

# Testing and Experimental Evaluation of QBF Based Bounded Semantics Model Checking with VERDS Version 1.30

## 1 Introduction

This document contains an experimental evaluation of the QBF-based bounded semantics model checking implemented in *verds* version 1.30<sup>1</sup>. The evaluation is relative to the BDD-based model checking implemented in NuSMV version 2.5.0<sup>2</sup>, based on two types of random boolean programs (with variable number of global and local variables)<sup>3</sup>. The experimental evaluation is done on a Linux platform with 128 GB memory. For convenience, the QBF-based bounded semantics model checking implemented in *verds* is referred to as *verds*-QBF, and the invocation of NuSMV with the option `-dcx` for not generating counterexamples is referred to as NuSMV.

*Brief Summary* Based on the test cases, the experimental evaluation shows that the bounded semantics model checking does not have advantage in verifying any of the properties that start with *AG*. On the other hand, the bounded semantics model checking has advantages in various degrees, ranging from a few percentage to a large percentage of test cases in different sets of test cases, with respect to the other verification and falsification problems (including falsification of *AG* properties). In summary, the bounded semantics model checking has advantage in more than 50 percent of the test cases, which are well distributed among verification and falsification of universal properties. In this sense, bounded semantics model checking and boolean diagram model checking may be considered complementary with their own advantages.

## 2 Types of Random Programs and Types of Properties

Two sets of random programs and 24 properties are formulated for the experimental evaluation.

*Programs with Concurrent Processes* The parameters of the first set of random boolean programs are as follows:

- a: number of processes
- b: number of all variables
- c: number of share variables
- d: number of local variables in a process

<sup>1</sup> <http://lcs.ios.ac.cn/~zwh/verds/>

<sup>2</sup> <http://nusmv.irst.itc.it/>

<sup>3</sup> <http://lcs.ios.ac.cn/~zwh/verds/>

The shared variables are initially set to a random value in  $\{0, 1\}$ , and the local variables are initially set to 0. For each process, the shared variables and the local variables are assigned the negation of a variable randomly chosen from these variables.

*Examples* An example of such programs in the input format of *NuSMV* is shown in Figure 1 of Appendix A. The corresponding program in the input format of *verds* (called *verds* verification model, or VVM for short) is shown in Figure 2 of Appendix A.

*Programs with Concurrent Sequential Processes* The parameters of the second set of random boolean programs are as follows, in addition to  $a, b, c, d$  specified above.

t: number of transitions in a process  
p: number of parallel assignments in each transition

Besides the  $b$  boolean variables, for each process, there is a local variable representing program locations, with  $e$  possible values. The shared variables are initially set to a random value in  $\{0, 1\}$ , and the local variables are initially set to 0. For each transition of a process,  $p$  pairs of shared variables and local variables are randomly chosen among the shared variables and the local variables, such that the first element of such a pair is assigned the negation of the second element of the pair. Transitions are numbered from 0 to  $t - 1$ , and are executed consecutively, and when the end of the sequence of the transitions is reached, it loops back to the execution of the transition numbered 0.

*Examples* An example of such programs in the input format of *NuSMV* is shown in Figure 3 of Appendix A. The corresponding program in VVM is shown in Figure 4 of Appendix A.

*Types of Properties* 24 properties are formulated for testing. The properties involve AG, AF properties and properties specified with different combinations of the operators with one or two levels of nesting (with two levels of nesting when *AX* or *EX* is involved). Properties  $p_{01}$  to  $p_{12}$  are shown as follows, in which  $v_i$  are global variables. Properties  $p_{13}$  to  $p_{24}$  are similar to  $p_{01}$  to  $p_{12}$  with the difference of  $\wedge$  and  $\vee$  replaced by respectively  $\vee$  and  $\wedge$ .

$p_{01}$ :	$AG(\bigvee_{i=1}^c v_i)$
$p_{02}$ :	$AF(\bigvee_{i=1}^c v_i)$
$p_{03}$ :	$AG(v_1 \rightarrow AF(v_2 \wedge \bigvee_{i=3}^c v_i))$
$p_{04}$ :	$AG(v_1 \rightarrow EF(v_2 \wedge \bigvee_{i=3}^c v_i))$
$p_{05}$ :	$EG(v_1 \rightarrow AF(v_2 \wedge \bigvee_{i=3}^c v_i))$
$p_{06}$ :	$EG(v_1 \rightarrow EF(v_2 \wedge \bigvee_{i=3}^c v_i))$
$p_{07}$ :	$A(v_1 U A(v_2 U \bigvee_{i=3}^c v_i))$
$p_{08}$ :	$A(v_1 U E(v_2 U \bigvee_{i=3}^c v_i))$
$p_{09}$ :	$A(v_1 U A(v_2 R \bigvee_{i=3}^c v_i))$
$p_{10}$ :	$A(v_1 U E(v_2 R \bigvee_{i=3}^c v_i))$
$p_{11}$ :	$A(AX v_1 R AX A(v_2 U \bigvee_{i=3}^c v_i))$
$p_{12}$ :	$A(EX v_1 R EX E(v_2 U \bigvee_{i=3}^c v_i))$

### 3 Experimental Data

For the programs with concurrent processes, we let  $b$  vary over the set of values  $\{12, 24, 36\}$ ,  $a = 3$ ,  $c = b/2$ , and  $d = c/a$ . For the programs with concurrent sequential processes, we let  $b$  vary over the set of values  $\{12, 16, 20\}$ ,  $a = 2$ ,  $c = b/2$ ,  $t = c$ ,  $p = 4$ , and  $d = c/a$ . Each of the 20 properties is tested on 20 test cases for each value of  $b$ .

