S$: A set of initial states	Labeled Transition System (LTS)

– $F \subseteq S$: A set of acceptance states

ω-Automaton

LTS +	$F \subseteq S$	Büchi-Condition	BA

Büchi-Condition:

An **accepting run** of A is a run π of A, such that

$inf(\pi) \cap F \neq \emptyset$.

ω-Automaton

LTS +	$F \subseteq S$	Büchi-Condition	BA
LTS +	$F \subseteq 2^S$	Generalized B.-Condition	GBA

Generalized B.-Condition:

An **accepting run** of A is a run π of A, such that

for each $f \in F$, $\inf(\pi) \cap f \neq \emptyset$.

ω-Automaton

LTS +	$F \subseteq S$	Büchi-Condition	BA
LTS +	$F \subseteq 2^S$	Generalized B.-Condition	GBA
LTS +	$F \subseteq 2^S$	Muller-Condition	MA

Muller-Condition:

An **accepting run** of A is a run π of A, such that there exists $f \in F$, $\inf(\pi)=f$.

ω-Automaton

LTS +	$F \subseteq S$	Büchi-Condition	BA
LTS +	$F \subseteq 2^S$	Generalized B.-Condition	GBA
LTS +	$F \subseteq 2^S$	Muller-Condition	MA
LTS +	$F \subseteq 2^S \times 2^S$	Streett-Condition	SA

Streett-Condition:

An **accepting run** of A is a run π of A, such that

for each $(f,g) \in F$, $inf(\pi) \cap f \neq \emptyset \rightarrow inf(\pi) \cap g \neq \emptyset$.

ω-Automaton

LTS +	$F \subseteq S$	Büchi-Condition	BA
LTS +	$F \subseteq 2^S$	Generalized B.-Condition	GBA
LTS +	$F \subseteq 2^S$	Muller-Condition	MA
LTS +	$F \subseteq 2^S \times 2^S$	Streett-Condition	SA
LTS +	$F \subseteq 2^S \times 2^S$	Rabin-Condition	RA

Rabin-Condition:

An **accepting run** of A is a run π of A, such that there exists $(f,g) \in F$, $inf(\pi) \cap f \neq \emptyset \wedge inf(\pi) \cap g = \emptyset$.

ω-Automaton

LTS +	$F \subseteq S$	Büchi-Condition	BA
LTS +	$F \subseteq 2^S$	Generalized B.-Condition	GBA
LTS +	$F \subseteq 2^S$	Muller-Condition	MA
LTS +	$F \subseteq 2^S \times 2^S$	Streett-Condition	SA
LTS +	$F \subseteq 2^S \times 2^S$	Rabin-Condition	RA
LTS +	$F: S \rightarrow N$	Parity-Condition	PA

Parity-Condition:

An **accepting run** of A is a run π of A, such that the minimum of $\{ F(s) \mid s \in inf(\pi) \}$ is even.

Expressivity

Büchi Automata $\subseteq$ (1)

Generalized Büchi Automata $\subseteq$ (2)

Streett Automata $\subseteq$ (3)

Muller Automata $\subseteq$ Büchi Automata (4)

Büchi $\subseteq$ Parity $\subseteq$ Rabin $\subseteq$ Muller

Expressivity (1)

Büchi Automata $\subseteq$ Generalized Büchi Automata

BA = $<\Sigma, S, \Delta, I, F>$

GBA = $<\Sigma, S, \Delta, I, \{F\}>$

Expressivity (2)

Generalized Büchi Automata $\subseteq$ Streett Automata

GBA = $<\Sigma,S,\Delta,I,F>$, F={f1,f2,...,fn}

SA = $<\Sigma,S,\Delta,I,F'>$,
F'={(S,f1),(S,f2),...,(S,fn)}

Expressivity (3)

Streett Automata $\subseteq$ Muller Automata

SA = $<\Sigma,S,\Delta,I,F>$, $F=\{(f1,g1),(f2,g2),\ldots,(fn,gn)\}$

MA = $<\Sigma,S,\Delta,I,F'>$, $F'=h1 \cap h2 \cap \ldots \cap hn$

$h1 = \{ Y \mid Y \cap f1 = \varnothing \} \cup \{ Y \mid Y \cap g1 \neq \varnothing \}$

