Learning One-Clock Timed Automata

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Model/Automaton learning

- Machine learning

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- Machine learning

a sample set

$M = \{(x, y) | x \in X, y \in Y\}$

learn →

Model

$f : X \rightarrow Y$

$f(x) = y, \forall x \in X$

predict or identify $f(x)$

for all $x \in X$

Model/Automaton learning

- Machine learning

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- Model/Automaton learning

Σ is an alphabet

$X = \Sigma^*$ set of words

$Y = \{+, -\}$ or other set of labels

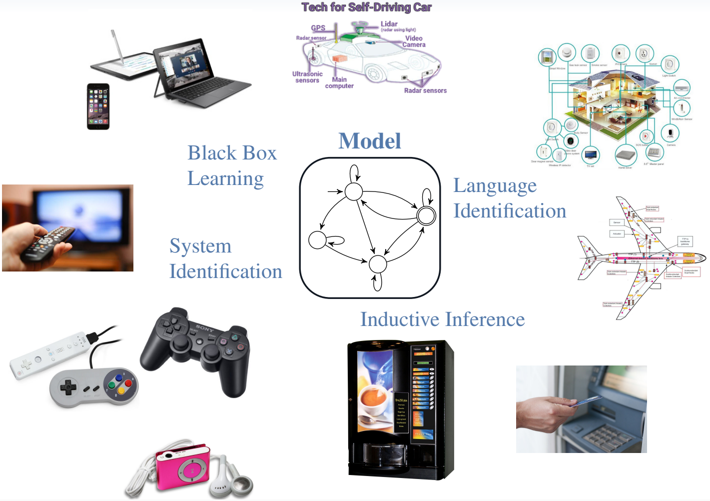
learn →

Model

f is a language

$L \subseteq \Sigma^*$

The model is a kind of Automaton


Tech for Self-Driving Car
GPS
Lidar
(radar using light)
Radar sensor
Video Camera
Ultrasonic sensors
Main computer
Radar sensors
Model
Black Box Learning
System Identification
Language Identification
Inductive Inference

L^* : Classic automaton learning of DFA

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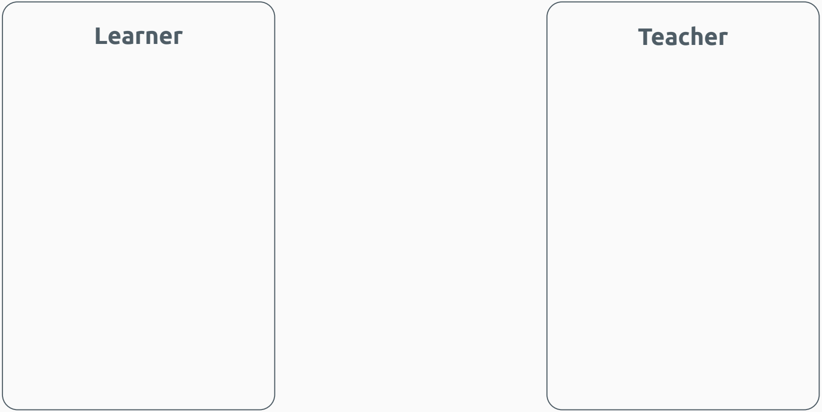

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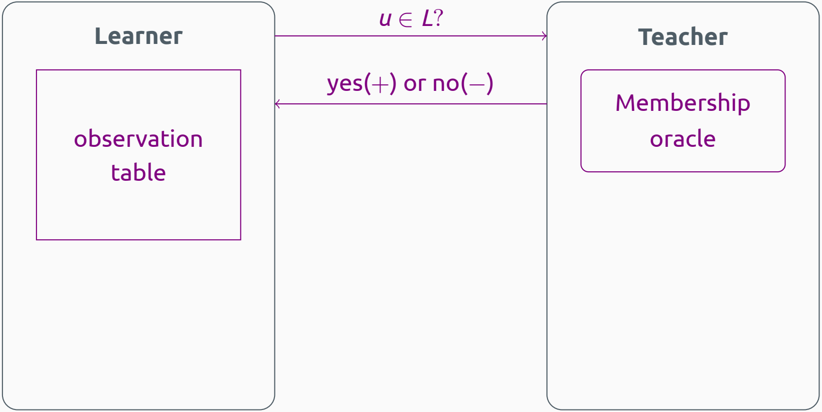