For brevity, for each type of programs and each type of properties, a summary of the experimental data is presented. Additional details are presented in Appendix B.

The data are shown as follows, where  $N$  is the number of test cases,  $T$  is the number of test cases in which the property is true,  $F$  is the number of test cases in which the property is false,  $adv$  is the number of cases in which *verds*-QBF has an advantage. The table on the top is the experimental data for the first type of programs, and the table on the bottom is that for the second type of programs. The experimental data are obtained using the command “*verds*-QBF filename” for bounded semantics model checking with *verds*-QBF and using the command “NuSMV -dcx filename” for model checking with *NuSMV*.

property	adv/T	adv/F	adv/N
$p_{01}$	-	60/60	60/60
$p_{02}$	60/60	-	60/60
$p_{03}$	0/3	43/57	43/60
$p_{04}$	0/60	-	0/60
$p_{05}$	46/53	1/7	47/60
$p_{06}$	53/60	-	53/60
$p_{07}$	60/60	-	60/60
$p_{08}$	60/60	-	60/60
$p_{09}$	50/52	5/8	55/60
$p_{10}$	60/60	-	60/60
$p_{11}$	13/13	45/47	58/60
$p_{12}$	60/60	-	60/60
$p_{13}$	-	60/60	60/60
$p_{14}$	3/3	55/57	58/60
$p_{15}$	0/8	38/52	38/60
$p_{16}$	0/60	-	0/60
$p_{17}$	46/56	1/4	47/60
$p_{18}$	53/60	-	53/60
$p_{19}$	5/5	54/55	59/60
$p_{20}$	16/21	30/39	46/60
$p_{21}$	3/3	56/57	59/60
$p_{22}$	3/3	56/57	59/60
$p_{23}$	-	60/60	60/60
$p_{24}$	24/31	11/29	35/60
sum			1190/1440

  

$p_{01}$	0/53	2/7	2/60
$p_{02}$	60/60	-	60/60
$p_{03}$	0/10	1/50	1/60
$p_{04}$	0/60	-	0/60
$p_{05}$	0/46	0/14	0/60
$p_{06}$	0/60	-	0/60
$p_{07}$	60/60	-	60/60
$p_{08}$	60/60	-	60/60
$p_{09}$	36/54	0/6	36/60
$p_{10}$	53/60	-	53/60
$p_{11}$	33/47	0/13	33/60
$p_{12}$	52/60	-	52/60
$p_{13}$	-	60/60	60/60
$p_{14}$	4/4	4/56	8/60
$p_{15}$	0/10	1/50	1/60
$p_{16}$	0/60	-	0/60
$p_{17}$	0/48	0/12	0/59
$p_{18}$	0/60	-	0/60
$p_{19}$	8/8	52/52	60/60
$p_{20}$	13/18	27/42	40/60
$p_{21}$	4/4	54/56	58/60
$p_{22}$	4/4	55/56	59/60
$p_{23}$	-	59/60	59/60
$p_{24}$	8/16	29/44	37/60
sum			739/1440

## 4 Summary

Two types of programs are tested. Based on the test cases, the experimental evaluation shows that the bounded semantics model checking does not have advantage in verifying any of the properties that start with *AG*. On the other hand, the bounded verification has advantage in various degrees with respect to the other verification and falsification problems (including falsification of *AG* properties).

*Note* The efficiency also depends very much on the QBF-solving techniques. External QBF-solvers may be used to increase the efficiency of the verification. For ACTL formulas, special considerations are possible [2], and the use of SAT-solving techniques may be more efficient for this kind of problems.

## References

1. A. Cimatti, E. M. Clarke, F. Giunchiglia, M. Roveri. NUSMV: A New Symbolic Model Verifier. CAV 1999: 495-499.
2. W. Zhang. Bounded Semantics of CTL and SAT-based Verification. Lecture Notes in Computer Science 5885 (ICFEM 2009):286-305. Springer-Verlag. 2009.

## A Examples of SMV and VVM Programs

Examples of SMV programs and VVM programs are presented. The contents are as follows:

*Random concurrent processes* A random SMV program with 6 global variables, 3 processes and 2 local variables for each of the processes, and the corresponding VVM program;

*Random concurrent sequential processes* A random SMV program with 6 global variables, 2 processes and 4 local variables (of which 3 are boolean variables) for each of the processes, and the corresponding VVM program.

```
MODULE main
VAR  v1:boolean; v2:boolean; v3:boolean;
    v4:boolean; v5:boolean; v6:boolean;
    v7:boolean; v8:boolean;
    p1:process p1t(v1,v2,v3,v4,v5,v6);
    p2:process p2t(v1,v2,v3,v4,v5,v6);
ASSIGN init(v1):=1; init(v2):=0; init(v3):=1;
    init(v4):=1; init(v5):=1; init(v6):=1;
    init(v7):=0; init(v8):=0;
    next(v1):=!v5; next(v2):=!v6; next(v3):=!v4;
    next(v4):=!v4; next(v5):=!v7; next(v6):=!v3;
    next(v7):=!v1; next(v8):=!v1;
SPEC  AG((v1=1|v2=1|v3=1|v4=1|v5=1|v6=1))

MODULE p1t(v1,v2,v3,v4,v5,v6)
VAR  v9:boolean; v10:boolean;
ASSIGN init(v9):=0; init(v10):=0;
    next(v1):=!v6; next(v2):=!v3; next(v3):=!v4;
    next(v4):=!v10; next(v5):=!v3; next(v6):=!v3;
    next(v9):=!v10; next(v10):=!v1;

MODULE p2t(v1,v2,v3,v4,v5,v6)
VAR  v11:boolean; v12:boolean;
ASSIGN init(v11):=0; init(v12):=0;
    next(v1):=!v3; next(v2):=!v6; next(v3):=!v12;
    next(v4):=!v2; next(v5):=!v5; next(v6):=!v3;
    next(v11):=!v11; next(v12):=!v5;
```