Expressivity (4)

Muller Automata $\subseteq$ Büchi Automata

MA = $<\Sigma,S,\Delta,I,F>$, F={f1,f2,...,fn}

BA = $<\Sigma,S',\Delta',I',F'>$

Only need to consider:

MA = $<\Sigma,S,\Delta,I,F>$, F={f1}

Expressivity (4a)

MA

A= $<\Sigma,S,\Delta,I,F>$, F={f1,…,fm}

A1 = $<\Sigma,S,\Delta,I,\{f1\}>$, …, Am = $<\Sigma,S,\Delta,I,\{fm\}>$

L(A) = L(A1) $\cup$ … $\cup$ L(Am)

Let MA A = $<\Sigma,S,\Delta,I,F>$ where F={f}.

How to construct a BA B= $<\Sigma,S',\Delta',I',F'>$ such that L(B)=L(A)

Expressivity (4b)

MA = $<\Sigma,S,\Delta,I,F>$; F={f}; f={s1,…,sn}

$S' = S \cup S \times \{0,\ldots,n\}$

$\Delta' = \Delta \cup \{(s,a,(s',0)) \mid (s,a,s') \in \Delta\} \cup$

$\{((s,i),a,(s',i)) \mid (s,a,s') \in \Delta,\ s,s' \in f,\ i=0,\ldots,n-1\} \cup$

$\{((s,0),a,(s',1)) \mid (s,a,s') \in \Delta,\ s=s1,\ s' \in f\} \cup \ldots \cup$

$\{((s,n-1),a,(s',n)) \mid (s,a,s') \in \Delta,\ s=sn,\ s' \in f\} \cup$

$\{((s,n),a,(s',0)) \mid (s,a,s') \in \Delta,\ s' \in f\}$

BA = $<\Sigma,S',\Delta',I,Sx\{n\}>$

Expressivity (Directly From BA to MA)

Büchi Automata $\subseteq$ Muller Automata

BA B= $<\Sigma,S,\Delta,I,F>$ with a Büchi-condition F

MA A= $<\Sigma,S,\Delta,I,F'>$ with

Muller-condition: $F' = \{ f \mid f \cap F \neq \emptyset \}$

Expressivity

√ Büchi Automata $\subseteq$

√ Generalized Büchi Automata $\subseteq$

√ Streett Automata $\subseteq$

√ Muller Automata $\subseteq$ Büchi Automata

Büchi $\subseteq$ Parity $\subseteq$ Rabin $\subseteq$ Muller

Expressivity (1)

Büchi Automata $\subseteq$ Parity Automata

BA = $<\Sigma,S,\Delta,I,F>$

PA = $<\Sigma,S,\Delta,I,F'>$ with F'(s)=0 when s in F

F'(s)=1 otherwise.

The minimum of $\{ F'(s) \mid s \in inf(\pi) \}$ is even iff

$inf(\pi) \cap F \neq \emptyset$

Expressivity (2)

Parity Automata $\subseteq$ Rabin Automata

$PA = <\Sigma,S,\Delta,I,\{ f_0,f_1,\ldots,f_{2n}\} >$

$RA = <\Sigma,S,\Delta,I,F>$

$F= \{ (f_0,\emptyset),(f_2,f_1),(f_4,f_1\cup f_3),\ldots,(f_{2n}, f_1 \cup\ldots \cup f_{2n-1}) \}$

Expressivity (3)