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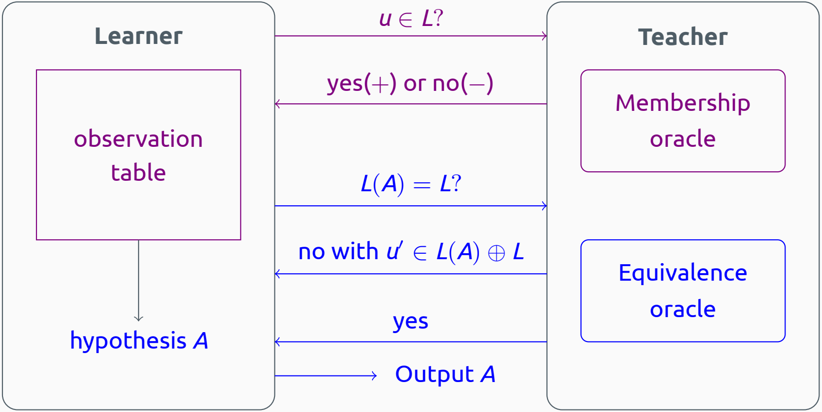

Motivation

- More recent work extends L^* algorithm to different models
  - Mealy machines [9], I/O automata [1], register automata [6], NFA [3], Büchi automata [7], symbolic automata [8, 4] and MDP [10], etc..

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- More recent work extends L^* algorithm to different models
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- Motivation
  - How to actively learn a timed model for a real-time system?
- Related work
  - Active learning of event-recording automata [5].
  - Passive identification of timed automata in the limit via fitting a labelled sample $S = (S_+, S_-)$ [12].
  - Passive learning of timed automata via Genetic Programming and testing [11].

Outline

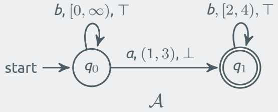
Basic idea

- Learning (regular) timed-automata with a single clock.
- Challenges
  - State now includes both location and clock value.
  - Determining the guard condition on transitions.
  - Determining reset information on transitions.
  - (related to the previous points) Matching time observed from outside to internal clock used on the guards.
- Solutions of learning deterministic one-clock timed automata (DOTA).
  - ☺ A normalization map from delay timed words (outside) to logical timed words (inside).
  - ☺ Utilize a partition function to map *logical-timed* values to finite intervals (similar to learning symbolic automata).
  - ☺ First consider the case of a smart teacher who can tell the learner reset informations. Then drop the assumption (i.e. reduction to a normal teacher) by guessing reset information.

Learning from a smart teacher

- The DOTA $\mathcal{A}$ recognizes the target language $\mathcal{L}$.
- $\Sigma = \{a, b\}$; $\mathcal{B} = \{\top, \bot\}$ where $\top$ is for reset, $\bot$ otherwise.

Example



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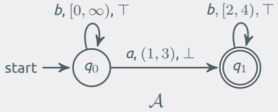

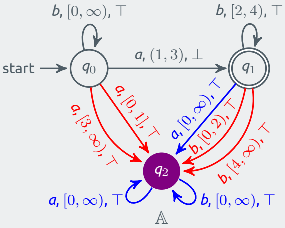

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- **Delay timed words** $(\Sigma \times \mathbb{R}_{\geq 0})^*$: outside observations;
  e.g. $\omega = (b, 0)(a, 1.1)(b, 1)$ is an accepting timed words.

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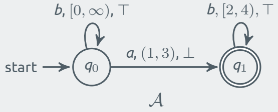

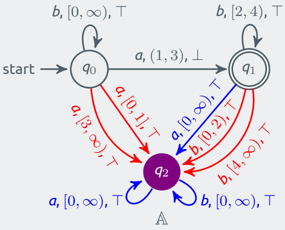

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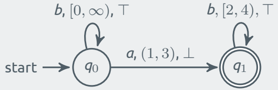
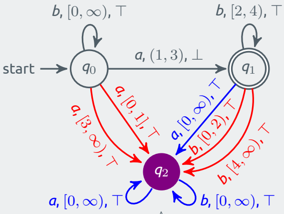
- **Delay timed words** $(\Sigma \times \mathbb{R}_{\geq 0})^*$: outside observations;
  e.g. $\omega = (b, 0)(a, 1.1)(b, 1)$ is an accepting timed words.
- **Reset-logical timed words** $(\Sigma \times \mathbb{R}_{\geq 0} \times \mathcal{B})^*$: inside logical actions;
  e.g. $\gamma_r = (b, 0, \top)(a, 1.1, \bot)(b, 2.1, \top)$ is the reset-logical counterpart of ω.
  Logical counterpart $\gamma = (b, 0)(a, 1.1)(b, 2.1)$.