Fig. 1. A Program with Concurrent Processes

```

VVM
VAR   v1:(0..1); v2:(0..1); v3:(0..1); v4:(0..1); v5:(0..1); v6:(0..1);
INIT  v1=1; v2=0; v3=1; v4=1; v5=1; v6=1;
PROC  p0:p0t(v1,v2,v3,v4,v5,v6);
      p1:p1t(v1,v2,v3,v4,v5,v6); p2:p2t(v1,v2,v3,v4,v5,v6);
SPEC  AG((v1=1|v2=1|v3=1|v4=1|v5=1|v6=1))

MODULE p0t(v1,v2,v3,v4,v5,v6)
VAR   v7:(0..1); v8:(0..1);
INIT  v7=0; v8=0;
TRANS TRUE: (v1,v2,v3,v4,v5,v6,v7,v8) :=
           (1-v5,1-v6,1-v4,1-v4,1-v7,1-v3,1-v1,1-v1);

MODULE p1t(v1,v2,v3,v4,v5,v6)
VAR   v9:(0..1); v10:(0..1);
INIT  v9=0; v10=0;
TRANS TRUE: (v1,v2,v3,v4,v5,v6,v9,v10) :=
           (1-v6,1-v3,1-v4,1-v10,1-v3,1-v3,1-v10,1-v1);

MODULE p2t(v1,v2,v3,v4,v5,v6)
VAR   v11:(0..1); v12:(0..1);
INIT  v11=0; v12=0;
TRANS TRUE: (v1,v2,v3,v4,v5,v6,v11,v12) :=
           (1-v3,1-v6,1-v12,1-v2,1-v5,1-v3,1-v11,1-v5);

```

**Fig. 2.** A Program with Concurrent Processes in VVM

```

MODULE main
VAR    v1:boolean; v2:boolean; v3:boolean;
      v4:boolean; v5:boolean; v6:boolean;
      cp:{c0,c1,c2,c3,c4,c5}; v7:boolean; v8:boolean; v9:boolean;
      p1:process plt(v1,v2,v3,v4,v5,v6);
ASSIGN init(v1):=1; init(v2):=0; init(v3):=1;
      init(v4):=1; init(v5):=1; init(v6):=1;
      init(cp)=c0; init(v7)=0; init(v8)=0; init(v9)=0;
      next(cp):=case cp=c0:c1; cp=c1:c2;
                  cp=c2:c3; cp=c3:c4; cp=c4:c5; cp=c5:c0; esac;
      next(v1):=case cp=c0:!v9; cp=c3:!v6; cp=c5:!v8; 1:v1; esac;
      next(v2):=case cp=c0:!v4; cp=c3:!v9; 1:v2; esac;
      next(v3):=case cp=c2:!v6; cp=c3:!v8; cp=c4:!v1; 1:v3; esac;
      next(v4):=case cp=c1:!v8; cp=c2:!v2; cp=c4:!v5; 1:v4; esac;
      next(v5):=case cp=c0:!v8; cp=c1:!v2;
                  cp=c2:!v8; cp=c3:!v7; 1:v5; esac;
      next(v6):=case cp=c1:!v1; cp=c5:!v4; 1:v6; esac;
      next(v7):=case cp=c0:!v6; cp=c2:!v1;
                  cp=c4:!v6; cp=c5:!v3; 1:v7; esac;
      next(v8):=case cp=c4:!v9; 1:v8; esac;
      next(v9):=case cp=c1:!v7; cp=c5:!v2; 1:v9; esac;
SPEC  AG((v1=1|v2=1|v3=1|v4=1|v5=1|v6=1))

MODULE plt(v1,v2,v3,v4,v5,v6)
VAR    cp:{c0,c1,c2,c3,c4,c5}; v10:boolean; v11:boolean; v12:boolean;
ASSIGN init(cp)=c0; init(v10)=0; init(v11)=0; init(v12)=0;
      next(cp):=case cp=c0:c1; cp=c1:c2;
                  cp=c2:c3; cp=c3:c4; cp=c4:c5; cp=c5:c0; esac;
      next(v1):=case cp=c0:!v4; cp=c2:!v11; cp=c5:!v11; 1:v1; esac;
      next(v2):=case cp=c4:!v3; 1:v2; esac;
      next(v3):=case cp=c0:!v2; cp=c1:!v11;
                  cp=c2:!v4; cp=c5:!v2; 1:v3; esac;
      next(v4):=case cp=c3:!v3; 1:v4; esac;
      next(v5):=case cp=c1:!v1; cp=c2:!v10; 1:v5; esac;
      next(v6):=case cp=c2:!v2; cp=c4:!v1; cp=c5:!v4; 1:v6; esac;
      next(v10):=case cp=c0:!v11; cp=c1:!v6;
                  cp=c3:!v6; cp=c4:!v5; 1:v10; esac;
      next(v11):=case cp=c3:!v5; cp=c4:!v12; 1:v11; esac;
      next(v12):=case cp=c0:!v6; cp=c1:!v2;
                  cp=c3:!v1; cp=c5:!v5; 1:v12; esac;