Rabin Automata $\subseteq$ Muller Automata

RA = $<\Sigma,S,\Delta,I,F>$, F={(f1,g1),(f2,g2),…,(fn,gn)}

MA = $<\Sigma,S,\Delta,I,F'>$, F'=h1 $\cup$ h2 $\cup$ … $\cup$ hn

h1 = $\{ Y\subseteq S \mid Y\cap f1\neq\emptyset, Y\cap g1=\emptyset\}$

Expressivity

√ Büchi Automata $\subseteq$

√ Generalized Büchi Automata $\subseteq$

√ Streett Automata $\subseteq$

√ Muller Automata $\subseteq$ Büchi Automata

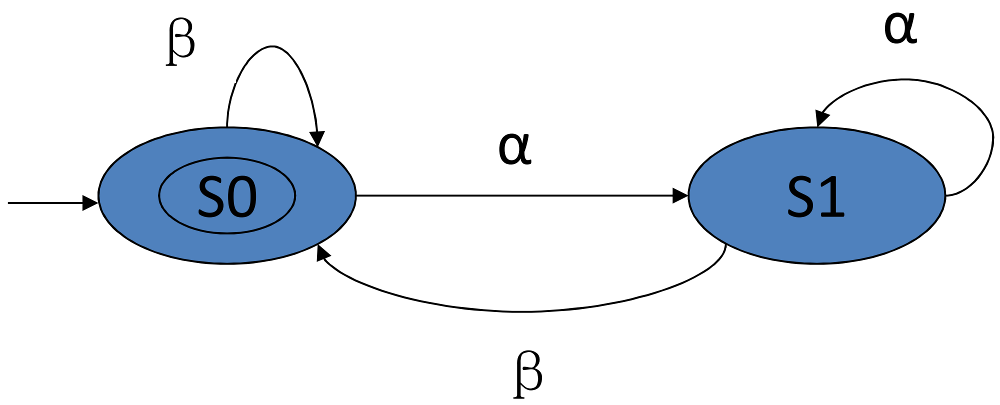
√ Büchi $\subseteq$ Parity $\subseteq$ Rabin $\subseteq$ Muller

Deterministic ω-Automaton

Deterministic LTS + Acceptance Condition

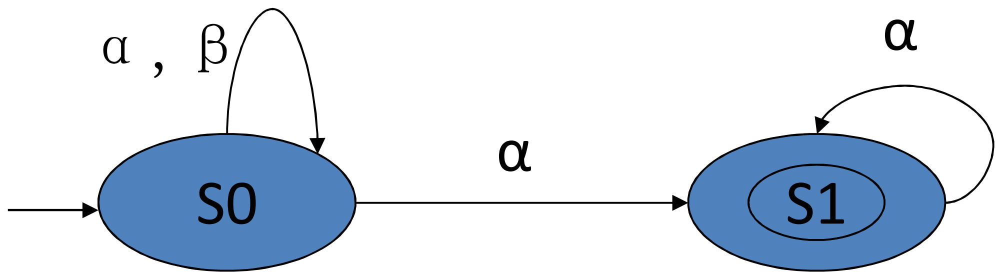
Deterministic ω-Automaton

D:



$$L(B) = (\alpha^*\beta)^\omega$$

ND:



$$L(A) = L(\neg B) = (\alpha+\beta)^*\alpha^\omega$$

Deterministic ω-Automaton

- Closed under complementation
  - Deterministic Muller
  - Deterministic Streett
  - Deterministic Rabin
  - Deterministic Parity
- Not closed under complementation
  - Deterministic Büchi
  - Deterministic Generalized Büchi

Deterministic ω-Automaton

A Büchi automaton is equivalent to, respectively,

a deterministic Muller automaton,

a deterministic Rabin automaton, and

a deterministic Streett automaton, and

a deterministic parity automaton.

(V) Summary

- 标号迁移系统(LTS)
- Büchi自动机(BA)
- 泛Büchi自动机(GBA)
- ω-自动机(BA,GBA,MA,SA,RA,PA)

表达能力问题

表达方式问题

练习：

1。

给定GBA A=<Σ,S,Δ,I,{f_1,…,f_n}>.
构造BA B=<Σ,S',Δ',I',F'> 使得 L(B)=L(A).

2。
给定GBA A_1, A_2. 定义GBA的交和并运算.
即
a) 定义$A_1 \cap A_2$ 使得$L(A_1 \cap A_2) = L(A_1) \cap L(A_2)$;
b) 定义$A_1 \cup A_2$ 使得$L(A_1 \cup A_2) = L(A_1) \cup L(A_2)$;