Example



$\mathcal{A}$



$\mathbb{A}$

Learning from a smart teacher

- Given a DOTA $\mathbb{A}$, $L_r(\mathbb{A})$ represents the recognized reset-logical timed language of $\mathbb{A}$; $L(\mathbb{A})$ represents the logical timed language.

Theorem

Given two DOTAs $\mathbb{A}$ and $\mathcal{H}$, if $L_r(\mathbb{A}) = L_r(\mathcal{H})$, then $\mathcal{L}(\mathbb{A}) = \mathcal{L}(\mathcal{H})$.

Learning from a smart teacher

- Given a DOTA $\mathbb{A}$, $L_r(\mathbb{A})$ represents the recognized reset-logical timed language of $\mathbb{A}$; $L(\mathbb{A})$ represents the logical timed language.
- Guiding principle : learning the (delayed) timed language of a DOTA $\mathbb{A}$ can be reduced to learning the reset-logical timed language of $\mathbb{A}$.

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- Guiding principle : learning the (delayed) timed language of a DOTA $\mathbb{A}$ can be reduced to learning the reset-logical timed language of $\mathbb{A}$.
- Smart teacher setting : membership queries are logical timed words, teacher responds with reset information.

Theorem

Given two DOTAs $\mathbb{A}$ and $\mathcal{H}$, if $L_r(\mathbb{A}) = L_r(\mathcal{H})$, then $\mathcal{L}(\mathbb{A}) = \mathcal{L}(\mathcal{H})$.

Definition (Reset-logical-timed observation table)

A reset-logical-timed observation table for a DOTA $\mathbb{A}$ is a 7-tuple $\mathbf{T} = (\Sigma, \mathbf{\Sigma}, \mathbf{\Sigma_r}, \boldsymbol{S}, \boldsymbol{R}, \boldsymbol{E}, f)$ where Σ is the finite alphabet; $\mathbf{\Sigma} = \Sigma \times \mathbb{R}_{\geq 0}$ is the infinite set of logical-timed actions; $\mathbf{\Sigma_r} = \Sigma \times \mathbb{R}_{\geq 0} \times \mathcal{B}$ is the infinite set of reset-logical-timed actions; $\boldsymbol{S}, \boldsymbol{R} \subset \mathbf{\Sigma}_r^*$ and $\boldsymbol{E} \subset \mathbf{\Sigma}^*$ are finite sets of words, where $\boldsymbol{S}$ is called the set of prefixes, $\boldsymbol{R}$ the boundary, and $\boldsymbol{E}$ the set of suffixes. Specifically,

- $\boldsymbol{S}$ and $\boldsymbol{R}$ are disjoint, i.e., $\boldsymbol{S} \cup \boldsymbol{R} = \boldsymbol{S} \uplus \boldsymbol{R}$;
- The empty word is by default both a prefix and a suffix, i.e., $\epsilon \in \boldsymbol{E}$ and $\epsilon \in \boldsymbol{S}$;
- f: $(\boldsymbol{S} \cup \boldsymbol{R}) \cdot \boldsymbol{E} \mapsto \{-, +\}$ is a classification function such that for a reset-logical-timed word γ_r, $\gamma_r \cdot e \in (\boldsymbol{S} \cup \boldsymbol{R}) \cdot \boldsymbol{E}$, $f(\gamma_r \cdot e) = -$ if $\Pi_{\{1,2\}}\gamma_r \cdot e$ is invalid [1], otherwise if $\Pi_{\{1,2\}}\gamma_r \cdot e \notin L(\mathbb{A})$, $f(\gamma_r \cdot e) = -$, and $f(\gamma_r \cdot e) = +$ if $\Pi_{\{1,2\}}\gamma_r \cdot e \in L(\mathbb{A})$;

1. The projection of an n-tuple x onto its first two components is denoted by $\Pi_{\{1,2\}}\mathrm{x}$, which extends to a sequence of tuples as $\Pi_{\{1,2\}}(\mathrm{x}_1, \ldots, \mathrm{x}_k) = \left(\Pi_{\{1,2\}}\mathrm{x}_1, \ldots, \Pi_{\{1,2\}}\mathrm{x}_k\right)$.