```

**Fig. 3.** A Program with Concurrent Sequential Processes



```

VVM
VAR   v1:(0..1); v2:(0..1); v3:(0..1); v4:(0..1); v5:(0..1); v6:(0..1);
INIT  v1=1; v2=0; v3=1; v4=1; v5=1; v6=1;
PROC  p0:p0t(v1,v2,v3,v4,v5,v6); p1:p1t(v1,v2,v3,v4,v5,v6);
SPEC  AG((v1=1|v2=1|v3=1|v4=1|v5=1|v6=1))

MODULE p0t(v1,v2,v3,v4,v5,v6)
VAR   cp:{c0,c1,c2,c3,c4,c5}; v7:(0..1); v8:(0..1); v9:(0..1);
INIT  cp=c0; v7=0; v8=0; v9=0;
TRANS cp=c0: (v2,v7,v5,v1,cp):=(1-v4,1-v6,1-v8,1-v9,c1);
        cp=c1: (v9,v5,v6,v4,cp):=(1-v7,1-v2,1-v1,1-v8,c2);
        cp=c2: (v5,v7,v3,v4,cp):=(1-v8,1-v1,1-v6,1-v2,c3);
        cp=c3: (v2,v1,v5,v3,cp):=(1-v9,1-v6,1-v7,1-v8,c4);
        cp=c4: (v4,v3,v7,v8,cp):=(1-v5,1-v1,1-v6,1-v9,c5);
        cp=c5: (v6,v7,v9,v1,cp):=(1-v4,1-v3,1-v2,1-v8,c0);

MODULE p1t(v1,v2,v3,v4,v5,v6)
VAR   cp:{c0,c1,c2,c3,c4,c5}; v10:(0..1); v11:(0..1); v12:(0..1);
INIT  cp=c0; v10=0; v11=0; v12=0;
TRANS cp=c0: (v1,v3,v12,v10,cp):=(1-v4,1-v2,1-v6,1-v11,c1);
        cp=c1: (v3,v12,v10,v5,cp):=(1-v11,1-v2,1-v6,1-v1,c2);
        cp=c2: (v5,v3,v1,v6,cp):=(1-v10,1-v4,1-v11,1-v2,c3);
        cp=c3: (v4,v12,v11,v10,cp):=(1-v3,1-v1,1-v5,1-v6,c4);
        cp=c4: (v2,v10,v11,v6,cp):=(1-v3,1-v5,1-v12,1-v1,c5);
        cp=c5: (v12,v1,v3,v6,cp):=(1-v5,1-v11,1-v2,1-v4,c0);

```

**Fig. 4.** A Program with Concurrent Sequential Processes in VVM

## B Details of Experimental Evaluation

This section contains experimental evaluation based on 2 types of random boolean programs: programs with concurrent processes and programs with concurrent sequential processes. The maximum time for *verds*-QBF is set to 300 seconds. The sign – in the tables either means timeout or the data are not available.

### B.1 Programs with Concurrent Processes

For the programs with concurrent processes, we let  $b$  vary over the set of values  $\{12, 24, 36\}$ ,  $a = 3$ ,  $c = b/2$ , and by locating each process with the same number of local variables, we obtain  $d = (b-c)/a = c/a$ . Each property is tested initially on 20 test cases for each  $b \in \{12, 24, 36\}$ . The details are shown as follows, where the left part of the table is the data obtained for model checking with *NuSMV*, using the command “NuSMV -dcx filename”. The right part of the table is the data obtained for model checking with *verds*-QBF, using the command “verds -QBF filename”. Let  $\tau(b, i)$  be the time for verification of the  $i$ -th ( $i \in \{1, \dots, 20\}$ ) test case with  $b$  variables, and let *lt.aver* and *rt.aver* denote the *aver* value in respectively the left part and the right part of the table. The explanation of the meaning of different columns of the data is as follows.

$$\begin{aligned} \min(b) &= \min_{1 \leq i \leq 20} \tau(i, b) \\ \max(b) &= \max_{1 \leq i \leq 20} \tau(i, b) \\ \text{aver}(b) &= \sum_{1 \leq i \leq 20} \tau(i, b) / 20 \\ \text{ratio}(b) &= \text{lt.aver}(b) / \text{rt.aver}(b) \end{aligned}$$

The *ratio* is marked with 0.00 when the *max* of the right part of the table is not available (the time exceeds 300 seconds). NuSMV has advantage in cases where *ratio* < 1.00 (possibly except the case of *p09*).

$p_{01}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	3.91
24	0.4	18.4	4.3	0.0	0.0	0.0	332.79
36	27.8	986.3	295.9	0.0	0.2	0.1	5690.86

$p_{02}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	7.87
24	0.4	15.7	3.8	0.0	0.0	0.0	1158.66
36	26.1	945.7	282.3	0.0	0.0	0.0	62037.63

$p_{03}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	-	-	0.00
24	0.4	19.9	4.5	0.0	-	-	0.00
36	28.5	1226.8	335.6	0.2	-	-	0.00

$p_{04}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	-	-	-	0.00
24	0.5	15.7	3.8	-	-	-	0.00
36	26.3	924.6	280.9	-	-	-	0.00

$p_{05}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	-	-	0.00
24	0.4	18.9	4.2	0.0	-	-	0.00
36	26.7	1231.3	346.1	0.1	-	-	0.00

$p_{06}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	1.09
24	0.4	16.4	4.0	0.0	0.5	0.2	25.96
36	29.3	970.8	300.5	0.1	8.5	1.7	177.54

$p_{07}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	8.39
24	0.4	16.0	3.9	0.0	0.0	0.0	1191.78
36	25.9	982.9	297.3	0.0	0.0	0.0	57720.62

$p_{08}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	7.67
24	0.4	16.1	3.9	0.0	0.0	0.0	1416.16
36	27.4	956.2	300.2	0.0	0.0	0.0	61900.18

$p_{09}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	3.06
24	0.4	19.4	4.3	0.0	5.2	0.4	10.82
36	26.3	1023.8	296.4	0.0	-	-	0.00

