# Verification of Concurrent Programs

## Decidability, Complexity, Reductions.

Ahmed Bouajjani

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    Memory model (sequential consistency, causal delivery, etc.
- Infrastructures
  - Performances $\Rightarrow$ Store buffers, cashes, replicas, etc.

    Relaxed memory models, weak consistency criteria.

    (action reordering, lossyness, duplication, etc.)

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  - Complexity (huge number of action orders),
    Undecidability (some commutations allow to encode TM! – queues).

# Questions

- Limits of decidability?
- Complexity?
- Basic (conceptual/technical) tools?
- General and efficient algorithmic approaches?

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  - Also useful for reasoning about weak memory models: modeling of the effects of various kind of relaxations.

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- Code-to-code translations to:
  - Sequential programs: getting rid of concurrency
  - Concurrent programs over SC: getting rid of relaxed memory models/weak consistency models
- Separation of the issues:
  - As general as possible, regardless from the decidability issue
  - Independent from the used data types
  - Holds for unbounded control parameters: recursion depth, number of processes/created tasks, size of buffers, etc.
  - Precise reduction, under well defined conditions on the control features in programs/classes of computations
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  *Finite data domains, ...*
- When is this possible? How?

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- Sequentialization under Context-bounding [Lal, Reps, 08]
  - Each thread has a finite number of execution rounds
  - Bounded Input/Output interfaces: memory states at the starting/ending points of each round
  - Assume-Guarantee approach: Guess the Input states (nondeterministic assignments), produce the Output states, Check composability
  - Code-to-code translation to a sequential program

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  - Reduction to state reachability (coverability) in Petri nets
  - Based on a finite-state abstraction of the interface of each thread
    - ⋆ A thread generates a context-free set $S$ of sequences of thread creation events, but not all created threads must contribute to a computation.
    - ⋆ $\Rightarrow$ It is sound to close the set $S$ by the sub-word relation.
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- General sequentialization schema: tree traversal + bounded interfaces [B., Emmi, Parlato, 11] (Bounded tree-width behaviors)

# Many other works

- Asynchronous programs
  - Synchronous procedure calls + Asynchronous task creation
  - Tasks are run until completion

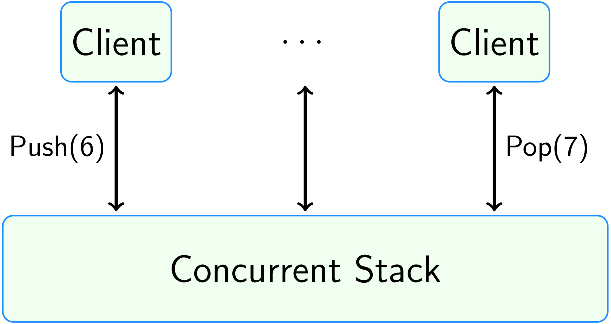
  [Sen, Viswanathan, 06], [Jhala, Majumdar, 07], ...

- Asynchronous programs + priorities & preemption
  [Atig, B., Touili, 08], [Emmi, Qadeer, Lal, 12]

- Recursively parallel programs [B., Emmi, 12]
  Cilk, X10, ...

- ...

# Libraries of Concurrent Objects




Specification given by a regular language
Example of a valid sequence: `Push(6)Push(7)Pop(7)`

# Linearizability [Herlihy, Wing, 90]

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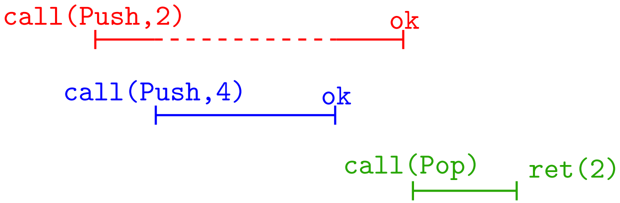

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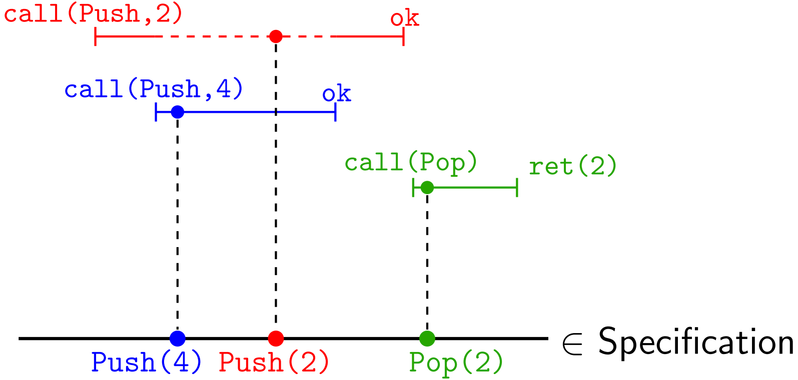

# Checking Linearizability

- Fixed number of finite-state threads [Alur, McMillan, Peled, 96]
  - Reduction to a problem of the form:

    $$\mathit{MostGeneralClient}[\mathit{Impl}] \subseteq \mathit{Closure}(\mathit{Spec})$$

  - $\Rightarrow$ Non-Linearizability $\rightsquigarrow$ a state reachability problem:

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  - Static Linearizability:

    *Linearization points are fixed in the code, except for read-only methods.*

    - ⋆ Most of implementations of concurrent objects satisfy this condition.

      *Linearization point = commit point*

    - ⋆ Reduction (of non-Static Linearizability) to control state reachability.
    - ⋆ P/EXPSPACE-complete for fixed/unbounded number of threads.

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- Decidability (finite data domain): Reduction to reachability in Petri nets (using Parikh image computations for Specification closure)

# Weak Memory Models: State Space Reachability

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- Other Works: abstraction/symbolic analysis/bounded model checking:
  - [Kuperstein, Vechev, Yahav, 11]
  - [Linden, Wolper, 10-11]
  - [Abdulla, Atig, Chen, Leonardson, Rezine, 12]
  - [Alglave, Kroening, Nimal, Tautchnig, 13]

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- Other Work
  - Testing: [Burckhardt, Musuvathi, CAV'08], [Burnim, Stergiou, Sen, 11]
  - Upper-approximate analysis: [Alglave, Maranget, 11]
  - Stronger criterion: Triangular data races [Owens, 10]

# Conclusion / questions

- A lot remains to be understood concerning decidability frontiers, complexity, and reducibility to problems such as state reachability in basic models.
- In particular: correctness over weak memory models, correctness criteria in the distributed case (papers in POPL'14), etc.
- Generic reductions for general classes of programs and general families of correctness criteria ?
- Sequentialization (What is pushdown representable?) is related to the notion of "bounded tree-width" [La Torre, Parlato, Madhusudan, 11].
- We need a general framework for reasoning about order constraints and their violations: What is Petri net representable (Petrifiable)?