Learning from a smart teacher

- Reduced
  - $\forall s, s' \in \boldsymbol{S} : s \neq s'$ implies $row(s) \neq row(s')$;
- Closed
  - $\forall r \in \boldsymbol{R}, \exists s \in \boldsymbol{S} : row(s) = row(r)$;
- Consistent
  - $\forall \gamma_r, \gamma_r' \in \boldsymbol{S} \cup \boldsymbol{R}$, $row(\gamma_r) = row(\gamma_r')$ implies $row(\gamma_r \cdot \boldsymbol{\sigma_r}) = row(\gamma_r' \cdot \boldsymbol{\sigma_r}')$, for all $\boldsymbol{\sigma_r}, \boldsymbol{\sigma_r}' \in \boldsymbol{\Sigma_r}$ satisfying $\gamma_r \cdot \boldsymbol{\sigma_r}, \gamma_r' \cdot \boldsymbol{\sigma_r}' \in \boldsymbol{S} \cup \boldsymbol{R}$ and $\Pi_{\{1,2\}}\boldsymbol{\sigma_r} = \Pi_{\{1,2\}}\boldsymbol{\sigma_r}'$;
- Evidence-closed
  - $\forall s \in \boldsymbol{S}$ and $\forall e \in \boldsymbol{E}$, the reset-logical-timed word $\pi(\Pi_{\{1,2\}} s \cdot e)$ belongs to $\boldsymbol{S} \cup \boldsymbol{R}$[2].

2. For the sake of simplicity, we define a function π that maps a logical-timed word to its unique reset-logical-timed counterpart in membership queries.

Learning from a smart teacher

$\mathbf{T}$	ϵ	$\cdots$
ϵ	$-$	
$(a, 1.1, \bot)$	$+$	
$(a, 0, \top)$	$-$	
$(b, 0, \top)$	$-$	
$(a, 1.1, \bot)(a, 0, \top)$	$-$	
$(a, 1.1, \bot)(b, 0, \top)$	$-$	

Learning from a smart teacher

$\mathbf{T}$	ϵ	$\cdots$
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$(a, 1.1, \bot)(a, 0, \top)$	$-$	
$(a, 1.1, \bot)(b, 0, \top)$	$-$	

The rows ϵ and $(a, 1.1, \bot)$ are boxed and labelled ***S***.

The prefixes set ***S*** indicates the locations

Learning from a smart teacher

S

$\mathbf{T}$	ϵ	$\cdots$
ϵ	$-$	
$(a, 1.1, \bot)$	$+$	
$(a, 0, \top)$	$-$	
$(b, 0, \top)$	$-$	
$(a, 1.1, \bot)(a, 0, \top)$	$-$	
$(a, 1.1, \bot)(b, 0, \top)$	$-$	

R

The prefixes set ***S*** indicates the locations

The boundary ***R*** indicates the transitions

Learning from a smart teacher

S ***E***

$\mathbf{T}$	$\epsilon \quad \cdots$
ϵ	$-$
$(a, 1.1, \bot)$	$+$
$(a, 0, \top)$	$-$
$(b, 0, \top)$	$-$
$(a, 1.1, \bot)(a, 0, \top)$	$-$
$(a, 1.1, \bot)(b, 0, \top)$	$-$

R

The prefixes set ***S*** indicates the locations

The boundary ***R*** indicates the transitions

The suffixes set ***E*** distinguishes the locations

Learning from a smart teacher

S

E

$\mathbf{T}$	ϵ $\cdots$
ϵ	$-$
$(a, 1.1, \bot)$	$+$
$(a, 0, \top)$	$-$
$(b, 0, \top)$	$-$
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$(a, 1.1, \bot)(b, 0, \top)$	$-$