$p_{10}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	6.44
24	0.4	17.9	4.1	0.0	0.0	0.0	1020.89
36	29.6	1129.0	332.9	0.0	0.0	0.0	54570.15

$p_{11}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	2.20
24	0.5	20.4	4.6	0.0	0.3	0.1	62.90
36	29.4	1038.1	326.0	0.0	3.8	0.6	546.90

$p_{12}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	5.74
24	0.4	16.2	4.0	0.0	0.0	0.0	952.73
36	30.2	955.2	312.7	0.0	0.0	0.0	53910.67

$p_{13}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	10.62
24	0.4	15.4	4.0	0.0	0.0	0.0	1213.17
36	26.9	923.4	285.1	0.0	0.0	0.0	58776.45

$p_{14}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	2.10
24	0.4	16.8	4.0	0.0	0.2	0.0	81.22
36	27.2	1046.8	311.0	0.0	8.3	1.1	294.33

$p_{15}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	-	-	0.00
24	0.4	20.0	4.5	0.0	-	-	0.00
36	29.9	1303.9	340.1	0.3	-	-	0.00

$p_{16}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	-	-	-	0.00
24	0.4	15.9	4.0	-	-	-	0.00
36	26.9	932.9	282.9	-	-	-	0.00

$p_{17}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.3	0.0	0.47
24	0.5	19.9	4.3	0.0	-	-	0.00
36	29.1	1305.9	348.2	0.1	-	-	0.00

$p_{18}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	1.07
24	0.4	16.6	4.0	0.0	0.5	0.2	25.23
36	28.3	975.7	304.6	0.1	8.6	1.8	168.33

$p_{19}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	3.54
24	0.4	18.3	4.2	0.0	0.0	0.0	722.72
36	31.6	1077.6	352.7	0.0	0.0	0.0	35099.05

$p_{20}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	3.33
24	0.5	18.4	4.4	0.0	-	-	0.00
36	28.5	1014.0	321.7	0.0	-	-	0.00

$p_{21}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	4.41
24	0.5	17.0	4.2	0.0	0.0	0.0	866.29
36	30.2	1007.1	328.9	0.0	0.0	0.0	40350.71

$p_{22}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	4.66
24	0.5	16.2	4.0	0.0	0.0	0.0	915.27
36	28.2	1115.0	343.5	0.0	0.0	0.0	42673.31

$p_{23}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.0	0.0	6.15
24	0.5	17.5	4.2	0.0	0.0	0.0	1034.65
36	31.5	1239.3	363.4	0.0	0.0	0.0	43526.82

$p_{24}$

b	min	max	aver	min	max	aver	ratio
12	0.0	0.0	0.0	0.0	0.1	0.0	1.74
24	0.4	17.4	4.3	0.0	-	-	0.00
36	27.6	964.1	299.9	0.4	-	-	0.00

A summary for  $b = 36$  is as follows.

m/p	min	max	aver	min	max	aver	ratio
$p_{01}$	27.8	986.3	295.9	0.0	0.2	0.1	5690.86
$p_{02}$	26.1	945.7	282.3	0.0	0.0	0.0	62037.63
$p_{03}$	28.5	1226.8	335.6	0.2	-	-	0.00
$p_{04}$	26.3	924.6	280.9	-	-	-	0.00
$p_{05}$	26.7	1231.3	346.1	0.1	-	-	0.00
$p_{06}$	29.3	970.8	300.5	0.1	8.5	1.7	177.54
$p_{07}$	25.9	982.9	297.3	0.0	0.0	0.0	57720.62
$p_{08}$	27.4	956.2	300.2	0.0	0.0	0.0	61900.18
$p_{09}$	26.3	1023.8	296.4	0.0	-	-	0.00
$p_{10}$	29.6	1129.0	332.9	0.0	0.0	0.0	54570.15
$p_{11}$	29.4	1038.1	326.0	0.0	3.8	0.6	546.90
$p_{12}$	30.2	955.2	312.7	0.0	0.0	0.0	53910.67
$p_{13}$	26.9	923.4	285.1	0.0	0.0	0.0	58776.45
$p_{14}$	27.2	1046.8	311.0	0.0	8.3	1.1	294.33
$p_{15}$	29.9	1303.9	340.1	0.3	-	-	0.00
$p_{16}$	26.9	932.9	282.9	-	-	-	0.00
$p_{17}$	29.1	1305.9	348.2	0.1	-	-	0.00
$p_{18}$	28.3	975.7	304.6	0.1	8.6	1.8	168.33
$p_{19}$	31.6	1077.6	352.7	0.0	0.0	0.0	35099.05
$p_{20}$	28.5	1014.0	321.7	0.0	-	-	0.00
$p_{21}$	30.2	1007.1	328.9	0.0	0.0	0.0	40350.71
$p_{22}$	28.2	1115.0	343.5	0.0	0.0	0.0	42673.31
$p_{23}$	31.5	1239.3	363.4	0.0	0.0	0.0	43526.82
$p_{24}$	27.6	964.1	299.9	0.4	-	-	0.00

*Number of cases in which verds-QBF has advantages* The data are also presented in a way that the number of cases in which *verds*-QBF has advantage and that in which NuSMV has advantage are explicitly shown, where  $N$  is the number of test cases,  $T$  is the number of test cases in which the property is true,  $F$  is the number of test cases in which the property is false,  $adv$  is the number of cases in which *verbs*-QBF has an advantage.

$p_{01}$

b	adv/T	adv/F	adv/N
12	-	20/20	20/20
24	-	20/20	20/20
36	-	20/20	20/20
sum	-	60/60	60/60

$p_{02}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	60/60	-	60/60

$p_{03}$

b	adv/T	adv/F	adv/N
12	0/2	10/18	10/20
24	0/1	17/19	17/20
36	-	16/20	16/20
sum	0/3	43/57	43/60

$p_{04}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
24	0/20	-	0/20
36	0/20	-	0/20
sum	0/60	-	0/60

$p_{05}$

b	adv/T	adv/F	adv/N
12	9/16	0/4	9/20
24	18/18	1/2	19/20
36	19/19	0/1	19/20
sum	46/53	1/7	47/60

$p_{06}$

b	adv/T	adv/F	adv/N
12	13/20	-	13/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	53/60	-	53/60

$p_{07}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	60/60	-	60/60

$p_{08}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	60/60	-	60/60

$p_{09}$

b	adv/T	adv/F	adv/N
12	14/14	3/6	17/20
24	17/18	2/2	19/20
36	19/20	-	19/20
sum	50/52	5/8	55/60

$p_{10}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	60/60	-	60/60

$p_{11}$

b	adv/T	adv/F	adv/N
12	3/3	15/17	18/20
24	7/7	13/13	20/20
36	3/3	17/17	20/20
sum	13/13	45/47	58/60

$p_{12}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	60/60	-	60/60

$p_{13}$

b	adv/T	adv/F	adv/N
12	-	20/20	20/20
24	-	20/20	20/20
36	-	20/20	20/20
sum	-	60/60	60/60

$p_{14}$

b	adv/T	adv/F	adv/N
12	2/2	16/18	18/20
24	1/1	19/19	20/20
36	-	20/20	20/20
sum	3/3	55/57	58/60



$p_{15}$

b	adv/T	adv/F	adv/N
12	0/5	6/15	6/20
24	0/2	16/18	16/20
36	0/1	16/19	16/20
sum	0/8	38/52	38/60

$p_{16}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
24	0/20	-	0/20
36	0/20	-	0/20
sum	0/60	-	0/60

$p_{17}$

b	adv/T	adv/F	adv/N
12	9/19	0/1	9/20
24	18/18	1/2	19/20
36	19/19	0/1	19/20
sum	46/56	1/4	47/60

$p_{18}$

b	adv/T	adv/F	adv/N
12	13/20	-	13/20
24	20/20	-	20/20
36	20/20	-	20/20
sum	53/60	-	53/60

$p_{19}$

b	adv/T	adv/F	adv/N
12	4/4	15/16	19/20
24	1/1	19/19	20/20
36	-	20/20	20/20
sum	5/5	54/55	59/60

$p_{20}$

b	adv/T	adv/F	adv/N
12	11/11	7/9	18/20
24	4/6	12/14	16/20
36	1/4	11/16	12/20
sum	16/21	30/39	46/60

$p_{21}$

b	adv/T	adv/F	adv/N
12	2/2	17/18	19/20
24	1/1	19/19	20/20
36	-	20/20	20/20
sum	3/3	56/57	59/60

$p_{22}$

b	adv/T	adv/F	adv/N
12	2/2	17/18	19/20
24	1/1	19/19	20/20
36	-	20/20	20/20
sum	3/3	56/57	59/60

$p_{23}$

b	adv/T	adv/F	adv/N
12	-	20/20	20/20
24	-	20/20	20/20
36	-	20/20	20/20
sum	-	60/60	60/60

$p_{24}$

b	adv/T	adv/F	adv/N
12	14/15	3/5	17/20
24	7/10	5/10	12/20
36	3/6	3/14	6/20
sum	24/31	11/29	35/60

## B.2 Programs with Concurrent Sequential Processes

For the programs with concurrent sequential processes, we let  $b$  vary over the set of values  $\{12, 16, 20\}$ ,  $a = 2$ ,  $c = b/2$ ,  $e = c$ ,  $f = 4$ , and  $d = (b - c)/a = c/a$ . Each property is tested on 20 test cases for each  $b \in \{12, 16, 20\}$ . The details are shown as follows.

$p_{01}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.2	0.1	-	-	-	0.00
16	0.6	7.8	2.6	0.3	-	-	0.00
20	8.9	79.1	37.4	-	-	-	0.00

$p_{02}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.2	0.1	0.0	0.0	0.0	26.91
16	0.7	3.7	1.8	0.0	0.0	0.0	258.64
20	9.2	82.1	37.8	0.0	0.0	0.0	3356.16

$p_{03}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.8	0.4	1.4	-	-	0.00
16	1.2	14.0	6.0	13.6	-	-	0.00
20	20.3	1028.9	256.0	-	-	-	0.00

$p_{04}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.4	0.2	-	-	-	0.00
16	0.9	5.4	2.7	-	-	-	0.00
20	13.8	150.1	61.1	-	-	-	0.00

$p_{05}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.7	0.3	1.0	-	-	0.00
16	1.7	29.8	8.7	129.9	-	-	0.00
20	83.9	2429.2	534.5	-	-	-	0.00

$p_{06}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.4	0.2	1.0	-	-	0.00
16	1.0	7.9	3.6	59.5	-	-	0.00
20	19.4	215.4	88.5	-	-	-	0.00

$p_{07}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.3	0.1	0.0	0.0	0.0	25.39
16	0.7	3.7	1.8	0.0	0.0	0.0	261.09
20	9.2	83.2	37.9	0.0	0.0	0.0	3387.83

$p_{08}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.3	0.2	0.0	0.0	0.0	31.56
16	0.8	6.0	2.7	0.0	0.0	0.0	382.11
20	14.9	149.2	65.6	0.0	0.0	0.0	6070.82

$p_{09}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.4	0.2	0.0	-	-	0.00
16	0.7	5.2	2.8	0.0	-	-	0.00
20	9.1	796.5	99.4	0.0	-	-	0.00