R

Body

The prefixes set ***S*** indicates the locations

The boundary ***R*** indicates the transitions

The suffixes set ***E*** distinguishes the locations

Body records whether automaton accepts logical timed words

Learning from a smart teacher

$\mathbf{T}$	ϵ	$\cdots$
ϵ	$-$	
$(a, 1.1, \bot)$	$+$	
$(a, 0, \top)$	$-$	
$(b, 0, \top)$	$-$	
$(a, 1.1, \bot)(a, 0, \top)$	$-$	
$(a, 1.1, \bot)(b, 0, \top)$	$-$	

The prefixes set ***S*** indicates the locations

The boundary ***R*** indicates the transitions

The suffixes set ***E*** distinguishes the locations

Body records whether automaton accepts logical timed words

accepts $(a, 1.1) \cdot \epsilon$ and gives the reset information $\bot$

does not accept $(a, 0) \cdot \epsilon$ and gives the reset information $\top$

Learning from a smart teacher

- Given a target timed language $\mathcal{L}$ which is recognized by a DOTA $\mathbb{A}$, let $n = |Q|$ be the number of locations of $\mathbb{A}$, $m = |\Sigma|$ the size of the alphabet, and κ the maximal constant appearing in the clock constraints of $\mathbb{A}$.

Theorem

The learning process with a smart teacher terminates and returns a DOTA which recognizes the target timed language $\mathcal{L}$.

Theorem

The complexity of the algorithm is $\mathcal{O}(mn^5\kappa^4)$ for number of membership queries, and $\mathcal{O}(mn^2\kappa^3)$ for number of equivalence queries.

Learning from a normal teacher

- In the normal teacher setting, the teacher responds to delay timed words, and no longer returns reset information in answers to membership and equivalence queries.
- The learner guesses the resets in order to convert between delay and logical timed words.

Learning from a normal teacher

- In the normal teacher setting, the teacher responds to delay timed words, and no longer returns reset information in answers to membership and equivalence queries.
- The learner guesses the resets in order to convert between delay and logical timed words.
- Basic process
  - At every round, guess all needed resets and put all resulting table candidates into a set *ToExplore*.
  - Take out one table instance from the set *ToExplore*.
  - The operations on the table are same to those in the situation with a smart teacher.

Learning from a normal teacher

- Termination and complexity
  - At every iteration, the learner selects the table instance which requires the least number of guesses.
  - The learner keeps the correct table instance of each iteration in *ToExplore* since he guesses all reset informations.
  - If $\mathbf{T} = (\Sigma, \boldsymbol{\Sigma}, \boldsymbol{\Sigma_r}, \boldsymbol{S}, \boldsymbol{R}, \boldsymbol{E}, f)$ is the final observation table for the correct candidate in the situation with a smart teacher, the learner can find it after checking $\mathcal{O}(2^{(|\boldsymbol{S}|+|\boldsymbol{R}|)\times(1+\sum_{e_i \in \boldsymbol{E}\setminus\{\epsilon\}}(|e_i|-1))})$ table instances in the worst situation with a normal teacher.
  - The process also may terminate and return a DOTA which is different to the one in the smart teacher situation.

Theorem

The learning process with a normal teacher terminates and returns a DOTA which recognizes the target timed language $\mathcal{L}$. It has exponential complexity in the number of membership and equivalence queries.

Experiment 1

Table 1 – Experimental results on random examples for the smart teacher situation.

Case ID	$\lvert\Delta\rvert_{mean}$	#Membership			#Equivalence			n_{mean}	t_{mean}
		N_{min}	N_{mean}	N_{max}	N_{min}	N_{mean}	N_{max}		
4_4_20	16.3	118	245.0	650	20	30.1	42	4.5	24.7
7_2_10	16.9	568	920.8	1393	23	31.3	37	9.1	14.6
7_4_10	25.7	348	921.7	1296	34	50.9	64	9.3	38.0
7_6_10	26.0	351	634.5	1050	35	44.7	70	7.8	49.6
7_4_20	34.3	411	1183.4	1890	52	70.5	93	9.5	101.7
10_4_20	39.1	920	1580.9	2160	61	73.1	88	11.7	186.7
12_4_20	47.6	1090	2731.6	5733	66	97.4	125	16.0	521.8
14_4_20	58.4	1390	2238.6	4430	79	107.7	135	16.0	515.5

Case ID : n_m_κ, consisting of the number of locations, the size of the alphabet and the maximum constant appearing in the clock constraints, respectively, of the corresponding group of $\mathcal{A}$'s.