$p_{10}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.6	0.2	0.0	74.0	3.8	0.06
16	1.1	9.5	4.1	0.0	40.5	2.3	1.76
20	25.6	414.4	122.7	0.0	22.6	1.3	92.22

$p_{11}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.6	0.2	0.0	-	-	0.00
16	0.8	7.4	2.7	0.0	-	-	0.00
20	11.1	192.0	62.8	0.0	-	-	0.00

$p_{12}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.3	0.2	0.0	-	-	0.00
16	0.9	8.1	3.4	0.0	-	-	0.00
20	21.9	239.0	102.7	0.0	-	-	0.00

$p_{13}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.3	0.1	0.0	0.0	0.0	27.35
16	0.7	5.7	2.4	0.0	0.0	0.0	332.91
20	14.3	143.3	64.2	0.0	0.0	0.0	5557.25

$p_{14}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.2	0.1	0.0	-	-	0.00
16	0.7	7.4	2.7	0.0	-	-	0.00
20	14.5	142.2	64.8	0.0	-	-	0.00

$p_{15}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.8	0.4	1.3	-	-	0.00
16	1.3	16.0	6.5	11.2	-	-	0.00
20	20.3	1007.9	292.9	-	-	-	0.00

$p_{16}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.4	0.2	-	-	-	0.00
16	1.0	5.5	2.7	-	-	-	0.00
20	14.0	151.4	61.3	-	-	-	0.00

$p_{17}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.7	0.3	1.1	-	-	0.00
16	1.8	27.4	9.0	140.8	-	-	0.00
20	82.6	2959.8	601.7	-	-	-	0.00

$p_{18}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.4	0.2	0.9	-	-	0.00
16	1.3	8.0	3.8	54.2	-	-	0.00
20	19.9	218.1	88.6	240.0	-	-	0.00

$p_{19}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.8	0.3	0.0	0.2	0.0	7.04
16	1.6	13.9	6.1	0.0	1.6	0.4	14.64
20	41.1	743.5	205.7	0.0	11.0	1.3	163.42

$p_{20}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.6	0.3	0.0	-	-	0.00
16	1.1	14.7	5.6	0.0	-	-	0.00
20	25.5	2615.4	376.0	0.0	-	-	0.00

$p_{21}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.5	0.2	0.0	0.2	0.0	6.01
16	1.1	10.8	4.5	0.0	2.8	0.4	11.00
20	29.9	351.3	137.5	0.0	11.1	1.1	121.29

$p_{22}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.5	0.2	0.0	0.5	0.1	4.05
16	1.0	9.8	3.9	0.0	1.8	0.3	14.39
20	25.6	285.0	101.5	0.0	7.5	0.7	138.13

$p_{23}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.6	0.2	0.0	0.1	0.0	15.48
16	1.3	13.6	5.7	0.0	0.3	0.0	141.37
20	37.5	635.0	190.8	0.0	120.8	6.2	31.00

$p_{24}$

b	min	max	aver	min	max	aver	ratio
12	0.1	0.4	0.2	0.0	-	-	0.00
16	0.9	6.6	3.4	0.0	-	-	0.00
20	16.6	239.9	87.8	0.0	-	-	0.00

A summary for  $b = 20$  is as follows.

m/p	min	max	aver	min	max	aver	ratio
$p_{01}$	8.9	79.1	37.4	-	-	-	0.00
$p_{02}$	9.2	82.1	37.8	0.0	0.0	0.0	3356.16
$p_{03}$	20.3	1028.9	256.0	-	-	-	0.00
$p_{04}$	13.8	150.1	61.1	-	-	-	0.00
$p_{05}$	83.9	2429.2	534.5	-	-	-	0.00
$p_{06}$	19.4	215.4	88.5	-	-	-	0.00
$p_{07}$	9.2	83.2	37.9	0.0	0.0	0.0	3387.83
$p_{08}$	14.9	149.2	65.6	0.0	0.0	0.0	6070.82
$p_{09}$	9.1	796.5	99.4	0.0	-	-	0.00
$p_{10}$	25.6	414.4	122.7	0.0	22.6	1.3	92.22
$p_{11}$	11.1	192.0	62.8	0.0	-	-	0.00
$p_{12}$	21.9	239.0	102.7	0.0	-	-	0.00
$p_{13}$	14.3	143.3	64.2	0.0	0.0	0.0	5557.25
$p_{14}$	14.5	142.2	64.8	0.0	-	-	0.00
$p_{15}$	20.3	1007.9	292.9	-	-	-	0.00
$p_{16}$	14.0	151.4	61.3	-	-	-	0.00
$p_{17}$	82.6	2959.8	601.7	-	-	-	0.00
$p_{18}$	19.9	218.1	88.6	240.0	-	-	0.00
$p_{19}$	41.1	743.5	205.7	0.0	11.0	1.3	163.42
$p_{20}$	25.5	2615.4	376.0	0.0	-	-	0.00
$p_{21}$	29.9	351.3	137.5	0.0	11.1	1.1	121.29
$p_{22}$	25.6	285.0	101.5	0.0	7.5	0.7	138.13
$p_{23}$	37.5	635.0	190.8	0.0	120.8	6.2	31.00
$p_{24}$	16.6	239.9	87.8	0.0	-	-	0.00

*Number of cases in which verds-QBF has advantages* The data are also presented in a way that the number of cases in which verds-QBF has advantage and that in which NuSMV has advantage are explicitly shown.

$p_{01}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
16	0/14	2/6	2/20
20	0/19	0/1	0/20
sum	0/53	2/7	2/60

$p_{02}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
16	20/20	-	20/20
20	20/20	-	20/20
sum	60/60	-	60/60

$p_{03}$

b	adv/T	adv/F	adv/N
12	0/6	0/14	0/20
16	0/3	1/17	1/20
20	0/1	0/19	0/20
sum	0/10	1/50	1/60

$p_{04}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
16	0/20	-	0/20
20	0/20	-	0/20
sum	0/60	-	0/60

$p_{05}$

b	adv/T	adv/F	adv/N
12	0/17	0/3	0/20
16	0/15	0/5	0/20
20	0/14	0/6	0/20
sum	0/46	0/14	0/60

$p_{06}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
16	0/20	-	0/20
20	0/20	-	0/20
sum	0/60	-	0/60

$p_{07}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
16	20/20	-	20/20
20	20/20	-	20/20
sum	60/60	-	60/60



$p_{08}$

b	adv/T	adv/F	adv/N
12	20/20	-	20/20
16	20/20	-	20/20
20	20/20	-	20/20
sum	60/60	-	60/60

$p_{09}$

b	adv/T	adv/F	adv/N
12	12/17	0/3	12/20
16	10/18	0/2	10/20
20	14/19	0/1	14/20
sum	36/54	0/6	36/60

$p_{10}$

b	adv/T	adv/F	adv/N
12	16/20	-	16/20
16	17/20	-	17/20
20	20/20	-	20/20
sum	53/60	-	53/60

$p_{11}$

b	adv/T	adv/F	adv/N
12	10/15	0/5	10/20
16	11/16	0/4	11/20
20	12/16	0/4	12/20
sum	33/47	0/13	33/60

$p_{12}$

b	adv/T	adv/F	adv/N
12	18/20	-	18/20
16	15/20	-	15/20
20	19/20	-	19/20
sum	52/60	-	52/60

$p_{13}$

b	adv/T	adv/F	adv/N
12	-	20/20	20/20
16	-	20/20	20/20
20	-	20/20	20/20
sum	-	60/60	60/60

$p_{14}$

b	adv/T	adv/F	adv/N
12	2/2	2/18	4/20
16	1/1	0/19	1/20
20	1/1	2/19	3/20
sum	4/4	4/56	8/60

$p_{15}$

b	adv/T	adv/F	adv/N
12	0/6	0/14	0/20
16	0/3	1/17	1/20
20	0/1	0/19	0/20
sum	0/10	1/50	1/60

$p_{16}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
16	0/20	-	0/20
20	0/20	-	0/20
sum	0/60	-	0/60

$p_{17}$

b	adv/T	adv/F	adv/N
12	0/19	0/1	0/20
16	0/15	0/5	0/20
20	0/13	0/6	0/19
sum	0/47	0/12	0/59

$p_{18}$

b	adv/T	adv/F	adv/N
12	0/20	-	0/20
16	0/20	-	0/20
20	0/20	-	0/20
sum	0/60	-	0/60

$p_{19}$

b	adv/T	adv/F	adv/N
12	4/4	16/16	20/20
16	2/2	18/18	20/20
20	2/2	18/18	20/20
sum	8/8	52/52	60/60

$p_{20}$

b	adv/T	adv/F	adv/N
12	6/7	7/13	13/20
16	5/6	8/14	13/20
20	2/5	12/15	14/20
sum	13/18	27/42	40/60

$p_{21}$

b	adv/T	adv/F	adv/N
12	2/2	17/18	19/20
16	1/1	18/19	19/20
20	1/1	19/19	20/20
sum	4/4	54/56	58/60

$p_{22}$

b	adv/T	adv/F	adv/N
12	2/2	17/18	19/20
16	1/1	19/19	20/20
20	1/1	19/19	20/20
sum	4/4	55/56	59/60

$p_{23}$

b	adv/T	adv/F	adv/N
12	-	20/20	20/20
16	-	20/20	20/20
20	-	19/20	19/20
sum	-	59/60	59/60

$p_{24}$

b	adv/T	adv/F	adv/N
12	5/8	6/12	11/20
16	2/3	11/17	13/20
20	1/5	12/15	13/20
sum	8/16	29/44	37/60

### B.3 Summary

The data are grouped into 12 groups, one for each type of properties where  $p_i$  and  $p_{i+12}$  are considered as of the same type. Since  $p_{05}$  and  $p_{06}$  are existential properties (and there is only one initial state in the models), we present and interpret the data according to the verification and falsification of  $\neg p_{05}$  and  $\neg p_{06}$ . The following is a summary of the experimental data.

properties	adv/T	adv/F	adv/N
$p_{01}, p_{13}$	0/53	182/187	182/240
$p_{02}, p_{14}$	127/127	59/113	186/240
$p_{03}, p_{15}$	0/31	83/209	83/240
$p_{04}, p_{16}$	0/240	0/0	0/240
$\neg p_{05}, \neg p_{17}$	2/37	92/202	94/239
$\neg p_{06}, \neg p_{18}$	0/0	106/240	106/240
$p_{07}, p_{19}$	133/133	106/107	239/240
$p_{08}, p_{20}$	149/159	57/81	206/240
$p_{09}, p_{21}$	93/113	115/127	208/240
$p_{20}, p_{22}$	120/127	111/113	231/240
$p_{21}, p_{23}$	46/60	164/180	210/240
$p_{21}, p_{24}$	144/167	40/73	184/240
sum	814/1247	1115/1632	1929/2880

The bounded semantics model checking does not have advantage in verifying any of the properties that start with  $AG$ . The bounded bounded semantics model checking has advantage in various degrees in verifying the other 8 types of properties (no data for verification are available for  $\neg p_{06}$ ) and in falsifying 11 types of properties (no data for falsification are available for  $p_{04}$ ). The details of the experimental data do not indicate how the sizes of problems may affect the advantage/disadvantage of bounded verification. Rather, the type of the verification problems matters.