$\lvert\Delta\rvert_{mean}$: the average number of transitions in the corresponding group.

#Membership & #Equivalence : the number of conducted membership and equivalence queries, respectively. N_{min} : the minimal, N_{mean} : the mean, N_{max} : the maximum.

n_{mean} : the average number of locations of the learned automata in the corresponding group.

t_{mean} : the average wall-clock time in seconds, including that taken by the learner and by the teacher.

Experiment 2

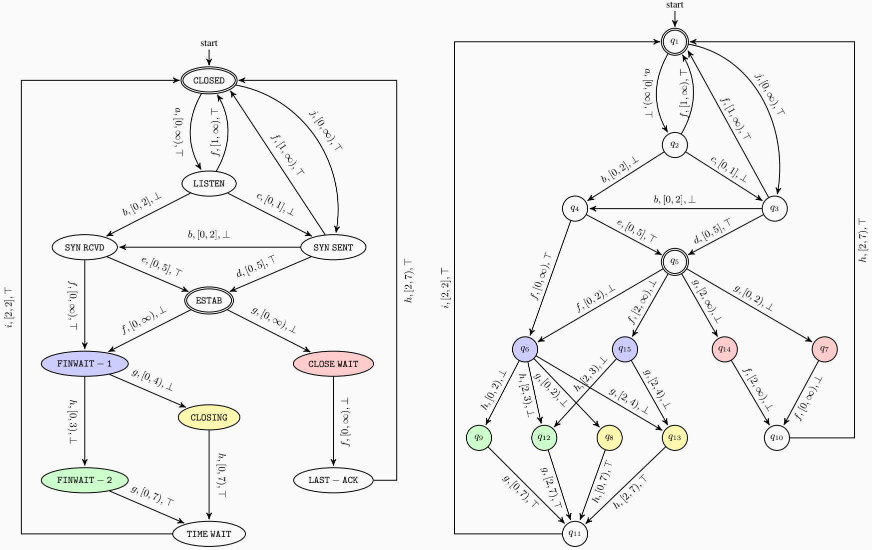

Figure 1 – Left : The functional specification of the TCP protocol with timing settings. Right : The learnt functional specification of the TCP protocol. Colors indicate the splitting of locations.

Table 2 – Experimental results on random examples for the normal teacher situation.

Case ID	$\|\Delta\|_{\text{mean}}$	#Membership			#Equivalence			n_{mean}	t_{mean}	$\#\mathbf{T}_{explored}$	#Learnt
		N_{min}	N_{mean}	N_{max}	N_{min}	N_{mean}	N_{max}				
3_2_10	4.8	43	83.7	167	5	8.8	14	3.0	0.9	149.1	10/10
4_2_10	6.8	67	134.0	345	6	13.3	24	4.0	7.4	563.0	10/10
5_2_10	8.8	75	223.9	375	9	15.2	24	5.0	35.5	2811.6	10/10
6_2_10	11.9	73	348.3	708	10	16.7	30	5.6	59.8	5077.6	7/10
4_4_20	16.3	231	371.0	564	27	30.9	40	4.0	137.5	8590.0	6/10

#Membership & #Equivalence : the number of conducted membership and equivalence queries with the cached methods, respectively. N_{min} : the minimal, N_{mean} : the mean, N_{max} : the maximum.

$\#\mathbf{T}_{explored}$: the average number of the explored table instances.

#Learnt : the number of the learnt DOTAs in the group (learnt/total).

Outline

Conclusion and future work

- Contributions
  - Give an active learning algorithm with a smart teacher for DOTAs. It is an efficient (polynomial) algorithm. (white-box or gray-box)
  - Give an active learning algorithm with a normal teacher for DOTAs. It has an exponential complexity increase. (black-box)

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  - Give an active learning algorithm with a normal teacher for DOTAs. It has an exponential complexity increase. (black-box)
- Future work
  - Extension to non-deterministic and multi-clock timed automata.
  - Improvements to efficiency of the algorithms.

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Thanks.